NAME:-______________________________________

SUBJECT:_STRUCTURL ANALYSIS

BRANCH:-AEE/A.E

STRUCTURAL ANALYSIS (15 - 20 M) 69/_

B·Sai Kumar
A_3 - civil

UNIT - I

BASIC CONCEPTS AND STATIC INDETERMINACY

Defination of structure:-

When any elastic body is subjected to external loading system displacement will be developed and internal resistance will be setup to resist against the displacement. Th type of elastic body is known as structure.

Ex:-

Mechanism:-

If no resistance is setup against the displacement and moves has a rigid body. It is known as mechanism. forming the sufficient no. of internal hinges segment of the structure will rotate infinately about internal hinges.

Ex:-

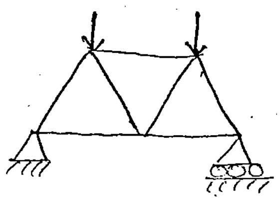
Classification of structure:-

structure
- Skeletal (1-D)
- Surface (2-D)
- Solid (3-D)

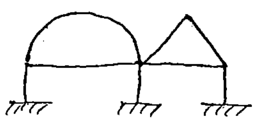
Skeletal structure:-

Structure which can be idealize into a series of c straight line and curved lines.

Ex:- Roof trusses, building frames



Roof struss.



Surface structure :-

Structure which can be idealise into plane (or) curved surfaces

Ex:- slabs, shells, and folded plates (thin economical elements)

Solid structure:-

Structure which can be neither a skeleton nor surface structure.

Ex:- Massive foundation, huge foundations, Machine foundations (Gas turbines, stream turbines etc).

Skeletal structure

- pin jointed structure
  - plane pin jointed
  - space pin jointed
- Rigid jointed structure
  - plane rigid Jointed
  - space rigid jointed.

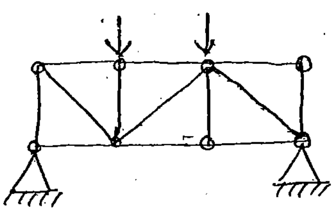
pin jointed structure:-

Members will be connected by pin (or) hinge joint such a way that all members will undergo axial forces only (compression (or) tension) when the load act at joints. Members are assumed to be straight. Members will not undergo any moment. Members will support by external force by developing the axial forces.

Plane pin jointed structure:-

plane pin jointed structures design force (or) axial force only.

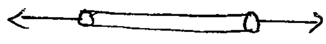
Ex:-



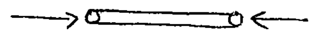
Space pin jointed structures:-

Design force (or) axial force only (same as plan pin jointed structure).

Ex:- space truss



Tension



compression.

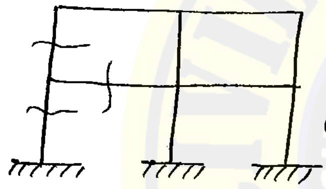
Rigid joint structures:-

Joint of a rigid jointed structures are assumed to be rigid and the angle between the members meeting at th joint remains un-change.

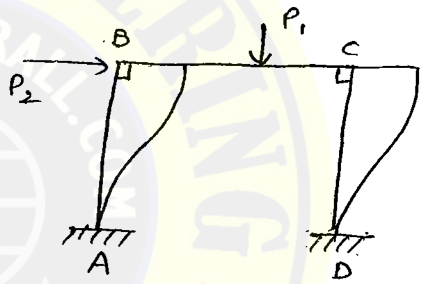
Rigid jointed structures will support the external loads by developing axial force, shear force, B·M and the twisting moment in a member.

plane rigid jointed structures:-

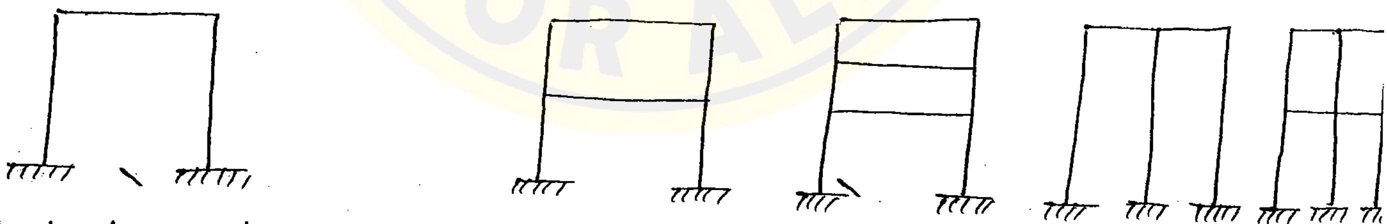
At a rigid joint same rotation and different displacement develops.



(Multi storey)



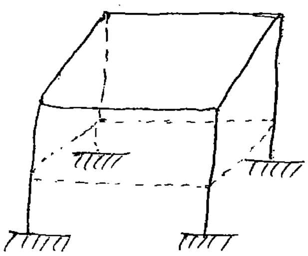
If a plane of loading and plane of a structure are same, design forces are axial force, shear force and BM

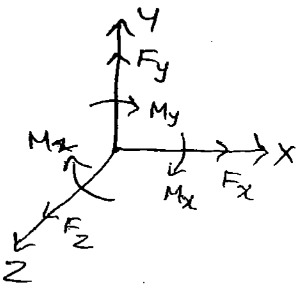


Single base single storey (SBSS) (SBDS) (SBTS) (DBSS) (DBD

Space rigid joint structure:-

In this plane of loading and plane of structure can be different.





$A \cdot F = F_z$

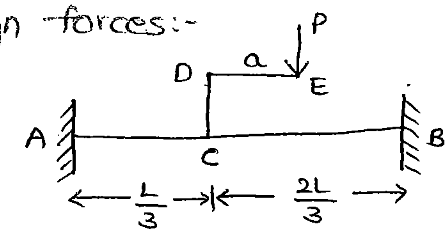
$S \cdot F = F_x, F$

$B \cdot M = M_x, M$

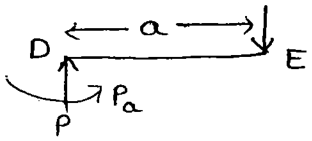
Twisting moment $= M_z$

Design forces:-

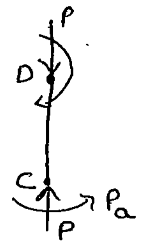
EX:-



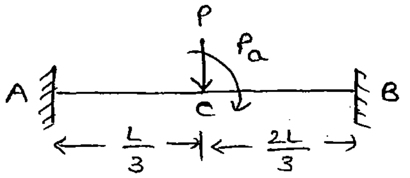
Free body diagram:



DE = S.F & B.M

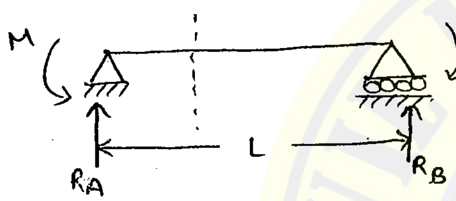


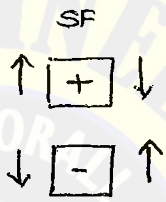
DC = Axial force and B.M



AB = S.F & B.M.

EX:-





$\Sigma V = 0$

$R_A + R_B = 0$

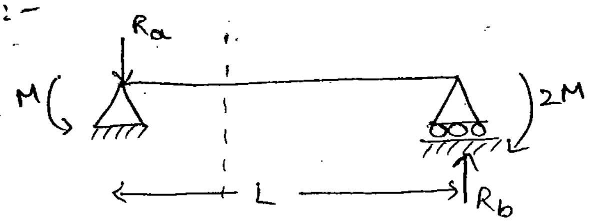
$\Sigma M_A = 0$

$-M + M - R_B \times L = 0$

$\therefore R_A = 0$

$\therefore R_B = 0$

EX:-



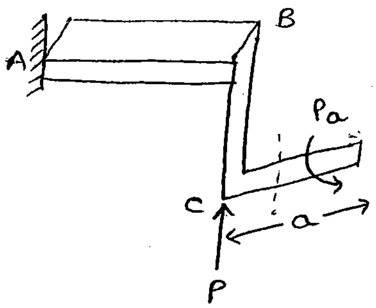
$R_A + R_B = 0$

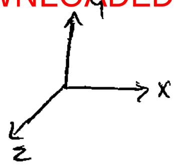
$\Sigma M_A = 0$

$-M + 2M - R_B \times L = 0$

$R_B = \frac{M}{L}$, $R_A = \frac{-M}{L}$

EX:-



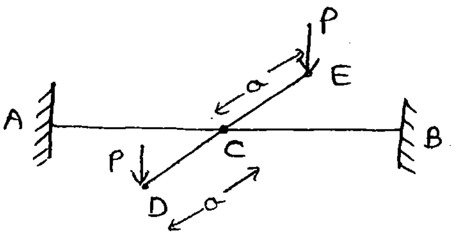


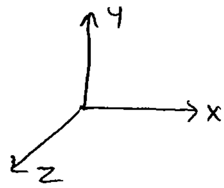
on DC = S.F & BM

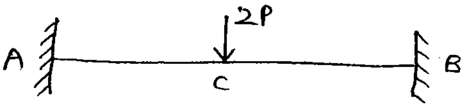
on CB = Aaxial force & BM

on BA = S.F, B.M, Twisti mome

EX:-

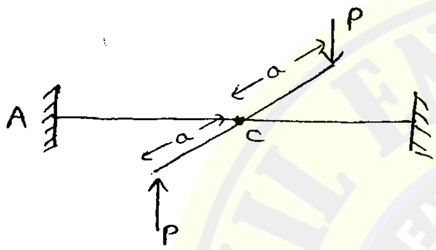


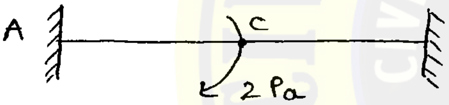


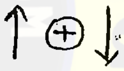


SF & B.M

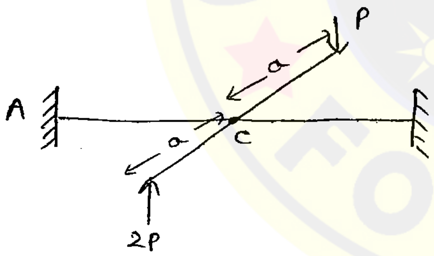
EX:-



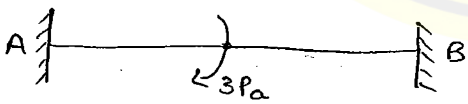




EX:-



S.F, B.M and T.M



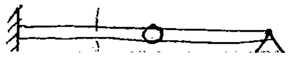
(P+2P)
=3Pa

Static indeterminacy:-

Statically determinant structure:-

It is the total no. of the reactive forces (support reactions are ≤ static equilibrium equation it is called a statically determinate structure).

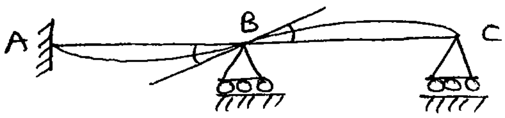
Ex:- cantilever beam, simply supported beam, overhang beam, three hinged arches, suspension cables and compound beam



Statically indeterminate structure :-

If total no. of unknown support reactions are greater than the equilibrium equation, it is called a statically indeterminate structure. Additional equations relative to comparetability conditions can be use to calculate the excess unknowns.

Ex:-



$\theta_{BA} = \theta_{BC}$

Fixed beam, continuous beam, propped beam, Rigid jointed frames, Redundant trusses.

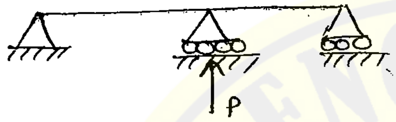
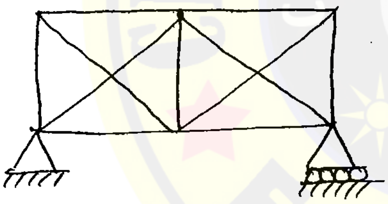


Fig: ~~P=~~ propped beam (Temporary additional support to reduce deflection and B.M).

Examples of Redundant :-

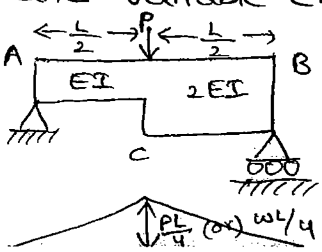


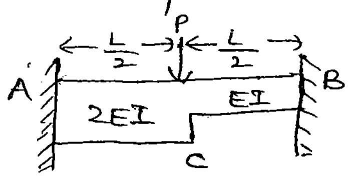
$M > 2J - 3$ (Redundant truss)

$M < 2J - 3$ (Deficient truss)

Difference between statically determinate and indeterminate structure :

Statically Determinate	Statically Indeterminate.
1. Equilibrium equations are sufficient to analyse	1. Insufficient additional comparability equation are used to analyse for excess unknowns.
2. B.M and S.F in a member are independent of flexural rigidity of the member i.e., type of a member and variable cross section	2. It is dependent

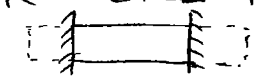




3. No stresses will be develop due to temparature changes and lack of fit.

3. Stresses due to temperature changes and lack of fit will be caused.

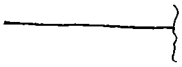
Ex:- $|\leftarrow L+\Delta L \rightarrow|$



$\Delta L \propto T$

$$\text{strain} = \frac{\Delta L}{L}$$

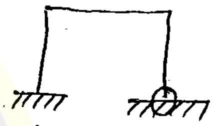
Support reactions(R):-

1. Free support  $R = 0$

2. Roller support  $R = 1$ (N (or) V) (Normal (or) vertical)

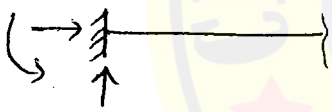
3. Hinge support (or) pin support :

$R = 2$ (V, H)

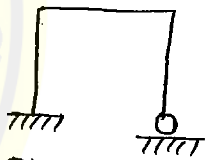


Fix ~~Rolle~~ Hinge

4. Fixed support

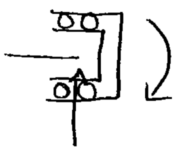


$R = 3$ (H, V, Moment)

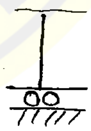


Fix Roller

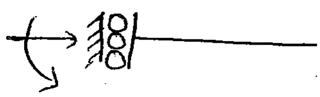
5. Horizontal shear hinge



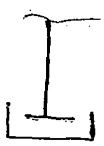
$R = 2$ (V, M)



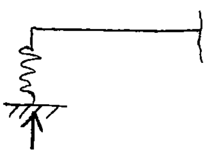
6. Verticle shear image



$R = 2$ (H, M)

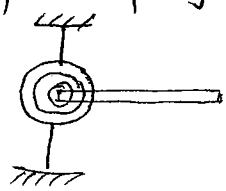


7. Elastic spring



$R = 1$ (V)

8. Flat spiral spring



$R = 2$ (V, M)
for verticle loading

$R = 3$ (V, M, T)
for general loading

EX:-

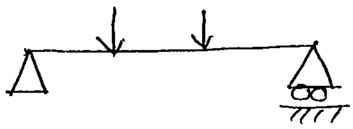


Fig:- verticle loading

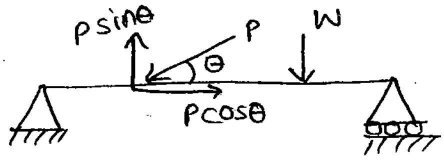


Fig:- General loading

Note:-

1. When beam is subjected to vertical loading, horizontal support should be neglect because beams are predominate to take verticle forces (Transverse force)

Equilibrium equations (r):-

1. Structure as a whole

(a) For plane structure, $r=3$ ($\Sigma X=0$, $\Sigma Y=0$, $\Sigma M=0$)

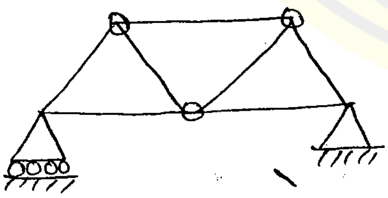
(b) For spatial structure, $r=6$ ($\Sigma X=0$, $\Sigma Y=0$, $\Sigma Z=0$, $\Sigma M_x=0$, $\Sigma M_y=0$, $\Sigma M_z=0$)

2. For a joints

(a) Each pin joint of a pin jointed plane frame (trusses)

$r=2$ ($\Sigma X=0$, $\Sigma Y=0$)

EX:-



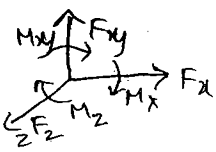
(b) For every joint of a pin jointed space frame, $r=3$ ($\Sigma X=0$, $\Sigma Y=0$, $\Sigma Z=0$)

(c) For every rigid joint of a rigid jointed frame

$r=3$ ($\Sigma X=0$, $\Sigma Y=0$, $\Sigma M=0$)

(d) For every joint of a every rigid joint space frame

$r=6$ ($\Sigma X=0$, $\Sigma Y=0$, $\Sigma Z=0$, $\Sigma M_x=0$, $\Sigma M_y=0$, $\Sigma M_z=0$)

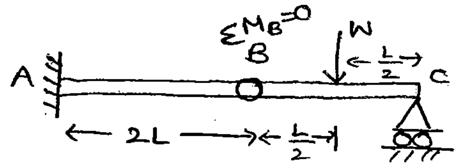


Note:-

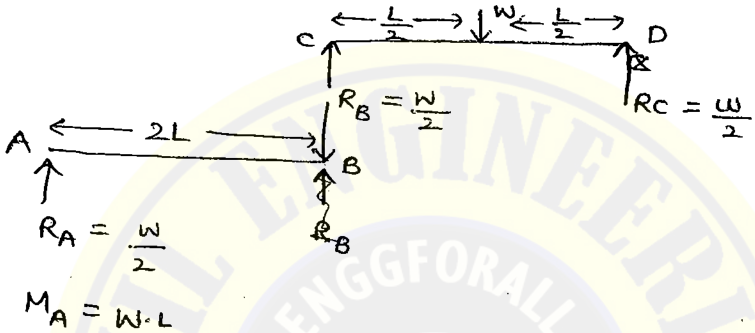
For a beam subjected to verticle loading no. of equilibrium eq ($r=2$) [$\Sigma Y=0$, $\Sigma M=0$]

Releases or Bonus equation :-

1. Internal hinge or pin or moment hinge :-



It is a mechanical device provided any where in the beam to make any one of the design force is zero. At the position of internal hinge and additional equilibrium equation $\Sigma M=0$ is available. It is capable of transparing the unbalance verticle forces for one part to the other part of a hinge.

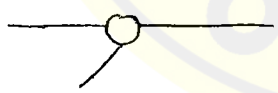


No. of releases = 1

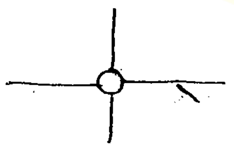
Note:-

1. If 'n' members are meeting at a internal hinge no. of releases equal to $(n-1)$

Ex:-

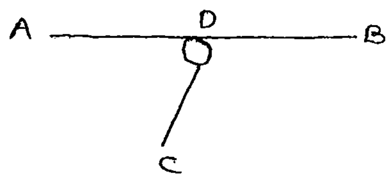


No. of releases = 2

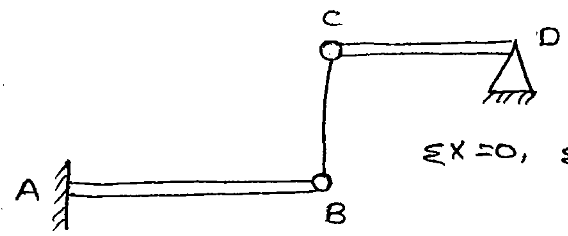


No. of releases = 3

2. If a internal hinge is provided tangentiall to the member, ACB must be treated as single member only.
Hence no. of realeases = 1



2.



$\Sigma X = 0, \ \Sigma M = 0$

Link is a straight bar with pinned ends. It is incapable of transfering the horizontal forces and moments from one side to the other side of a link. Therefore two additional equations $\Sigma X = 0$ and $\Sigma M = 0$ are available at a position of link.

If link is provided in a beam subjected to general loading no. of releases = 2 ($\Sigma X = 0$, $\Sigma M = 0$)

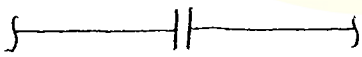
If link is provided in a beam subjected to verticle loading no. of releases = 1 ($\Sigma M = 0$)

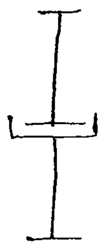
3. Horizontal shear release:-

Beams  No. of releases = 1 ($\Sigma X = 0$)

Frames 

4. verticle shear release:-

Beams 

Frames  No. of releases = 1 ($\Sigma Y = 0$)

Degree of static indeterminacy:-

$$D_s = D_{se} + D_{si} - \text{Releases}$$

D_{se} = external indeterminacy = $(R - r)$

D_{se} = It is with respectiue support reactions

D_{si} = Internal Indeterminacy (It is related to configuration of members)

$D_{si} = 0$, for beams

$D_{si} = m - (2j - 3)$ for pin jointed plane frames

$= m - (3j - 6)$ for pin jointed space frames

$= 3c$, for rigid jointed plane frames

$= 6c$, for rigid jointed space frames

m = no. of members

j = no. of joints

c = no. of cuts required to open the configuration of structure (no. of closed figures)

Ex:-

Degree of static equilibrium of a fixed beam.

$D_{se} = 4 - 2 = 2$

$D_{si} = 0$

$D_s = D_{se} + D_{si}$ − Releases

$= 2 + 0 - 0$

$\boxed{D_s = 2}$

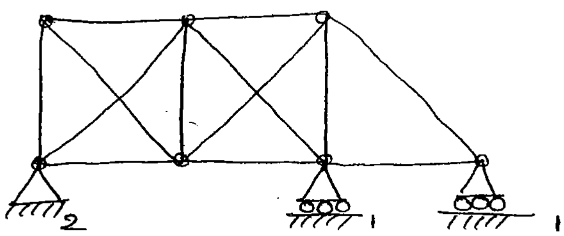
Ex:-

$D_{se} = 6 - 3 = 3$

$D_{si} \geq 0$

$\boxed{D_s = 3}$

Ex:-



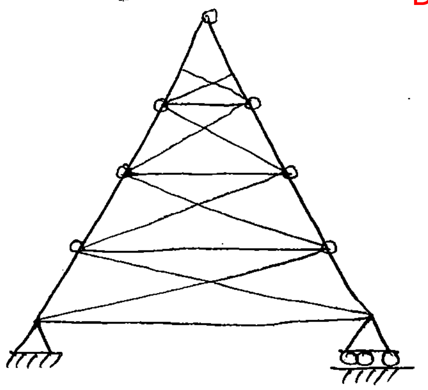
$D_{se} = 4 - 3 = 1$

$D_{si} = 13 - (2 \times 7 - 3) = 2$

$D_s = 1 + 2 = 3$

($\Sigma X, \Sigma Y, \Sigma M$ ...) equilibrium conditions

EX:-



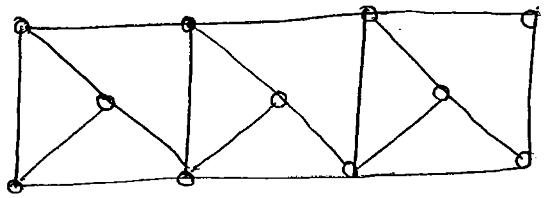
$D_{se} = 3 - 3 = 0$

$D_{si} = m - (2j - 3)$

$= 20 - (2\times 9 - 3)$

$= 5$

EX:-



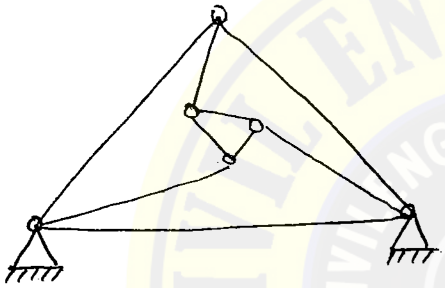
$D_{se} = 3 - 3 = 0$

$D_{si} = 19 - (2\times 11 - 3)$

$= 0$

$D_s = 0$

EX:-)



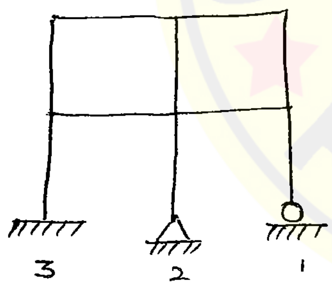
$D_{se} = 4 - 3 = \cancel{2}\ 1$

$D_{si} = 9 - (2\times 6 - 3)$

$= 0$

$D_s = 1$

EX:-)



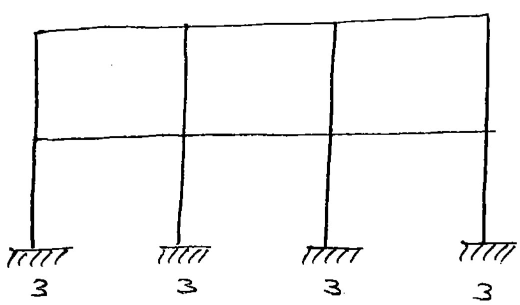
$D_{se} = 6 - 3 = 3$

$D_{si} = 3c$

$= 3\times 2 = 6$

$D_s = 3 + 6 = 9$

EX:- What is the degree of static indeterminacy of 3 bay 2 storey frame with fixed ends.



$D_{se} = 12 - 3 = 9$

$D_{si} = 3c$

$= 3\times (3) = 9$

$D_s = 9 + 9 = 18$

Note :-

1. If $D_s = 0$ then it is a statically determinate
2. If $D_s > 0$ then it is a statically indeterminate
3. If $D_{se} = 0$, then it is a externally determinate
4. If $D_{se} > 0$, then it is a externally indeterminate
5. If $D_{si} = 0$, then it is a Internally determinate
6. If $D_{si} > 0$, then it is a Internally indeterminate

Simplified formula for frames (without considering the releases)

1. $D_s = (M+R) - 2j$ for pin jointed plane frames
2. $D_s = (m+R) - 3j$ for pin jointed space frames
3. $D_s = (3m+R) - 3j$ for rigid jointed plane frames
4. $D_s = (6m+R) - 6j$ for rigid jointed space frames

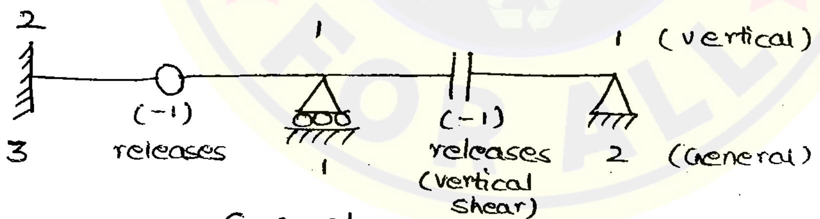
pin jointed plane frame:

$$D_s = D_{se} + D_{si}$$

$$= (R-3) + [m-(2j-3)]$$

$$D_s = (m+R) - 2j$$

Ex:-



General

$D_{se} = 6-3 = 3$

Releases $= -2$

$\overline{\quad 1 \quad}$

$\therefore D_s = 1$

vertical loading

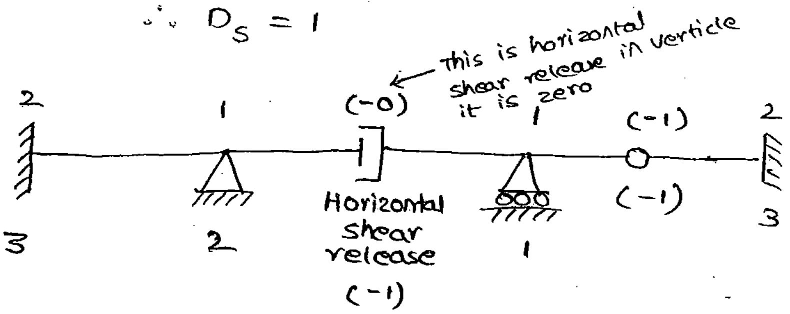
$D_{se} = 4-2 = 2$

Releases $= -2$

$\overline{\quad 0 \quad}$

$\therefore D_s = 0$

Ex:-



General loading

$D_{se} = 9-3 = 6$

Releases $= -2$

$D_s = 4$

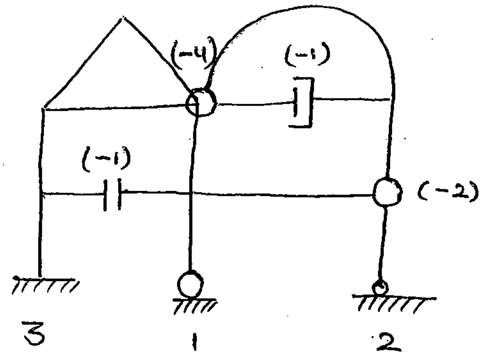
verticle loading

$D_{se} = 6-2 = 4$

Releases $= -1$

$D_s = 3$

EX:-



$D_{se} = 6-3 = 3$

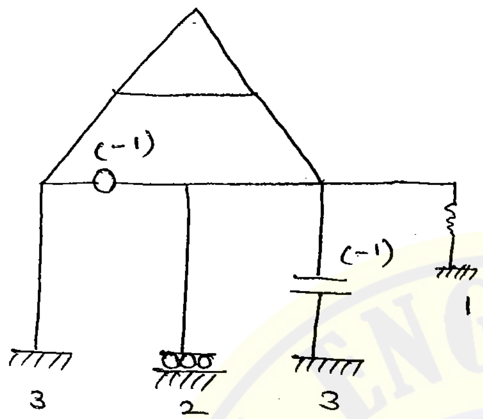
$D_{si} = 3\times4 = 12$

Releases $= -8$

7

$\boxed{\therefore D_s = 7}$

EX:-



$D_{se} = 9-3 = 6$

$D_{si} = 3\times2 = 6$

Releases $= -2$

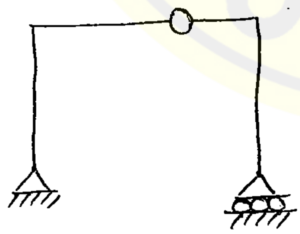
$D_s = 10$

Stability of structures:-

If a structure do not have a sufficient external constraint and internal constraints it will have a rigid body motion. when external load is applied.

General principles for coplanar unstable structures:-

1. If no. of support reactions are less than the equilibrium conditions a structure can be unstable.



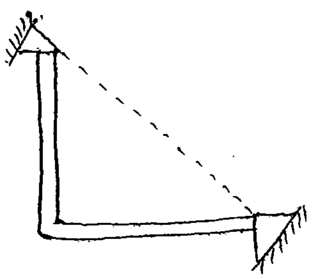
$R = 3$

$r = 4$

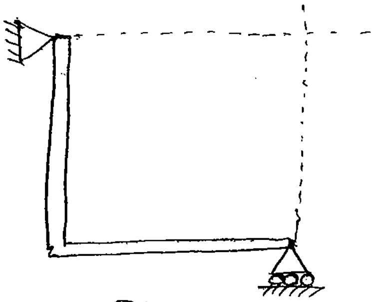
$\therefore R < r$

The structure can be unstable.

2. A structure can be unstable if line of actions of support reactions are concurrent. Even if no. of support reactions are greater than the equilibrium equations.



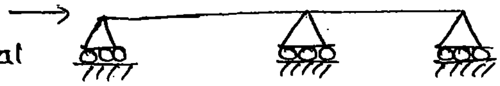
Unstable



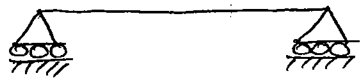
Stable structure

3. If support reactions are parallel to each other. A structure will have rigid body translation when a horizontal force is applied. such structure are called as Geometrically un-stable.

Applied Horizontal force

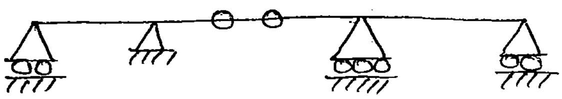


Unstable.



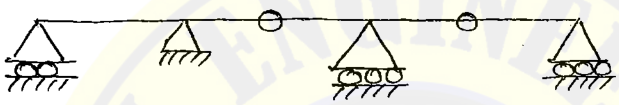
unstable

4. A structure may be locally unstable if more no. of releases placed in one member only of a structure.

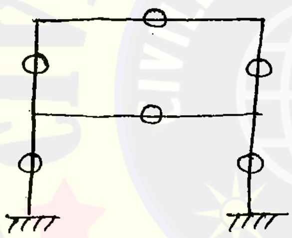


$D_s = 2$

Unstable



Stable

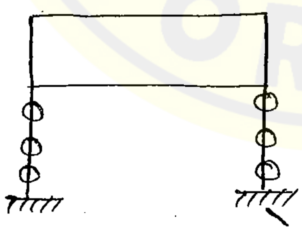


$D_{se} = 6-3 = 3$

$D_{si} = 3 \times 1 = 3$

$D_s = \underline{6}$

Stable



P.9 No:- 7

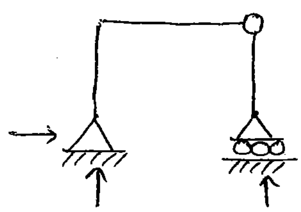
8. $D_{se} = 6-3 = 3$

$D_{si} = 0$

Releases $= 0$

$\overline{\quad 3 \quad}$

9.



$R = 3$

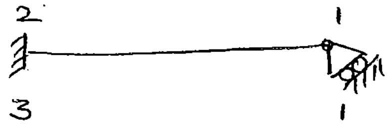
$r = 3+1 = 4$

$R < r$

Unstable.

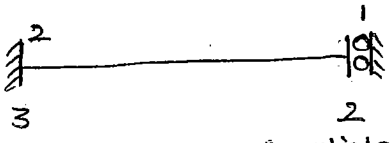
10.

	General	Verticle

(i) 

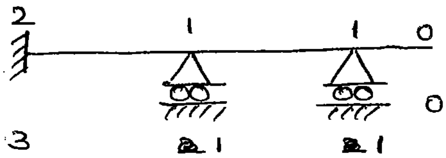
$D_{se} = 4-3=1$ $\quad$ $D_{se} = 3-2=1$

$D_s = 1$ $\quad$ $D_s = 1$

(ii) 

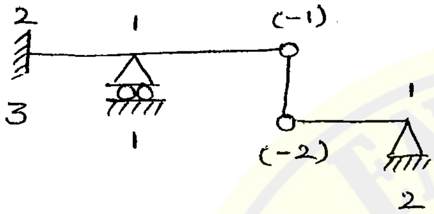
$D_{se} = 5-3=2$ $\quad$ $D_{se} = 3-2=1$

$D_s = 2$ $\quad$ $D_s = 1$

(iii) 

$D_{se} = 5-3=2$ $\quad$ $D_{se} = 4-2=2$

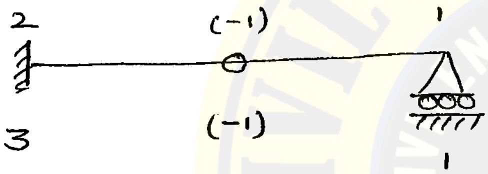
$D_s = 2$ $\quad$ $D_s = 2$

(iv) 

$D_{se} = 6-3=3$ $\quad$ $D_{se} = 4-2=2$

Releases $= -2$ $\quad$ Releases $= -1$

$\overline{\quad 1 \quad}$ $\quad$ $\overline{\quad 1 \quad}$

(v) 

$D_{se} = 4-3=1$ $\quad$ $D_{se} = 3-2=1$

Release $= -1$ $\quad$ Releases $= -1$

$\overline{\quad 0 \quad}$ $\quad$ $\overline{\quad 0 \quad}$

~~18~~. P.9 NO:-9

2. $\cancel{D_{se} = 6-3 = 3}$ $\quad$ $D_{se} = 4-3=1$

$\cancel{D_{si} = 3 \quad = 3}$ $\quad$ $D_{si} = m-(2j-3)$

$= 10-(2\times5-3)$

$= 3$

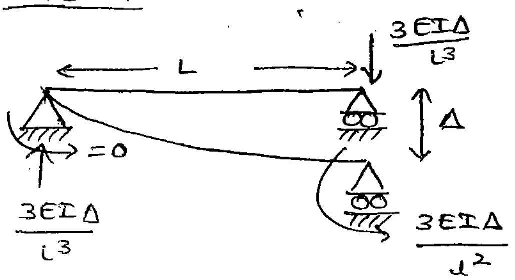
$D_s = \underline{\overline{1+3=4}}$

3. $D_{se} = 4-3=1$

$D_{si} = 12-(2\times7-3)$

$= 1$

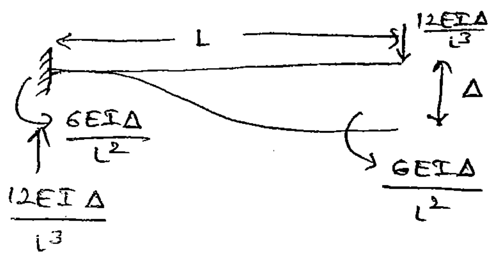
$D_s = 1+1=2$

5. 

$R = \frac{M}{L}$

$= \frac{3EI\Delta}{L^2} \times \frac{1}{L}$

$= \frac{3EI\Delta}{L^3}$



UNIT - 2

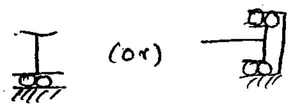
KINEMATIC INDETERMINACY

It deals with a motion of a body. It is also calle as Degree of freedom. Degree of kinematic indeterminacy (D_K is equal to the total number of independent joint displacem

1. A structure can be kinematically indeterminate if the no. o unknown displacements in a structure are greater than the no of compatibility equations.
2. Additional equations related to equilibrium conditions can be used to completely analyse a kinematic indeterminate struct

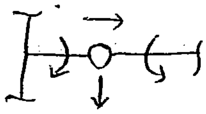
Degree of freedom for supports :-

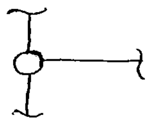
	Type of support	D_K
1.	Fixed support	0
2.	Hinged support	1 (θ)
3.	Roller support	2 ($\delta x, \theta$) (translation, rotation)
4.	Free support	3 ($\delta x, \delta y, \theta$)
5.	Horizontal shear hinge	1 (δx)
6.	Verticle shear hinge	1 (δy)



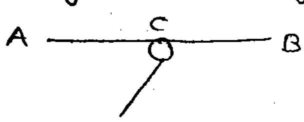
Degree of freedom for typical joints :-

1. Rigid joint  3 ($\delta x, \delta y, \theta$)

2.  4 (2θ's, δx, δy)

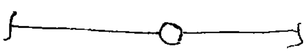
3.  5 (3θ's, δx, δy)

4. Internal hinge is tangential to the member



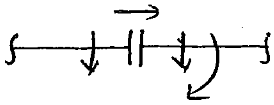
4 (2θ's, δx, δy)

5. Internal hinge in beams



4 (2θ's, δx, δy)

6. Horizontal shear release  4 $(2\delta x, \delta y, \theta)$

7. Verticle shear release  4 $(2\delta y, \delta x, \theta)$

Degree of freedom for frames :-

1. For each joint of a pin jointed plane frames, $D_K = 2$ $(\delta x, \delta y)$

2. For pin jointed space frames D.O.F for each joint = 3 $(\delta x, \delta y, \delta z)$

3. For each joint of a rigid jointed plane frames D.O.F = 3 $(\delta x, \delta y, \theta)$

4. For each joint of a rigid jointed space frame D.O.F = 6 $(\delta x, \delta y, \delta z, \theta_x, \theta_y, \theta_z)$

Note :-

1. At a roller support of a pin jointed plane frames D.O.F = 1 (δx)

2. At a position of the hinge support in a pin jointed structures D.O.F = zero (since no rotation is allowed)

Simplified formula for the D.O.F of frames :-

$$D_K = NJ - C$$

where

N = No. of D.O.F of a joint

N = 2 for pin joint of a pin jointed plane frames

N = 3 for pin joint of a pin jointed space frames

N = 3 for rigid joint of a rigid jointed plane frames

N = 6 for rigid joint of a rigid jointed space frames

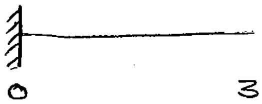
J = No. of joints

C = No. of reaction components

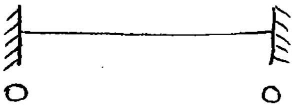
$C = R$ (considering the axial deformations} i.e., members are extensible)

$C = M + R$ (neglecting axial deformations} ie, members are inextensible or axially rigid)

EX:-

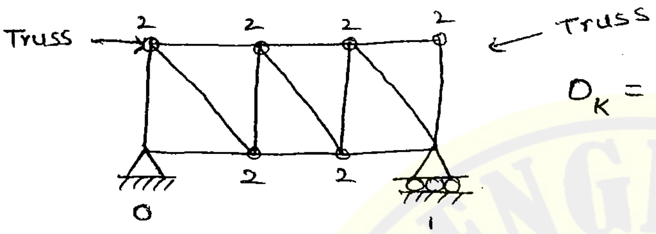


$D_K = 3$ $\quad D_S = 0$

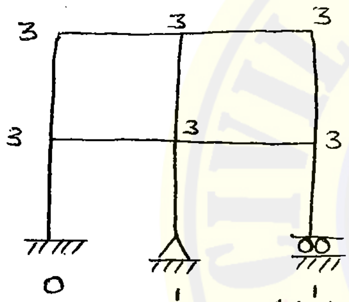


$D_K = 0$ $\quad D_S = 4-2 = 2$

$D_K = 6$ $\quad D_S = 6-3 = 3$



$D_K = 13$

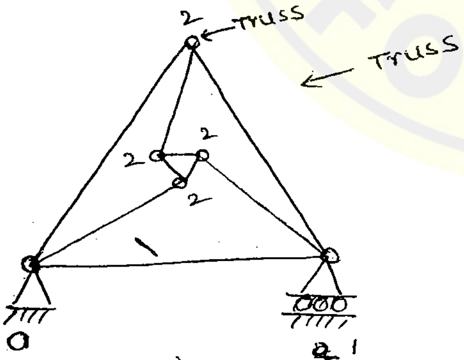


1) $D_K = 20$ (By considering axial deformation

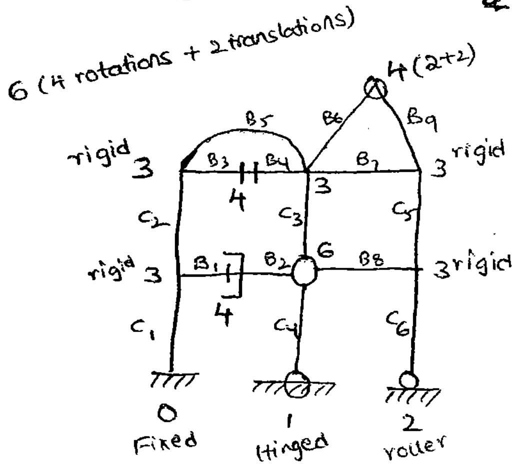
2) $D_K = 20 - \underset{\to \text{beams}}{4} = 16$ (Beams are axially rigid

3) $D_K = 20 - \underset{\to \text{coloums}}{6} = 14$ (columns are axially rigid

(Horizontal shear) 4) $D_K = 20 - 10 = 10$ (structure is axially rigid)



$D_K = 9$



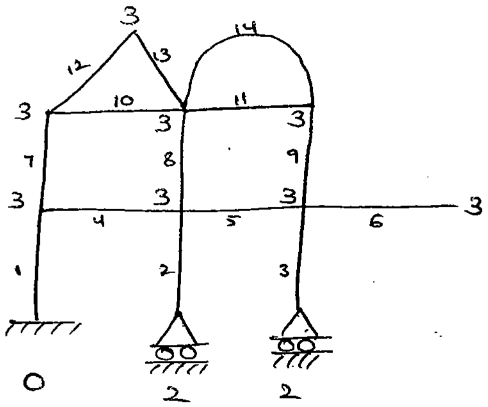
1) $D_K = 36$

2) $D_K = 36 - 9 = 27$ (Beams are axially rigid)

3) $D_K = 36 - 6 = 30$ (columns are axially rigid)

4) $D_K = 36 - 15 = 21$ (structure is axially rigid)

2.

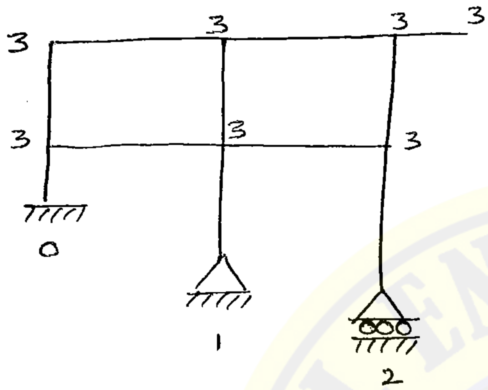


$D_K = 28$ (considering)

$D_K = 28 - 14$ (neglecting)

$= 14$

3.



Degree of static indeterminacy

$D_{se} = 6 - 3 = 3$

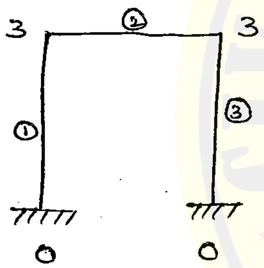
$D_{si} = 3C = 3(2) = 6$

$D_s = \overline{\underline{9}}$

Degree of kinematic indeterminacy

$D_K = 24 - 11 = 13$

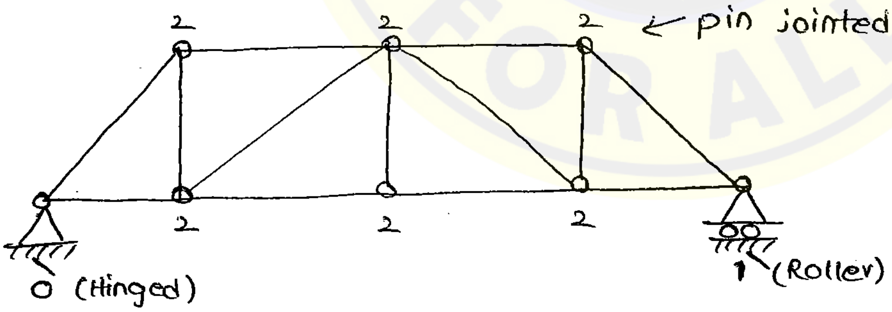
4.



$D_K = 6$ (considering)

$D_K = 6 - 3 = 3$ (neglecting)

5.

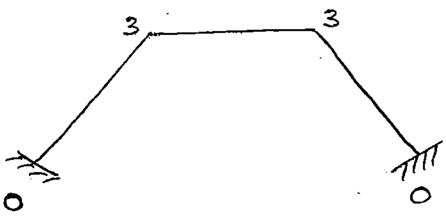


← pin jointed truss

$D_K = 13$

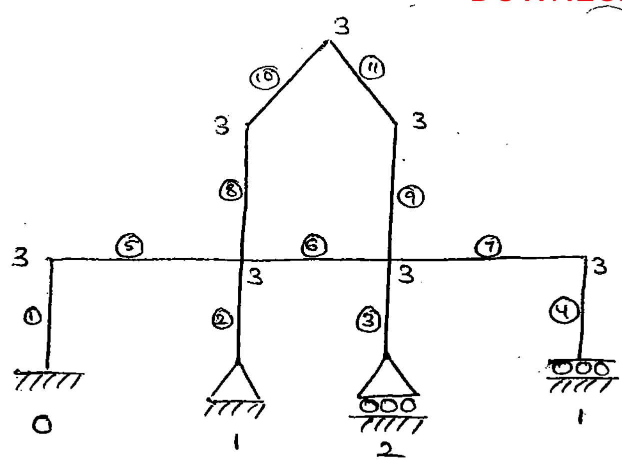
Classwork :-

1.



$D_K = 6 - 3 = 3$ (neglecting)

3.

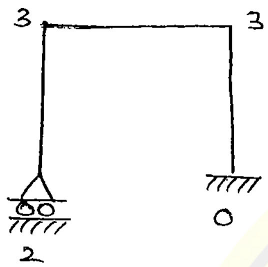


$D_K = 25 - 11$

$= 14$

P.g NO:- 16

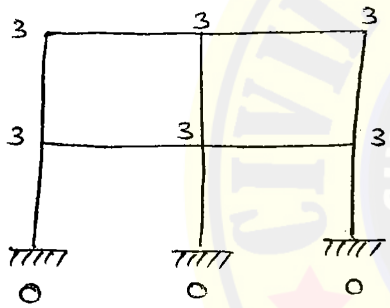
1.



$D_{se} = D_s = 4 - 3 = 1$

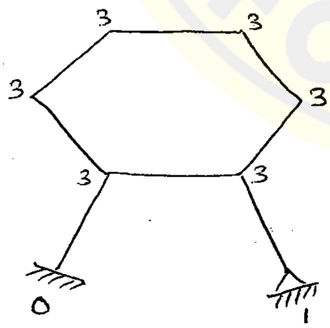
$D_K = 8 - 3 = 5$

2.



$D_K = 18$

Ex:-



$D_K = 19$

UNIT - III

Analysis of Determinate trusses:-

Need:-

1. When the span of the structure is large, heavy loads are coming on to the structure and depth of the structure is also more beam cannot be adviced to resist.
2. Steel trusses in which members can be straight bars, channels, can be used for the above purpose.

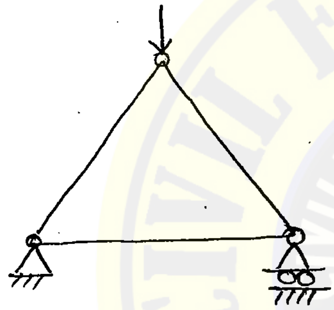
3. Trusses will be subjected either axial tension or compression.

4. Members of the truss will not be subjected to moment.

Pin jointed Frame (Trusses)

- Perfect frame ($m = 2j - 3$)
- Imperfect frame ($m \neq 2j - 3$)
  - Redundant frame ($m > 2j - 3$)
  - Deficiet frame ($m < 2j - 3$)

Ex:- perfect frame



$m = 3$

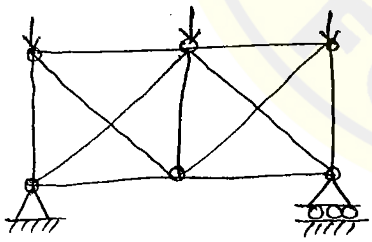
$2j - 3 = 2(3) - 3$

$= 3$

$\therefore m = 2j - 3$

$\therefore$ perfect frame

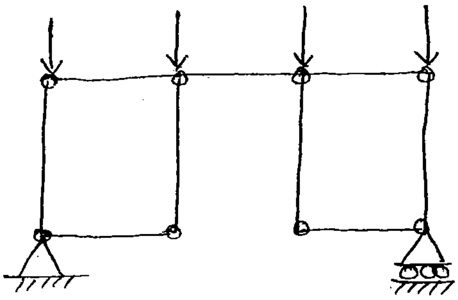
Ex:- Imperfect frame



$m = 11$

$2j - 3 = 2(6) - 3 = 09$

$\therefore m > 2j - 3$ (Redunant frame)



$m = 11$

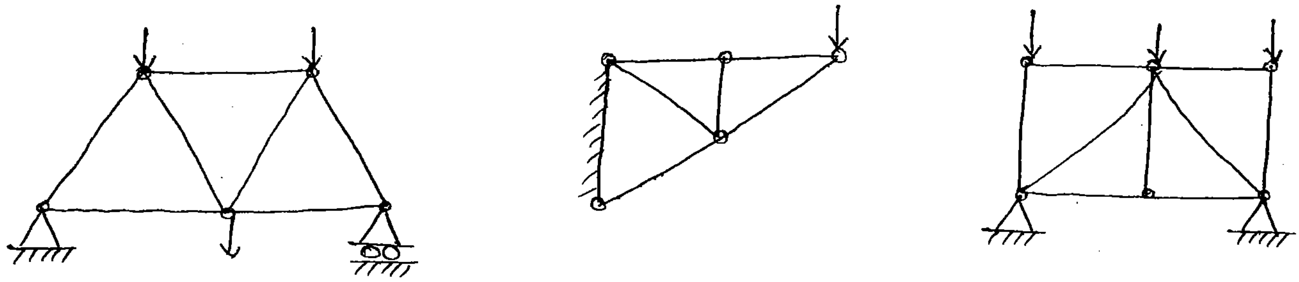
$2j - 3 = 2(8) - 3 = 13$

$\therefore m < 2j - 3$ (Deficiet truss)

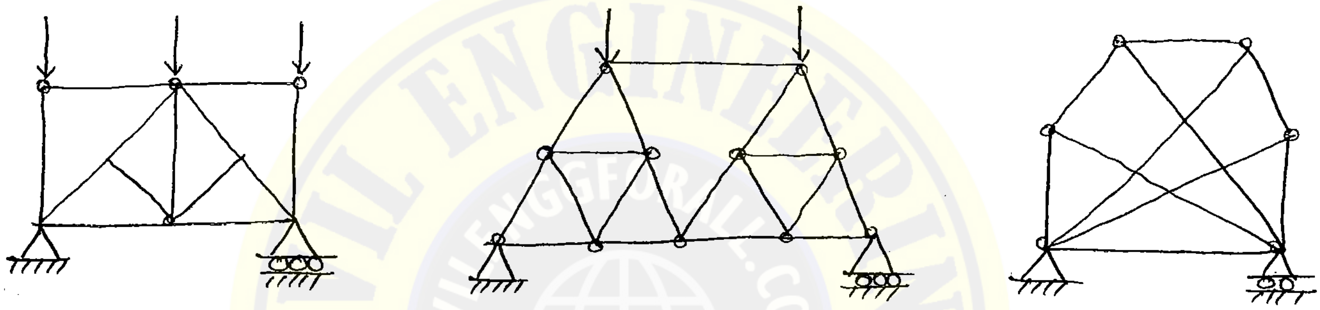
Trusses

- Simple truss
- complex truss

Examples of simple truss:-

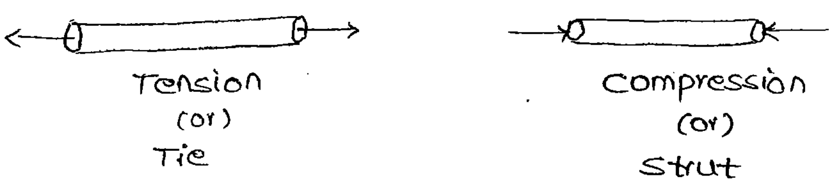


Examples of complex truss:-

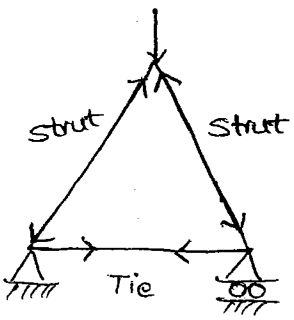


Assumptions made in the analysis of trusses:-

1. Hinge (or) pin joint at which members are connected are assumed to be frictionless ($B \cdot M = 0$)
2. Members of the truss are straight, not curved.
3. External load should act only at the joints.
4. Members of a truss will be subjected to either axial compression or tension.
5. self weight of the truss is ignored.
6.



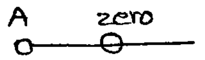
Ex:-

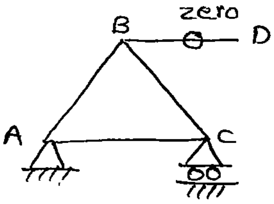


Rules for determining the members carry zero force :-

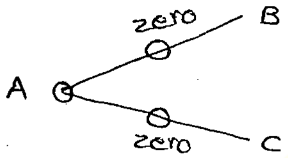
1. since a single member cannot form a joint the force in that member is always zero.

Ex:-

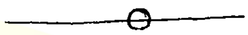




2. If two members are meeting at a joint this two members are non collinear the force in both the members are zero if there is no external force act at that joint.

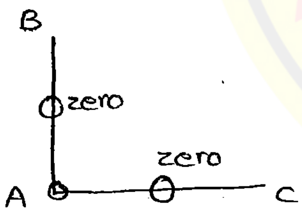


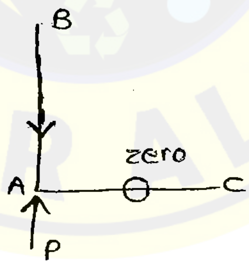
Non collinear



Members are in same line are called collinear.

If there is acting a external load on point 'A' then no force at point B and C is zero (0).



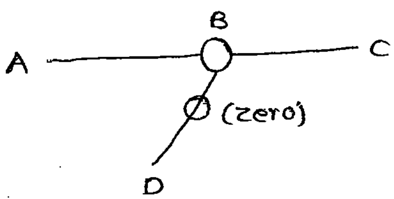


$F_{AB} = -P$

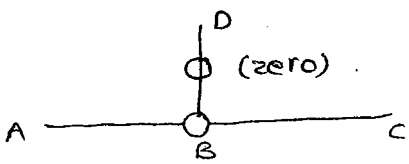
~~$F_{BA} = -P$~~

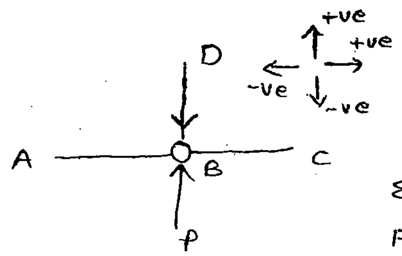
$F_{AC} = 0$

3. If three members are meeting at a joint two of them are collinear the force in third member only zero(0) if there is no external force act at that joint.



AB and BC are collinear and BD is not collinear, the force in BD = 0 (zero)





$F_{BD} = -P$

$F_{BA} = F_{BC}$

$\Sigma x = 0$ at B

$F_{BC} - F_{BA} = 0$

Methods for analysis :-

1. Method of joints
2. Method of sections (method of members)
3. Graphical method
4. Tension coefficient method

1. Method of joints :-

It is applicable if there are two unknown forces in the members of a truss at a joint.

procedure :-

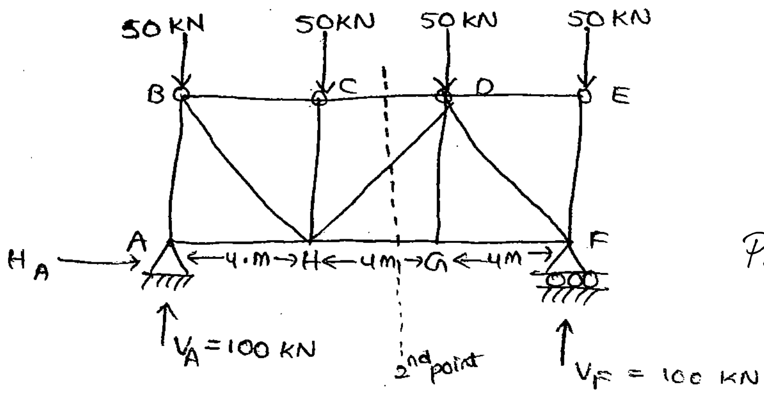
1. Calculate the support reactions due to applied forces by using $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$ (these three are for entire structure) (If necessary)
2. Select a joint at which there are two unknown forces in the members and draw the free body diagram of that joint.
3. By assuming each member carry a tensile force, keep the joint in equilibrium by applying $\Sigma X = 0$, $\Sigma Y = 0$.
4. Continue the step-2 and 3 to find out the forces in all the members of a truss.

Note:-

If there are 'n' no. of joints in a truss (n-1) joint are sufficient to keep it in equilibrium to calculate the forces in all the members of a truss.

2. Method of sections (Method of members):-

1. Calculate the support reaction of a truss due to applied forces. if necessary.



2. Pass a section through a choosen member and also through other two members such a way that the other two members should meet at a common joint. To make the moment due to other two members to be zero.

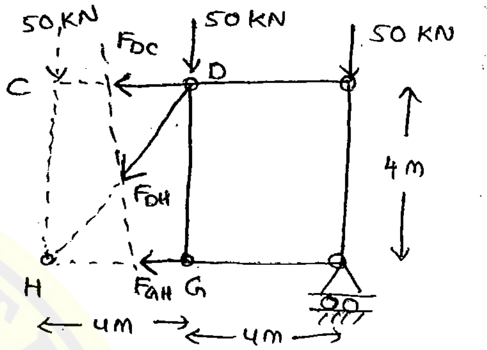
3. $\Sigma M = 0$ concept. It can be used to find out the forces in the horizontal members.

4. Dont cut more than three members at a time. All three cut members should not meet at a same joint.

5. $\Sigma M_H = 0$

$$-100 \times 8 + 50 \times 8 + 50 \times 4 - F_{DC} \times 4 = 0$$

$$F_{DC} = -50 \text{ kN}$$



Sub method: $\Sigma H = 0$ concept

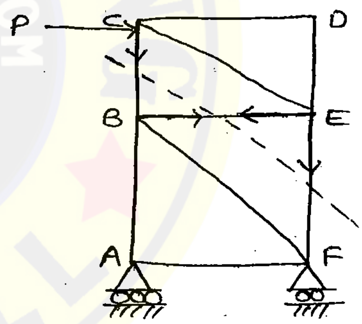
6. Pass a section through a choosen member (Horizontal) and other two members which do not have a horizontal component.

<u>Upper section:</u>

$\Sigma H = 0$

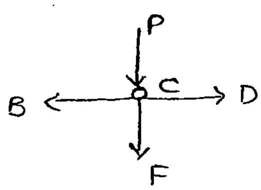
$+P - F_{EB} = 0$

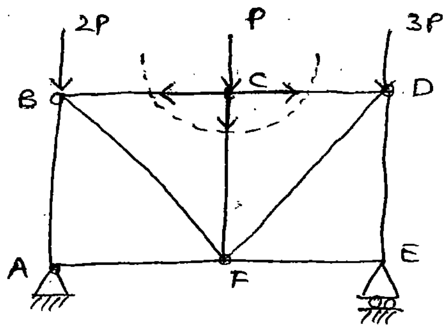
$F_{EB} = F_{BE} = P$



$\Sigma V = 0$ concept

7. pass a section through verticle member which is a choosen members and also other two member which do not have a verticle component.





$\Sigma y = 0$

$-P - F_{CF} = 0$

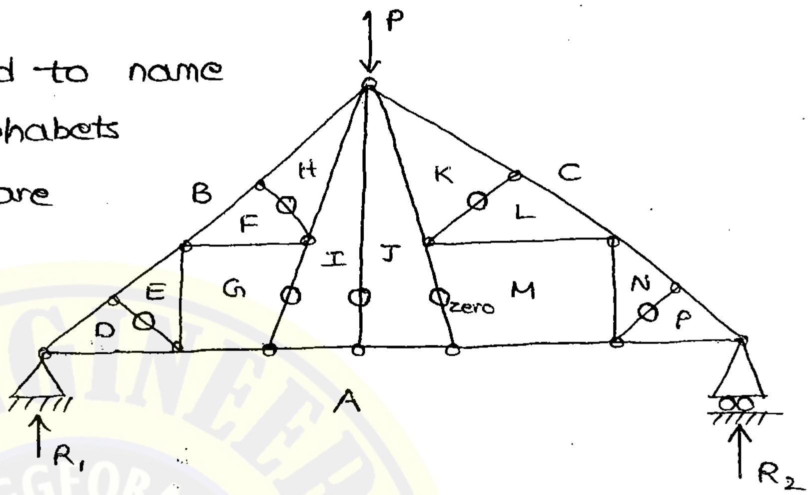
$\boxed{F_{CP} = -P}$

$\Sigma x = 0$

$\boxed{F_{CB} = F_{CD}}$

Graphical method:-

1. It is used to calculate the forces in the members of a complicated trusses graphically.
2. Mohr's graphical method is popular in calculating the forces in the members if more than four meet at a joint.
3. Fink truss or French roof truss can easily be analysed by graphical method.
4. Bow's notation is used to name a member by two alphabets
5. Total no. of members are carrying a zero force i.e., 7 members.



Tension coefficient method:-

1. Tension coefficient is defined as Force per unit length

$$t = \frac{T}{L}$$

T = Tension force

L = Length of a member.

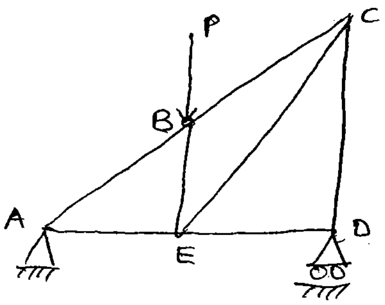
Units : N/mm

2. It is most suitable for analysis of spatial trusses

Note:-

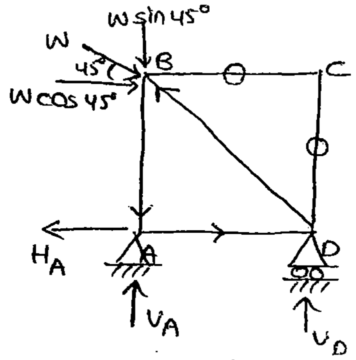
1. Method of sections is suitable for calculating the forces in the limited number of internal members.
2. Method of sections is useful to find out the forces in three unknown members at a time by cutting the section through these members.
3. The force in the member DE of a truss shown in fig. is '−P'

$$F_{BE} = -P$$



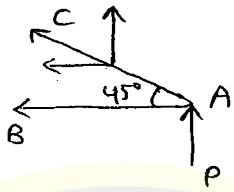
4. Force in the member 'BD' for the frame shown in fig. is '−W'

$$F_{BD} = -W$$



5. Force in a member 'AB' of a frame shown in the fig is '−P'

$$\boxed{F_{AB} = \bar{*}\,P}$$



$\Sigma x = 0$

$- F_{AB} - F_{AC}\cos 45^\circ = 0$

$F_{AB} = \frac{-F_{AC}}{\sqrt{2}}$

$\Sigma y = 0$

$P + F_{AC}\sin 45^\circ = 0$

$F_{AC} = \frac{-P}{\sqrt{}} \quad -P\sqrt{2}$

$\therefore\ F_{AB} = \frac{P\sqrt{2}}{\sqrt{2}} = P\ (\leftarrow)$

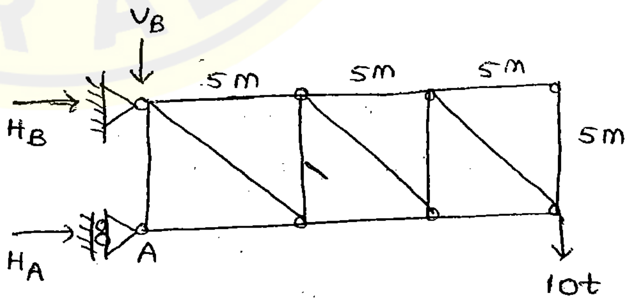
$\therefore\ F_{AB} = -P$

P.9 NO:- 21.

3. $\Sigma M_B = 0$

$-H_A \times 5 + 10 \times 15 = 0$

$H_A = 30t$



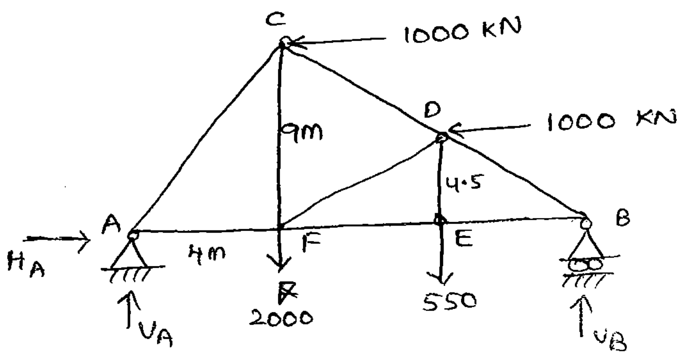
4. $\Sigma H = 0$

$H_A = 1000 + 1000$

$= 2000$ KN

$\Sigma V = 0$

$V_A + V_B = 2550$ KN



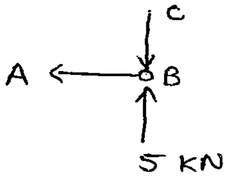
$\Sigma M_B = 0$

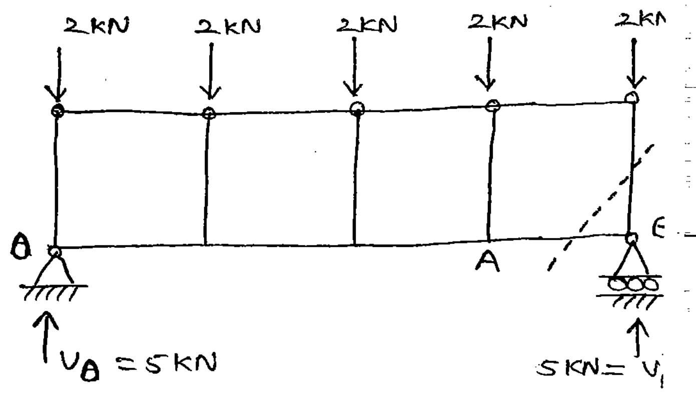
$16 V_A - 12(2000) - 6(550) - 9(1000) - 4.5(1000) = 0$

$16 V_A = 24000 + 3300 + 9000 + 4500$

$V_A = 2550$

5.

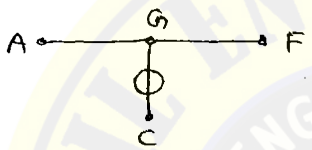


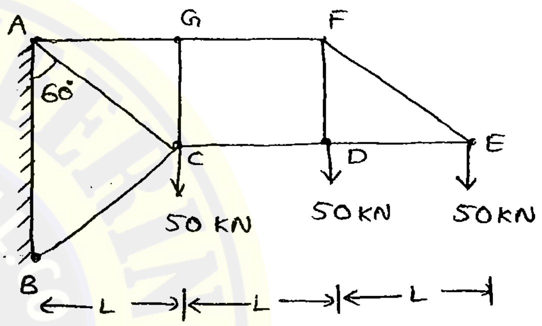


$F_{BC} = -5$ KN

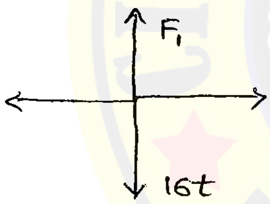
$F_{BA} = 0$

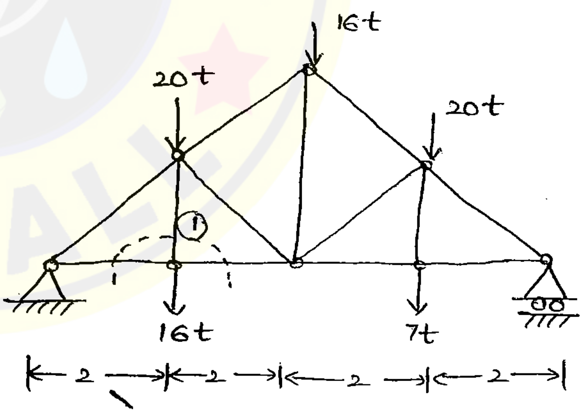
7. $F_{GC} = 0$





8.





$\Sigma y = 0$

$F_1 - 16 = 0$

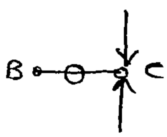
$F_1 = 16t$ (Tensile

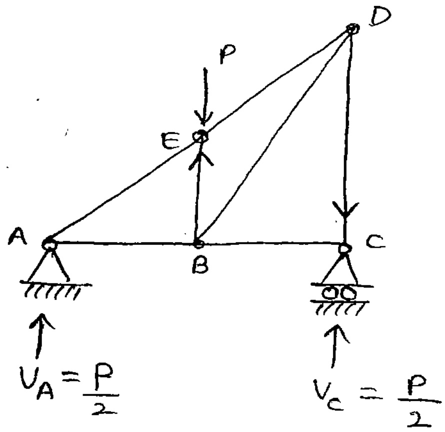
9. $F_{BE} = -P$

$F_{CD} = \frac{-P}{2}$

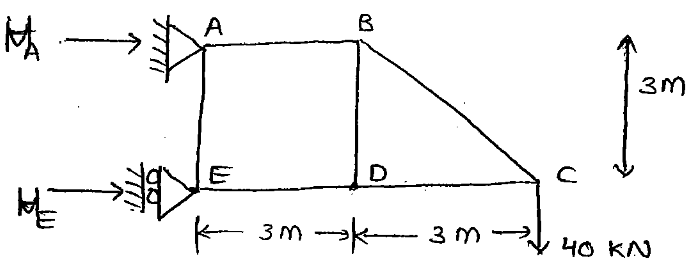
$F_{CB} = 0$

$\therefore\ P, \frac{P}{2}, 0$





10. $\Sigma M_A = 0$



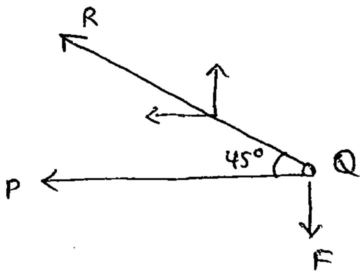
$-3H_E + 40 \times 6 = 0$

$H_E = 80$ KN ~~(compressive)~~

$F_{ED} = -80$ KN (compressive)

P.g NO:- 22

1. $\Sigma X = 0$



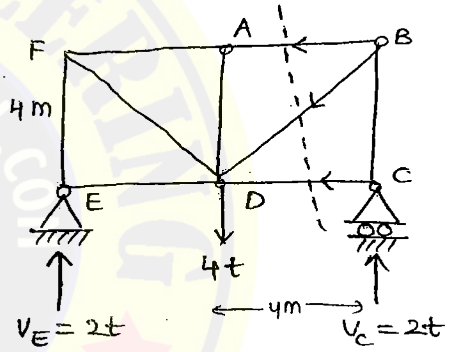
$-F_{QP} - F_{QR} \cos 45 = 0$

$\Sigma y = 0$

$F_{QR} \sin 45^\circ - F = 0$

$F_{QP} = F_{PQ} = -F$ (compressive)

2. <u>R.H.S</u>



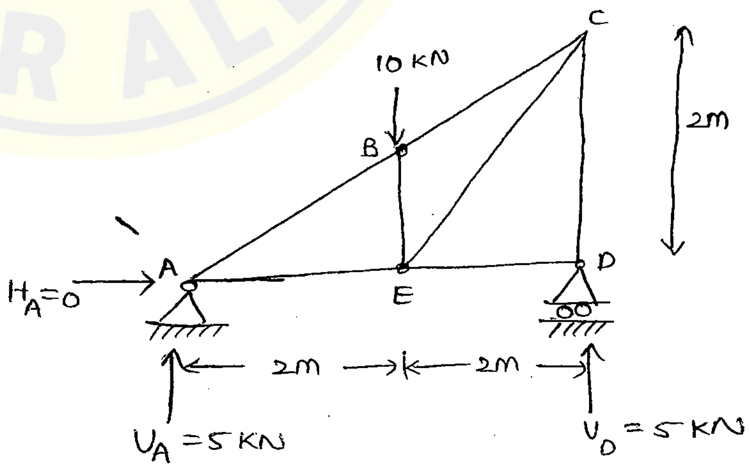
$\Sigma M_D = 0$

$-F_{BA} \times 4 - 2 \times 4 = 0$

$4 F_{BA} = -8t$

$F_{BA} = -2t$ (compressive)

3. $F_{BE} = -10$ KN

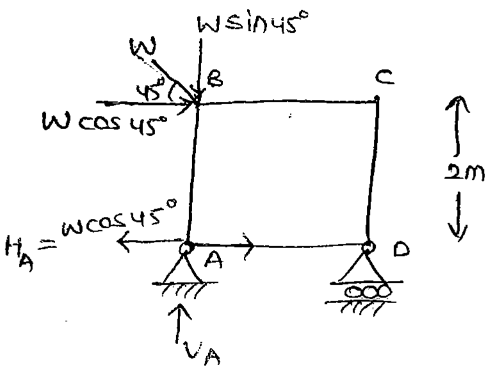


$F_{CD} = -5$ KN

$F_{ED} = 0$

$\therefore$ 10 KN, 5 KN, zero

5. $\Sigma M_D = 0$



$V_A \times 2 + W\cos 45^\circ \times 2 - W \sin 45^\circ \times 2 = 0$

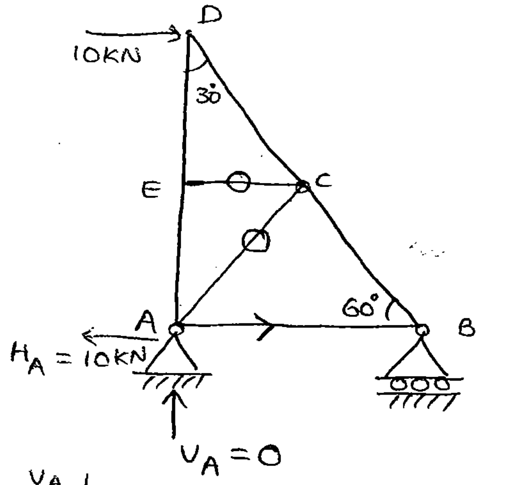
$\boxed{V_A = 0} \Rightarrow F_{AB} = \text{zero}$

$F_{AD} = W \cos 45^\circ = \frac{W}{\sqrt{2}}$

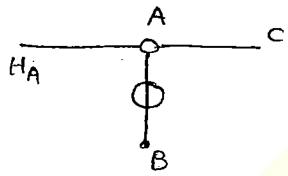
6. $F_{EC} = 0$

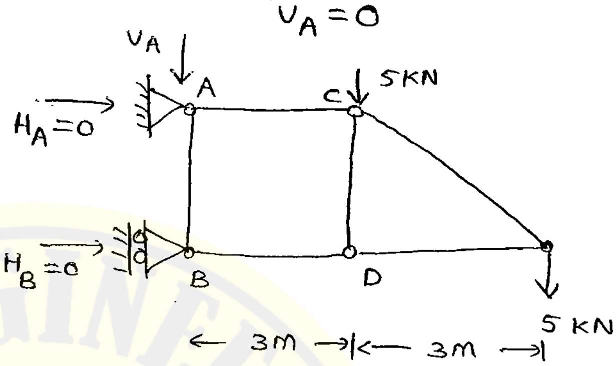
$F_{AC} = 0$

$F_{AB} = 10\ kN$



7. $F_{AB} = 0$



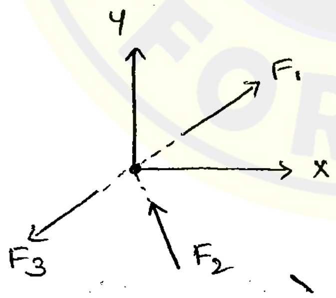


Four types of coplanar force system:-

1. Collinear force system

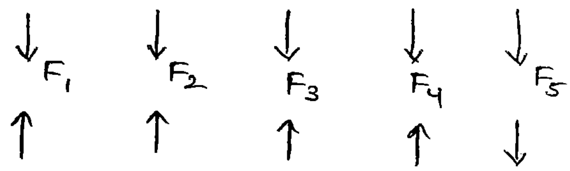
All forces should lie in same line of action.

2. Concurrent force system:-



All force should meet and start at a same point

3. parallel force system.



Line parallel

Unlike parallel.

4. Non concurrent Non parallel force system

UNIT - 4

BASIC METHODS OF STRUCTURAL ANALYSIS

Two methods are available in structural analysis to analyse the indeterminate structures.

1. Compatability method (or) Flexibility method (or) Force method

Compatability method:-

1. In this method redundant forces are unknowns.
2. Additional equations are obtained by considering the geometrical conditions imposed on the formations of the structure.
3. Flexibility is an amount of displacement caused due to unit force.

$$\therefore F = \frac{\Delta}{P}$$

$$\Delta = [F] \cdot \{P\}$$

P_1 P_2 → Redundant forces

4. The number of equations in flexibility method equal to the degree of static indeterminacy. since the redundants are support reactions.
5. These method is used for analysis of static indeterminate structure with lesser degree of static indeterminacy.

$$\therefore D_S < D_K$$

6. Various methods grouped under this category are
   a. Consistent deformation method.
   b. Clayperon's theorem of three moments
   c. Column analogy method
   d. Elastic centre method
   e. Maxwell - Mohr's equation
   f. Castigliano's theorem of minimum strain energy.

Equilibrium method (or) displacement (or) stiffness method:-

1. In this method displacement of the joints are taken as unknowns.
2. Equilibrium equations are expressed in terms of moments, rotations to get the actual joint displacements.

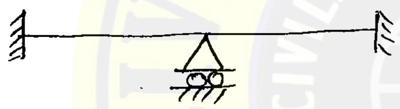
3. stiffness is an amount of force required to cause unit displacement

$$\therefore \quad K = \frac{P}{\Delta}$$

$$\{P\} = \{K\} \cdot \{\Delta\}.$$

4. The product of stiffness and flexibility is unity.
5. The number of equations in stiffness method equals to degree of freedom (D_K) as displacements are taken as unknowns.
6. These method is used for analysis of statically indeterminate structures if $D_K < D_S$.
7. Various methods grouped under this category are
   a. Moment distribution method
   b. slope deflection method
   c. Kani's method.

Ex:-1



Stiffness method

Ex:-2

Flexibility method

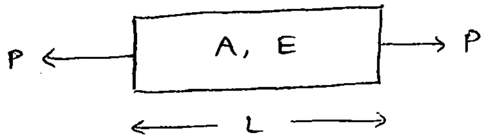
UNIT - 5

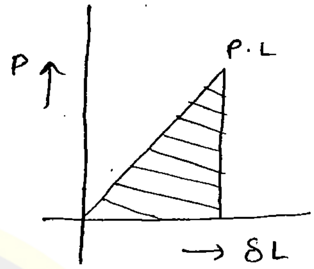
ENERGY PRINCIPLES

Energy principles are extensively used for determining the displacements in structures.

Strain Energy :-

It is an amount of workdone by an internal resistance against the deformation.





Resistance Vs deformation

$$\delta L = \frac{PL}{AE}$$

$$W = \frac{1}{2} \cdot P \cdot \delta L$$

$$U = \frac{1}{2} \cdot P \cdot \delta L$$

$$U = \frac{1}{2} P \cdot \delta L$$

$$= \frac{1}{2} P \left(\frac{PL}{AE}\right)$$

$$U = \frac{P^2 L}{2AE}$$

$$\sigma = \frac{P}{A}$$

$$P = \sigma \cdot A$$

$$U = \frac{1}{2} \frac{(\sigma A)^2 L}{AE}$$

$$= \frac{\sigma^2}{2E} \times AL$$

$$= \frac{\sigma^2}{2E} \times \text{volume}$$

$$= \frac{1}{2} \sigma \times \frac{\sigma}{E} \times \text{volume}$$

$$\boxed{U = \frac{1}{2} \sigma \times e \times \text{volume}}$$

Proof Resilience:-

It is a strain energy stored by an elastic body within the proportional limit. i.e., $\frac{\sigma^2}{2E} \times \text{volume}$.

Modulus of Resilience:-

It is a proof Resilience per unit volume

$$\therefore \frac{\sigma^2}{2E}$$

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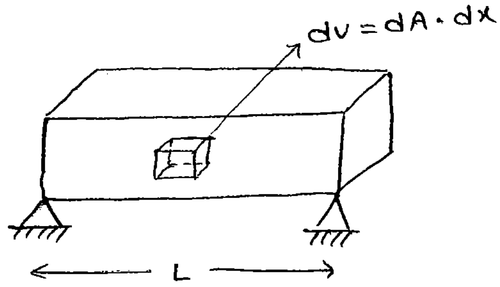
Strain energy due to Bending moment :-

$$\frac{M}{I} = \frac{\sigma}{y}$$



$$\delta v = \frac{\sigma^2}{2E} \cdot dA \cdot dx$$

$$U = \int_0^l \frac{\sigma^2}{2E} \cdot dA \cdot dx$$

$$= \int_0^l \left(\frac{M}{I} \cdot y\right)^2 \cdot \frac{1}{2E} \cdot dA \cdot dx$$

$$= \frac{1}{2E} \int_0^l \frac{M^2}{I^2} dx \, (y^2 \cdot dA)$$

$$U = \frac{1}{2E} \int_0^L \frac{M^2}{I^2} (I) \cdot dx$$

$$\boxed{U = \frac{1}{2EI} \int_0^L M^2 \cdot dx}$$

In general form strain energy, $U = \frac{1}{2EI} \int M_x^2 \cdot dx$

Strain energy due to shear stress:-

Strain energy due to s·s $= \frac{\tau^2}{2G} \times$ volume $= \int \frac{\tau^2}{2G} \cdot dv$

τ = shear stress

Strain energy due to Torsion :-

Strain energy due to Torsion $= \int_0^L \frac{T^2}{2GJ} \times dx = \frac{T^2 L}{2GJ}$

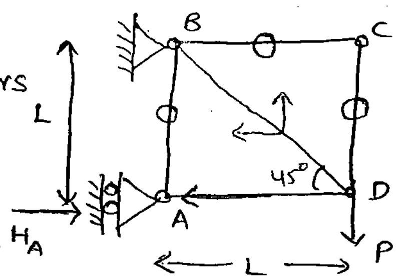
G = modulus of rigidity

T = Twisting moment

L = Length of a member

J = polar moment of inertia.

Ex:- Strain energy stored in the following pin jointed truss shown in fig.



A. AE = axial rigidity for all the members

$\Sigma M_B = 0$

$-H_A \times L + P \times L = 0$

$\boxed{H_A = P}$

At joint D :-

$\Sigma Y = 0$

$F_{DB} \sin 45^\circ - P = 0$

$F_{DB} = P\sqrt{2}$

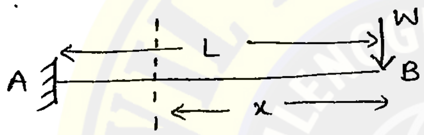
Member	P	L	AE	$\frac{P^2L}{2AE}$
AD	$-P$	L	AE	$\frac{P^2L}{2AE}$
DB	$P\sqrt{2}$	$\sqrt{2}L$	AE	$\frac{(2P^2)(\sqrt{2}L)}{2AE}$

$$U = \frac{P^2L}{2AE}\left(\frac{1+2\sqrt{2}}{\cancel{\sqrt{2}}}\right)$$

Strain energy due to Bending of beams:-

case: 1

Cantilever beam of span 'L' subjected to concentrated load of 'W'.



EI = constant

$$U = \frac{1}{2EI}\int M_x^2 \cdot dx$$

( ) −ve (Hogging)

( ) +ve (sagging)

$$M_x = -Wx$$

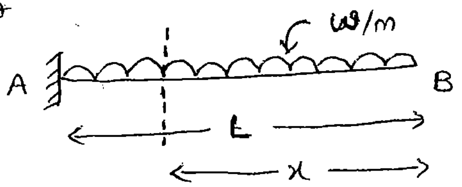
$$U = \frac{1}{2EI}\int_0^L (-Wx)^2 \cdot dx$$

$$= \frac{1}{2EI}\left[\frac{W^2x^3}{3}\right]_0^L$$

$$\boxed{U = \frac{W^2L^3}{6EI}}$$

Case : 2

Cantilever beam of span 'L' subjected to Uniformly distributed load. ω



$$M_x = -\omega x \cdot \frac{x}{2} = \frac{-\omega x^2}{2}$$

$$U = \frac{1}{2EI}\int_0^L \left(\frac{w^2x^4}{4}\right)\cdot dx$$

$$\boxed{U = \frac{w^2L^5}{40EI}}$$

The relation between the concentrated load and uniforml distributed load

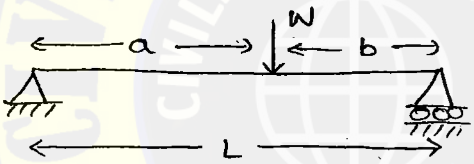
$$\boxed{W = w\cdot L}$$

Case : 3

A simply supported beam with concentrated load

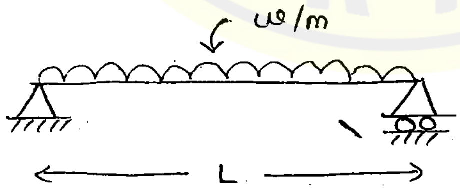
$$\boxed{U = \frac{W^2L^3}{96EI}}$$

Case: 4



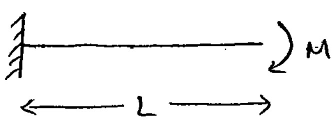
$$\boxed{U = \frac{W^2a^2b^2}{6EIL}}$$

Case: 5



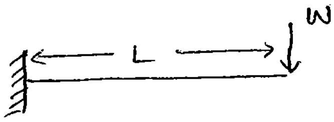
$$\boxed{U = \frac{w^2L^5}{240EI}}$$

Case: 6

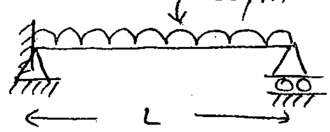


$$\boxed{U = \frac{M^2L}{2AE}}$$

Ex:-



Beam - 1



Beam - 2

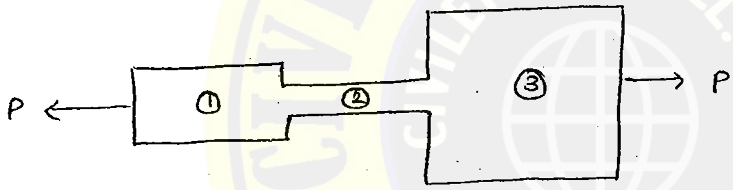
$$\frac{U_1}{U_2} = \frac{\frac{W^2 L^3}{6EI}}{\frac{w^2 L^5}{240 EI}}$$

$$= \frac{W^2 L^5}{6EI} \times \frac{240 EI}{w^2 L^5}$$

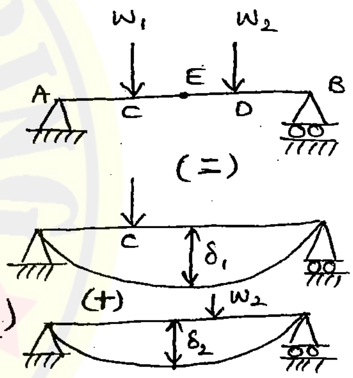
$$= 40$$

Castigliano's First theorem:-

1. It is used for analysis of statically determinate structures.
2. It is applicable for the structures if principle of super position is valid



$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$



$$(\delta_E = \delta_1 + \delta_2)$$

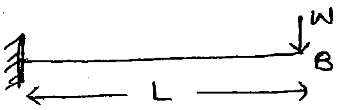
The deflection at any point due to external forces is the partial derivative of strain energy stored by a force at a same point.

$$\boxed{\delta = \frac{\partial U}{\partial W}}$$

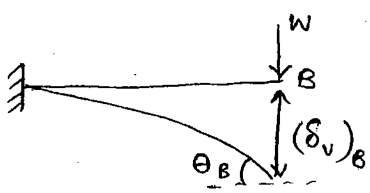
The rotation at any point due to applied moment is the partial derivative of strain energy stored with respect to the same moment at same point

$$\boxed{\theta = \frac{\partial U}{\partial M}}$$

Ex:-



$$(\delta_v)_B = ?$$

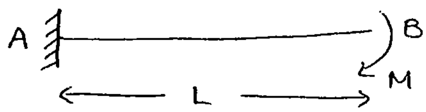


$$\boxed{\theta_B = \frac{WL^2}{2EI}}$$

$$U = \frac{W^2 L^3}{6EI}$$

$$(\delta_v)_B = \frac{\partial U}{\partial W} = \frac{2WL^3}{6EI} = \frac{WL^3}{3EI}$$

Ex:- 2)



$$\theta_B = \frac{\partial U}{\partial M}$$

$$U = \frac{M^2 L}{2EI}$$

$$\boxed{\theta_B = \frac{ML}{EI}}$$

In case of tapered sections (variable cross section)

deflection $\delta = \dfrac{\partial U}{\partial W} = \dfrac{\partial}{\partial W}\left(\dfrac{1}{2EI}\int M_x^2 \cdot dx\right)$

$$\delta = \frac{1}{2EI}\int 2M_x\left(\frac{\partial M_x}{\partial W}\right)dx$$

$$= \frac{1}{EI}\int M_x\left(\frac{\partial M_x}{\partial W}\right)dx$$

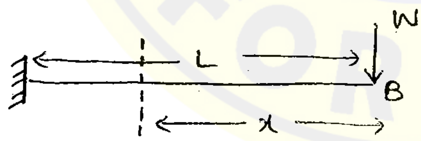
$$= \frac{1}{EI}\int M_x \cdot m_x \cdot dx$$

where

M_x = B·M at section-x due to applied load

m_x = B·M at section-x due to unit force applied at a poin[t] where deflection is desired.

Ex:- 3)



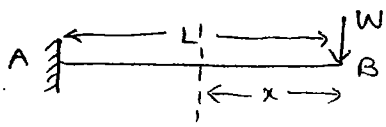
$$M_x = -Wx$$

$$\frac{\partial M_x}{\partial W} = -x$$

$$(\delta_v)_B = \frac{1}{EI}\int_0^L (-Wx)(-x)\cdot dx$$

$$= \frac{WL^3}{3EI}$$

Unit load method :-



$$M_x = -Wx$$

$$m_x = -x$$

$$(\delta_v)_B = \frac{1}{EI}\int M_x \cdot m_x \cdot dx$$

$$= \frac{1}{EI}\int_0^L (-Wx)(-x)\cdot dx$$

$$(\delta_v)_B = \frac{WL^3}{3EI}$$

Note:-

1. If there is no load acting at a point where deflection is desired, applied a dummy load or fictitious load at a point in the desired direction, and calculate the strain energy stored due to applied loads including dummy load.
2. The deflection at a point where the dummy load is applied is the partially derivative of strain energy and make dummy load = zero before integrating to get the deflection. This is called as Dummy load method.

Ex:-

$(\delta_v)_B = ?$

$$M_x = -Px - M$$

$$\frac{\partial M_x}{\partial P} = -x$$

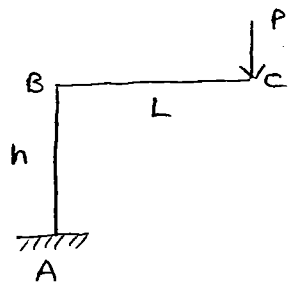
$$(\delta_v)_B = \frac{1}{EI}\int_0^L M_x\left(\frac{\partial M_x}{\partial P}\right)\cdot dx$$

$$= \frac{1}{EI}\int_0^L (\underbrace{-Px}_{=0}-M)(-x)\cdot dx$$

$$= \frac{1}{EI}\int_0^L M\cdot x\cdot dx$$

$$(\delta_v)_B = \frac{ML^2}{2EI}$$

Ex:-

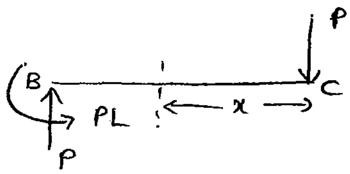


$(\delta_v)_c = ?$

$(\delta_H)_c = ?$

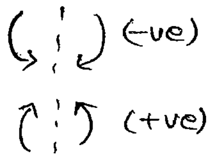
Verticle displacement at C :-

portion CB $(0 < x < L)$

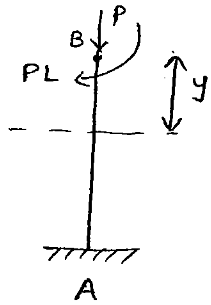


$$M_x = -Px$$

$$\frac{\partial M_x}{\partial P} = -x$$



portion BA $(0 < y < h)$



$$M_y = -PL$$

$$\frac{\partial M_y}{\partial P} = -L$$

$$(\delta_v)_c = \frac{1}{EI}\int_0^L (-Px)(-x)\,dx + \frac{1}{EI}\int_0^h (-PL)(-L)\cdot dy$$

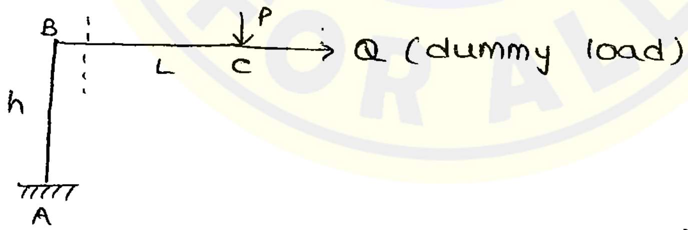
$$= \frac{1}{EI}\int_0^L Px^2\cdot dx + \frac{1}{EI}\int_0^h PL^2\cdot dy$$

$$= \frac{1}{EI}\left[\frac{PL^3}{3}\right] + \frac{1}{EI}\left[PL^2\cdot y\right]_0^h$$

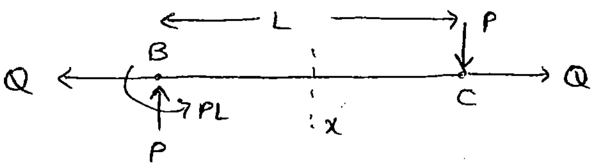
$$= \frac{PL^3}{3EI} + \frac{PL^2h}{EI}$$

$$\boxed{(\delta_v)_c = \frac{PL^3}{3EI}\left[1 + \frac{3h}{L}\right]}$$

Horizontal displacement at C :-



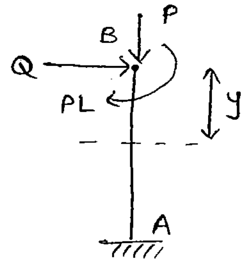
portion BC $(0 < x < L)$



$$M_x = -Px$$

$$\frac{\partial M_x}{\partial Q} = 0$$

portion BA $(0 < y < h)$



$$M_y = -Qy - PL$$

$$\frac{\partial M_y}{\partial Q} = -y$$

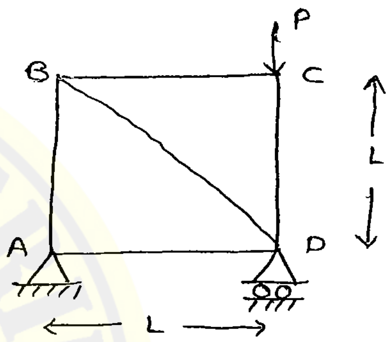
$$(\delta_H)_c = 0 + \frac{1}{EI}\int_0^H (\cancel{-Py} - PL)(-y)\,dy$$

$$= \frac{1}{EI}\int_0^H PLy \cdot dy$$

$$= \frac{1}{EI}\left[\frac{PLy^2}{2}\right]_0^H$$

$$\boxed{(\delta_H)_c = \frac{PLH^2}{2EI}}$$

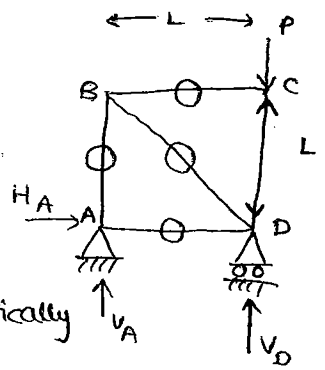
Analysis of displacement of joint of a determinate truss by using unit load method :-



1. calculate the forcesses in the member of a truss by method of joints or method of sections.
2. Let this forces be P_1, P_2, P_3 etc
3. By removing the applied loads and keeping unit force at a joint where displacement is to be calculated. calculate the forces in all the members of the truss. Let this forces be K_1, K_2, K_3 etc.
4. calculate the verticle (or) Horizontal displacement of a joint by using $\delta_v = \Sigma \frac{PKL}{AE}$

$$\delta_H = \Sigma \frac{PK'L}{AE}$$

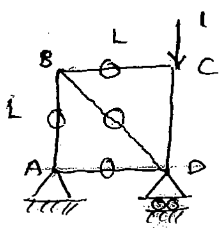
Ex:- What is the verticle displacement and Horizontal displacement at joint 'C'.



Member	P	K	L	$\frac{PKL}{AE}$
CD	$-P$	-1	L	$\frac{PL}{AE}$

Force in AB and BC is zero then automatically force in BD also zero.

$$\therefore (\delta_v)_c = \frac{PL}{AE}$$



$\Sigma V = 0$

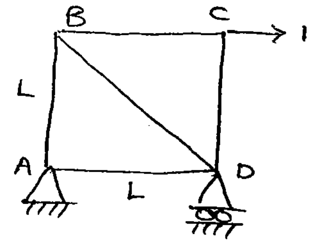
$V_A + V_D = P$

$\Sigma H = 0$

$\Sigma M_D = 0$

$V_A = 0$

Member	P	K'	L	$\frac{PK'L}{AE}$
CD	-P	0	L	0

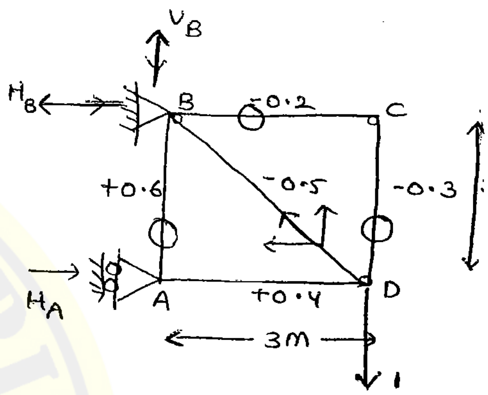


$$\therefore (\delta_H)_C = 0$$

5. If axial deformation of the members of a truss ar given the displacement of a joint can be calculated fro

$$\delta_V = \Sigma K \cdot \delta L$$

$$\delta_H = \Sigma K' \cdot \delta L$$



$$\Sigma M_A = 0$$

$$(1 \times 3) + (H_B \times 3) = 0$$

$$H_B = -1$$ (The direction of H_B is changed)

At joint D:-

$$\Sigma V = 0$$

$$F_{DB} \sin 45 - 1 = 0$$

$$F_{DB} = \sqrt{2}$$

Member	K	δL	$K \cdot \delta L$
DA	-1	0.4	
DB	$\sqrt{2}$	-0.5	

$$\therefore (\delta_V)_D = -1.107 \text{ mm}$$

6. If any member of truss is under temperature rise (or) fal and lack of fit the displacement of a joint can be calculated fo

$$\delta_V = \Sigma K \cdot \delta L$$

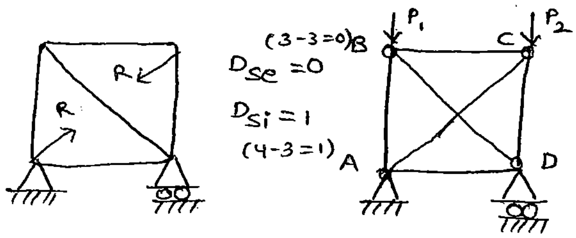
$$\delta_H = \Sigma K' \delta L$$

where

$$\delta L = L \alpha T$$

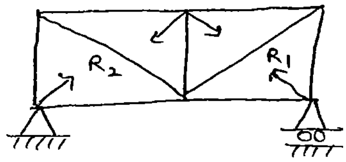
Castigliano's second theorem:-

1. It is used for the analysis of Redundant frames and statically indeterminate structure when supports do not yield.
2. In statically indeterminate structure the redundant forces of those which render to a total strain energy stored is a minimum.



$$\frac{\partial U}{\partial R}=0 \quad \text{and} \quad \frac{\partial U}{\partial M}=0$$

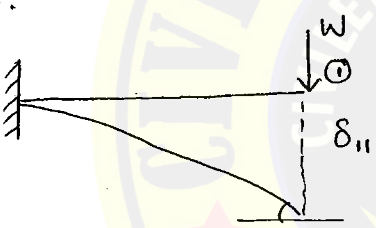
$$\frac{\partial U}{\partial R_1}=0 \quad \text{and} \quad \frac{\partial U}{\partial R_2}=0$$

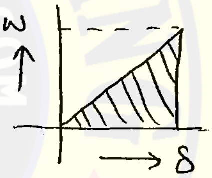


Classification of work:-

1. Real work:-

The amount of workdone by a load due to displacement caused by itself and its own direction is called as Real work.





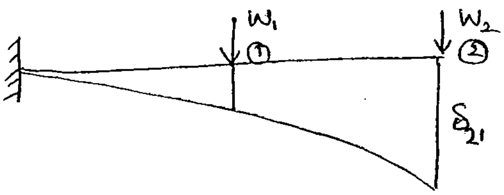
$$\text{Real work} = \frac{1}{2} W \delta_{11}$$

Note:-

Real work is same as the strain energy is stored in an elastic body due to gradually applied load

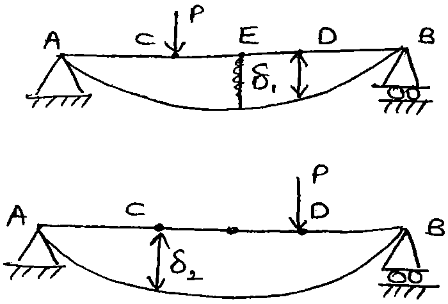
2. Virtual work:-

The workdone by a load for the displacement caused in its direction but displacement caused by some other load.



The virtual workdone by $W_2 = W_2 \times \delta_{21}$

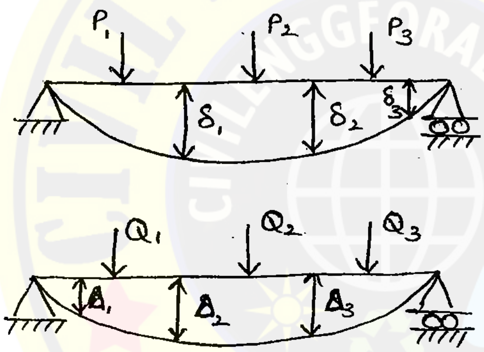
Maxwell's Law of Reciprocal deflection theorem:-



$$\boxed{\delta_1 = \delta_2}$$

Betti's theorem:-

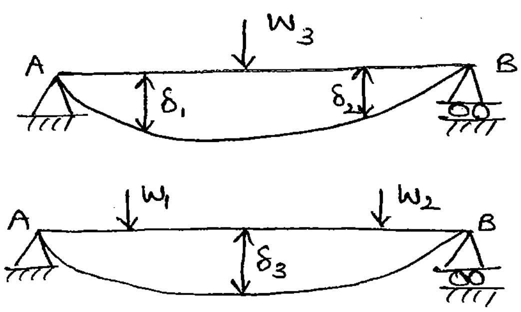
The virtual workdone by system of loads P_1, P_2, P_3 etc to due to the displacement caused by another system of loads Q_1, Q_2, Q_3 etc is equal to the virtual workdone by system of loads Q_1, Q_2, Q_3 etc due to the displacement caused by the loads P_1, P_2, P_3 etc.



$$\boxed{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 = Q_1\delta_1 + Q_2\delta_2 + Q_3\delta_3}$$

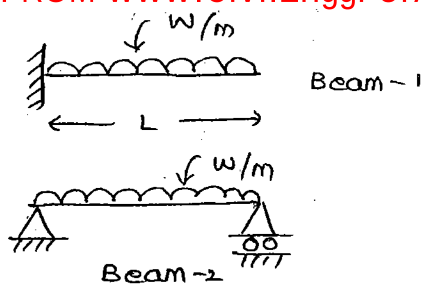
Maxwell's Bettis theorem:-

Virtual workdone by first loading due to displacement caused by second loading is same as the virtual workdone by second loading due to displacement caused by first loading.



$$\boxed{W_3\delta_3 = W_1\delta_1 + W_2\delta_2}$$

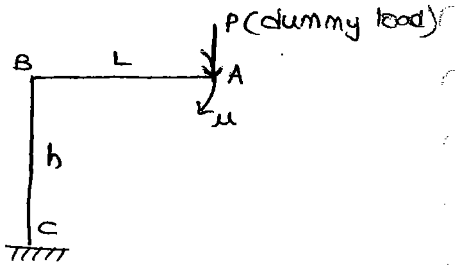
P.g NO:- 34



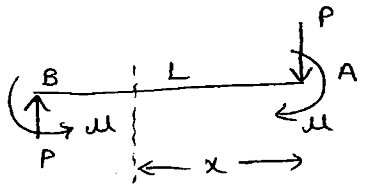
6. $$\frac{U_1}{U_2} = \frac{\frac{w^2L^5}{40EI}}{\left(\frac{w^2L^5}{240EI}\right)}$$

$$= 6$$

P.g NO:- 35



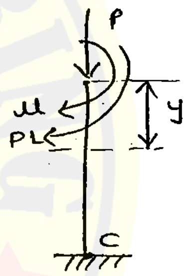
2. portion CB $(0 < x < L)$:-



$$M_x = -Px - \mu$$

$$\frac{\partial M_x}{\partial P} = -x$$

portion BC $(0 < y < h)$:-

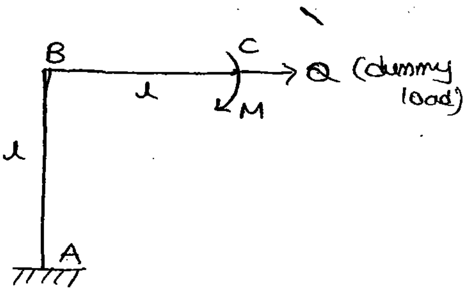


$$M_y = -PL - \mu$$

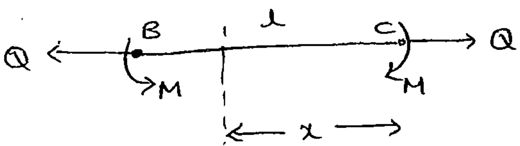
$$\frac{\partial M_y}{\partial P} = -L$$

$$(\delta_v)_A = \frac{1}{EI}\int_0^L (-\cancel{P}^{0}x - \mu)(-x)\cdot dx + \frac{1}{EI}\int_0^h (-\cancel{P}^{0}L - \mu)(-L)\cdot dy$$

$$\boxed{(\delta_v)_A = \frac{\mu L}{AEI}\left(h + \frac{L}{2}\right)}$$



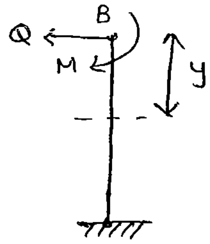
3. Portion CB $(0 < x < l)$:-



$$M_x = -M$$

$$\frac{\partial M_x}{\partial Q} = 0$$

portion BA $(0 < y < L)$:-



$$M_y = -Q \cdot y - M$$

$$\frac{\partial M_y}{\partial Q} = -y$$

$$(\delta_H)_c = 0 + \frac{1}{EI}\int_0^L (-\cancel{Q}^{\,0} \cdot y - M)(-y) \cdot dy$$

$$\boxed{(\delta_H)_c = \frac{ML^2}{2EI}}$$

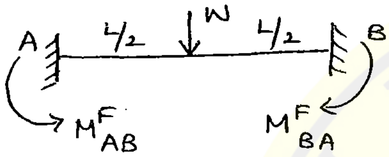
19-12-2014

UNIT-6

MOMENT DISTRIBUTION METHOD

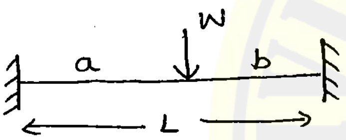
1. It is proposed by Hardy cross in 1930.
2. It is an Iterative method.
3. It is used for analysis of Indeterminate beams, rigid jointed frames but not suitable for pin jointed trusses.
4. It is less tedious when compare to slope deflection and kani's method.
5. It is a displacement or equilibrium or stiffness coefficient method.

Standard cases :-

1. 

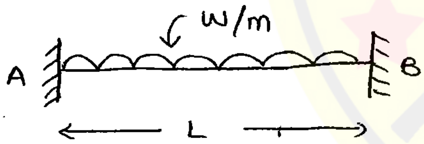
$$M^F_{AB} = \frac{-WL}{8}$$

$$M^F_{BA} = \frac{WL}{8}$$

2. 

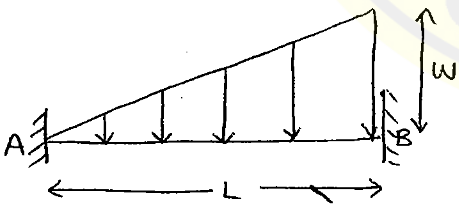
$$M^F_{AB} = \frac{-Wab^2}{L^2}$$

$$M^F_{BA} = \frac{Wa^2b}{L^2}$$

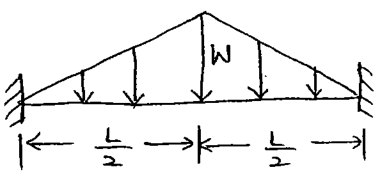
3. 

$$M^F_{AB} = \frac{-wL^2}{12}$$

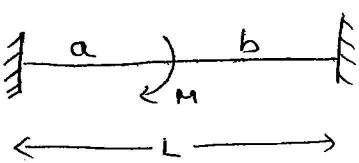
$$M^F_{BA} = \frac{wL^2}{12}$$

4. 

$$M^F_{AB} = \frac{-wL^2}{30} \qquad M^F_{BA} = \frac{wL^2}{20}$$

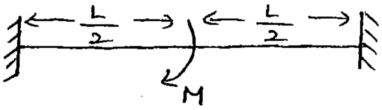
5. 

$$M^F_{AB} = \frac{-5}{96}wL^2 \qquad M^F_{BA} = \frac{5}{96}wL^2$$

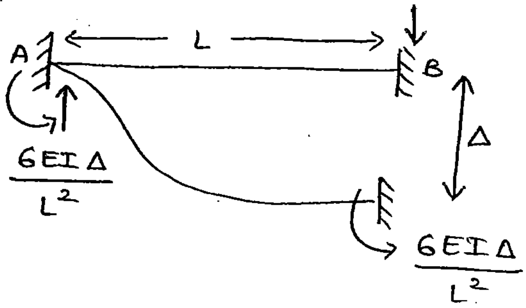
6. 

$$M^F_{AB} = \frac{M \cdot b(3a-L)}{L^2}$$

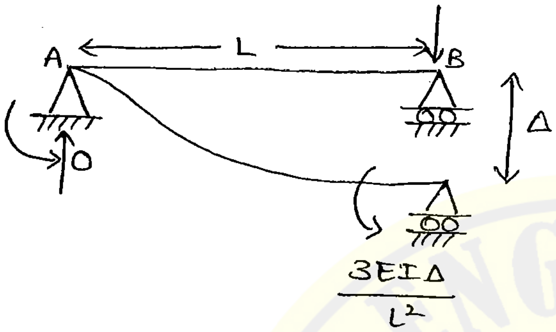
$$M^F_{BA} = \frac{M \cdot a(3b-L)}{L^2}$$

7. 

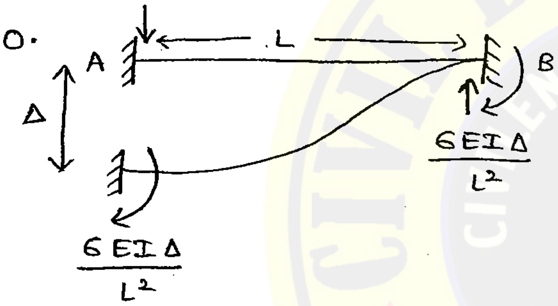
$M^F_{AB} = \frac{M}{4}$ $\quad M^F_{BA} = \frac{M}{4}$

8. 

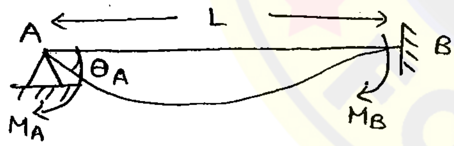
$M^F_{AB} = \frac{12EI\Delta}{L^3}$ $\quad M^F_{BA} = \frac{12EI\Delta}{L^3}$

9. 

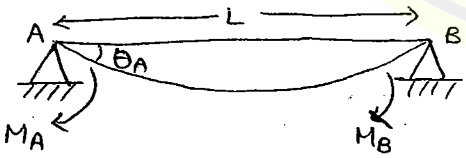
$M^F_{AB} = \frac{3EI\Delta}{L^3}$ $\quad M^F_{BA} = \frac{3EI\Delta}{L^3}$

10. 

$M^F_{AB} = \frac{12EI\Delta}{L^3}$ $\quad M^F_{BA} = \frac{12EI\Delta}{L^3}$

11. 

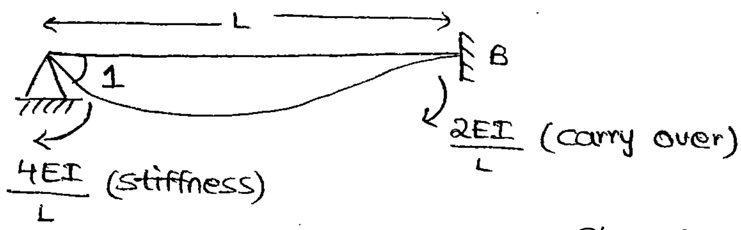
$M^F_{AB} = \frac{4EI\theta_A}{L}$ $\quad M^F_{BA} = \frac{2EI\theta_A}{L}$

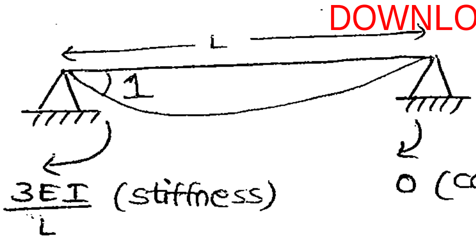
12. 

$M^F_{AB} = \frac{3EI\theta_A}{L}$ $\quad M^F_{BA} = 0$

Absolute stiffness (or) Stiffness factor (or) Rotational stiffness:

1. It is an amount of moment required to cause unit rotation without translation.





2. Rotational stiffness of a member when far end is fixed

$$\therefore \frac{4EI}{L}$$

3. Rotational stiffness of a member when far end is hinged or roller

$$\therefore \frac{3EI}{L}$$

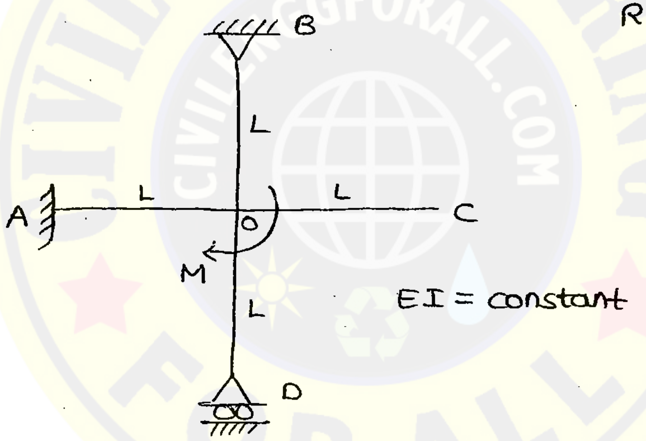
4. Rotational stiffness of a member when far end is free

$$\therefore 0$$

Rotational stiffness of a joint :-

It is the sum of the rotational stiffness of all the members meeting at a joint.

$$R.S = \frac{\text{Moment}}{\text{rotation}} = \frac{M}{\theta}$$



$$\text{Rotational stiffness of joint 'O'} = \frac{4EI}{L} + \frac{3EI}{L} + 0 + \frac{3EI}{L}$$

$$= \frac{10EI}{L}$$

$$\text{Rotation of a joint} = \frac{\text{Moment applied}}{\text{Rotational stiffness of a joint}}$$

** $\boxed{\text{Rotation of joint 'O'} = \frac{M}{\frac{10EI}{L}} = \frac{ML}{10EI}}$

Carry over factor:-

It is the ratio between moment induced at one end and moment applied at other end.

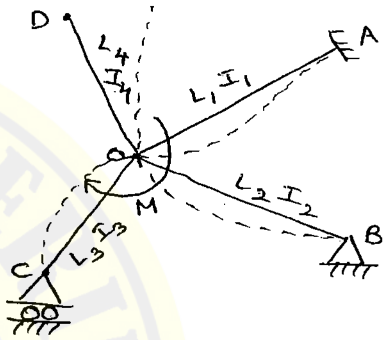
$$C.O.F = \frac{\text{Moment induced at one end}}{\text{Moment applied at other end}}$$

1. carry over factor for a member when far end is fixed equals to $\frac{1}{2}$

2. when far end is hinged or roller carry over factor $= 0$

Distribution theorem:-

1. Several prismatic members of different lengths are meeting at joint 'O'. and this joint is subjected to a moment of M



$$M_{OA} : M_{OB} : M_{OC} : M_{OD} = \frac{4EI_1}{L_1} : \frac{3EI_2}{L_2} : \frac{3EI_3}{L_3} : 0$$

$$= K_{OA} : K_{OB} : K_{OC} : K_{OD}$$

$$K_{OA} : K_{OB} : K_{OC} : K_{OD} = \frac{I_1}{L_1} : \frac{3}{4}\frac{I_2}{L_2} : \frac{3}{4}\frac{I_3}{L_3} : 0$$

* 2. When far end is fixed, relative stiffness $K = \frac{I}{L}$

* 3. When far end is hinged or roller, relative stiffness $K = \frac{3}{4}\frac{I}{L}$

* 4. When far end is free, relative stiffness = zero

Distibution factor:-

1. When a moment is applied at joint 'O' where several members are meeting the moment shared by each member i a certain ratio called as Distribution factor

$$D.F = \frac{K}{\Sigma K}$$

where

K = relative stiffness of a member for which distribution factor is to be calculated

ΣK = relative stiffness of all the members meeting at a joint.

Sum of the distribution factors of all the members meeting at a joint = 1

Ex:- The ratio of relative stiffness of a member when far end is fixed and when far end is hinged is [c]

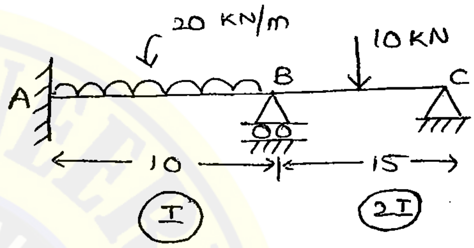
a) 0   b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) $\frac{1}{2}$

A. $D.F = \frac{K}{\Sigma K}$

$= \frac{K_1}{K_2}$

$= \frac{I/L}{\frac{3}{4} I/L} = \frac{4}{3}$

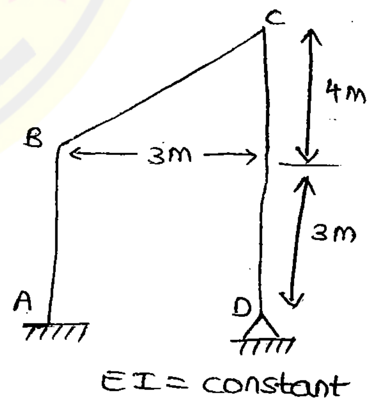
Ex:- Distribution factors of members BA and BC respectively are



A. Joint B :- (Assume B is fixed)

Member	K	$DF = \frac{K}{\Sigma K}$
BA	$\frac{I}{10}$	0.5
BC	$\frac{3}{4}\left(\frac{2I}{15}\right)$	0.5

Ex:- Calculate the D.F of the members at joints B and C



A. Joint B :-

Member	K	$DF = \frac{K}{\Sigma K}$
BA	$I/3$	$5/8$
BC	$I/5$	$3/8$

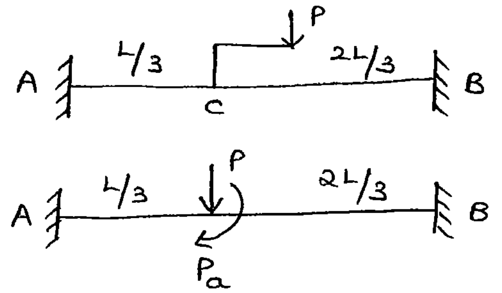
Joint C :-

Member	K	$DF = \frac{K}{\Sigma K}$
CB	$I/5$	$28/43$
CD	$\frac{3}{4} \cdot \frac{I}{7}$	$15/43$

Moment distribution method:-

procedure:-

1. Idealized the beam for the given loading



2. By making interior supports has fixed, calculate the relative stiffness of the members meeting at joints.

3. Calculate the distribution factors of all the members meetin at joints.

4. Assuming all the supports of a given beam or frame as fixed, calculate the F.E.M for applied loads.

5. If any end support of a given beam is hinged or rolle make the final moment to be zero at that support by adding or subtracting moment equals to F.E.M at that suppo These is called as Released moment

6. Half of the released moment is to be carry over to the far end of a member. It is called as carry over moment & calculate the total moments.

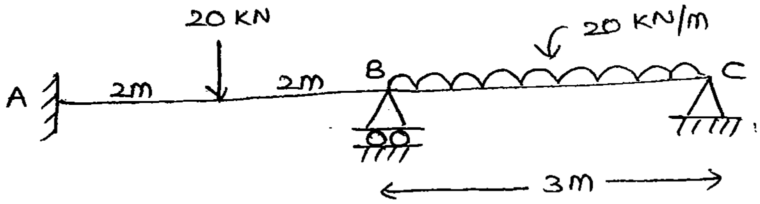
7. Calculate the unbalanced moment at a joint and distribute the balanced moment to all the members meeting at a joint in the ratio of distribution factors. These is also called as distributed moment.

8. Calculate the carry over moment and place the moment at far ends.

9. Repeat the steps 7 and 8, till the net moment is zero at a joint, and stop the Iteration with carry over moment

10. Calculate the final moments at each support.

Ex:- Calculate the final moments.



EI = constant

Assume 'B' is fixed

A. $D_{BA} = \dfrac{\frac{I}{4}}{\frac{I}{4} + \frac{3}{4}\cdot\frac{I}{3}} = \dfrac{K}{\Sigma K}$

$D_{BA} = 0.5$

$D_{BC} = 0.5$

$M^F_{AB} = \dfrac{-20\times4}{8} = -10$

$M^F_{BA} = 10$

$M^F_{BC} = \dfrac{-20\times3^2}{12} = -15$

$M^F_{CB} = 15$

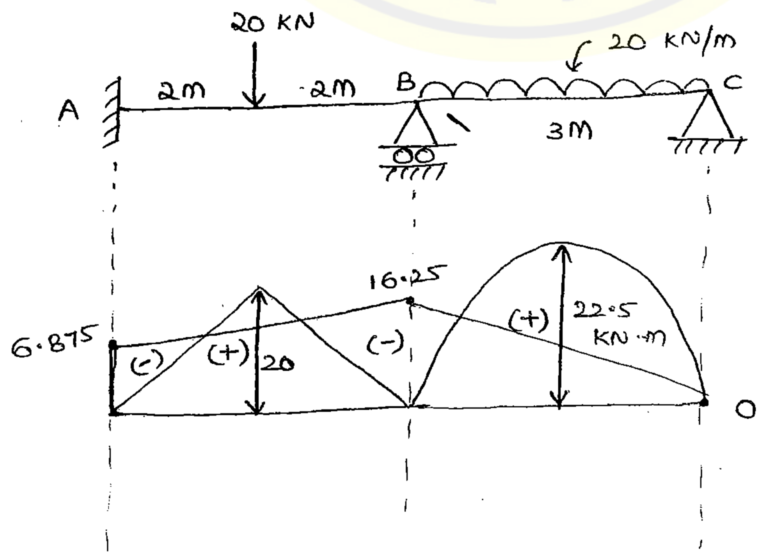
	AB	BA (0.5)	BC (0.5)	CB
F.E.M	−10	+10	−15	+15
Released Moment			−7.5 ← (½)	−15 ← (follow 5th step)
T.M	−10	+10	−22.5	0
D.M		6.25	6.25	
C.O.M	3.125 ← (½)			
F.M	−6.875	16.25	−16.25	0

$10 - 22.5 = -12.5$

$= -12.5\times0.5 = -6.25$

(if -ve came replace +ve)

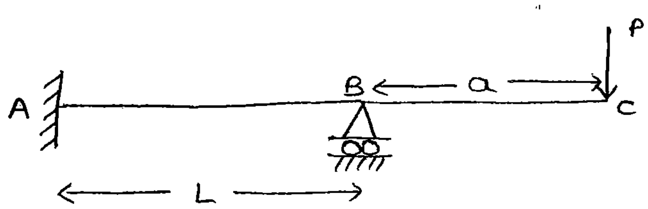
Ex:-

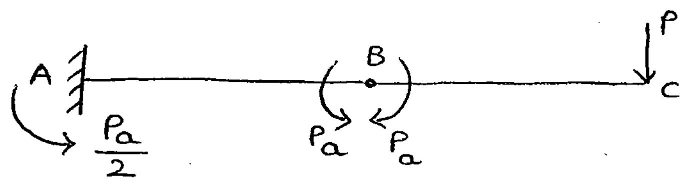


$\dfrac{20\times4}{4} = 20$ kN-m

$\dfrac{20\times3^2}{8} = 22.5$ kN-m

EX:- 2) Calculate the Moment at A ?

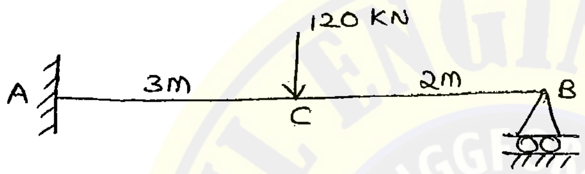




B is a roller. so i
is zero then place
anticlock wise mome
Then P_a is distribute
half to the A.

$$\therefore M_A = \frac{P_a}{2}$$

EX:-3) Calculate $M_A = ?$

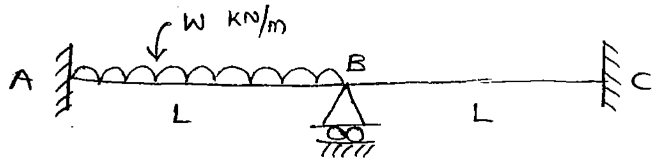


$$M^F_{AB} = \frac{-120 \times 3 \times 2^2}{5^2} = -57.6 \text{ KN-m} = \frac{-wab^2}{L^2}$$

$$M^F_{BA} = \frac{120 \times 3^2 \times 2}{5^2} = 86.4 \text{ KN-m} = \frac{wa^2b}{L^2}$$

AB	BA	
-57.6	86.4	Fixed end moments
- 43.2 ← $(\frac{1}{2})$	-86.4	
-100.8	0	Final moments

EX:- 4)



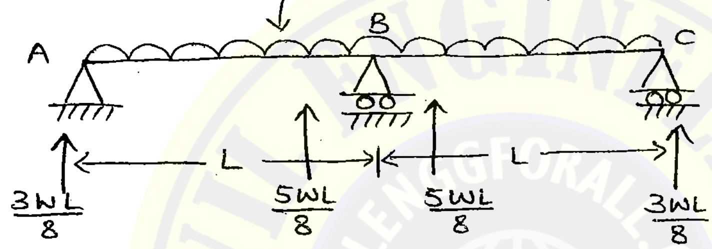
$$D_{BA} = \frac{\frac{3}{4}\frac{I}{L}}{\frac{3}{4}\frac{I}{L} + \frac{3}{4}\frac{I}{4}} = 0.5$$

$$D_{BC} = \frac{\frac{3}{4}\frac{I}{4}}{\frac{3}{4}\frac{I}{4} + \frac{3}{4}\frac{I}{4}} = 0.5$$

	AB	BA	BC	CB
		0·5	0·5	
F·E·M	$-\frac{wL^2}{12}$	$\frac{wL^2}{12}$	0	0
D·M		$-\frac{wL^2}{24}$	$-\frac{wL^2}{24}$	
C·O·M	$-\frac{wL^2}{48}$			$-\frac{wL^2}{48}$
Final moments	$-\frac{5wL^2}{48}$	$+\frac{wL^2}{24}$	$-\frac{wL^2}{24}$	$-\frac{wL^2}{48}$

Ex:-

(Assume B is fixed)



B·M at section - x :-

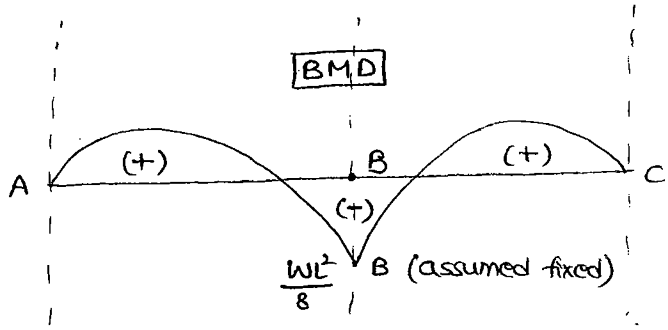
$$M_x = \frac{3}{8}wL \cdot x - \frac{wx^2}{2}$$

At $x=0$, $M_B = 0$

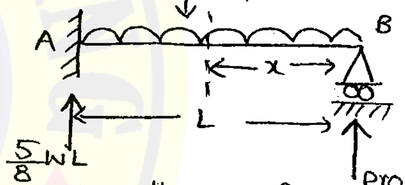
At $x = L$ $M_A = -\frac{wL^2}{8}$

$$R_A : R_B : R_C = \frac{3wL}{8} : \frac{10wL}{8} : \frac{3wL}{8}$$

$$= 3 : 10 : 3$$



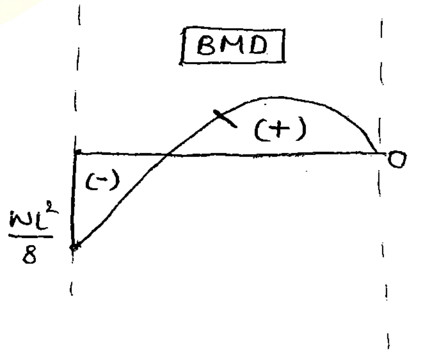
Ex:- for above pbm not same pbm



$$\frac{wL^4}{8EI} = \frac{PL^3}{3EI}$$

$P = \frac{3}{8}wL$

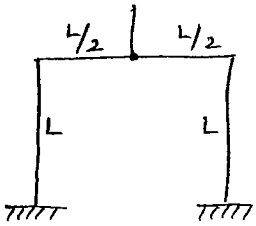
$$\boxed{P = \frac{3}{8}wL}$$



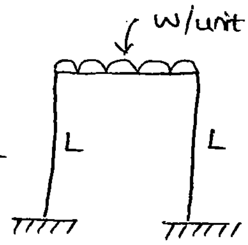
FRAMES

- Non sway
- sway
  - pure sway
  - Non pure sway

Non sway frames :-



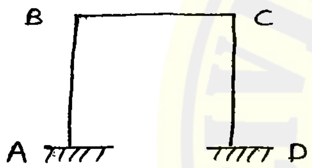
EI = constant



Sway frames :-

- sway occurs because of

1. Unsymmetrical loading :-

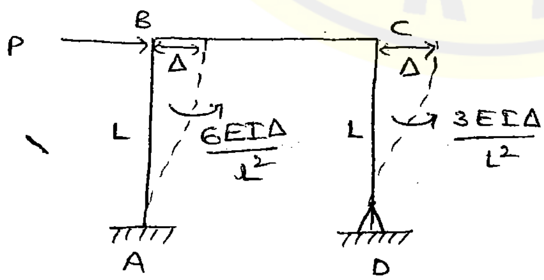


$$M^F_{BC} = \frac{-W \times \frac{L}{4} \times \left(\frac{3L}{4}\right)^2}{L^2}$$

$$M^F_{CB} = \frac{W \times \frac{3L}{4} \times \left(\frac{L}{4}\right)^2}{L^2}$$

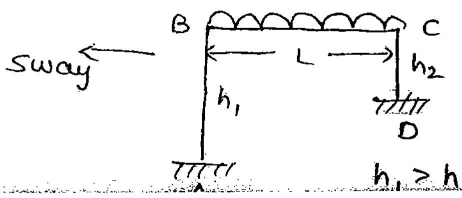
$M^F_{BC} > M^F_{CB}$ sway occurs towards lesser moment side

2. Unsymmetrical supports :-



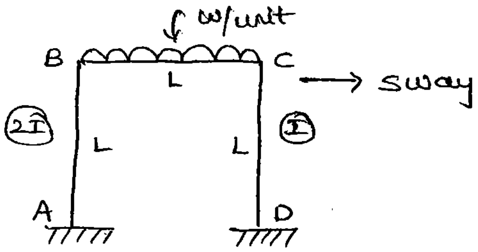
Now sway occurs towards right side because less moment developed at c due to the lateral displacement

3. keeping all the parameters are same if the height of the columns are different, columns will sway toward longer column side.

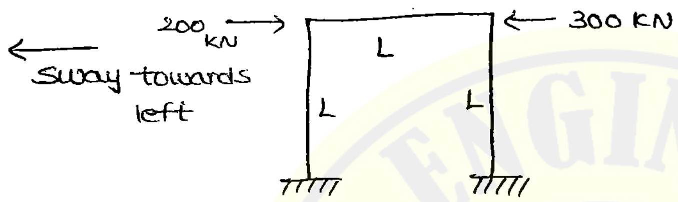


$h_1 > h_2$

4. Keeping the parameters of the frame same, if moment of Inertia of the columns are different, column will sway towards lesser moment of inertia side.

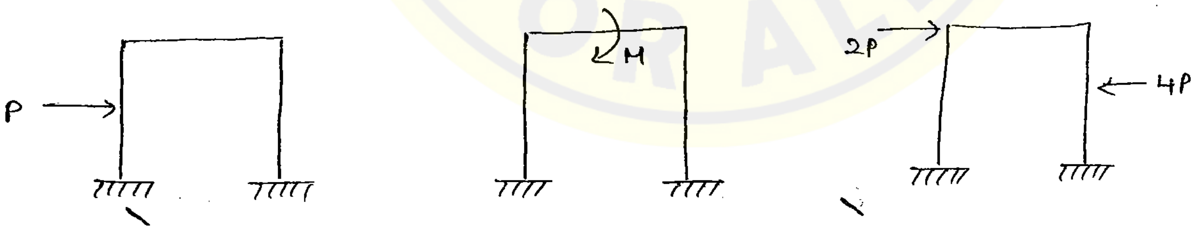


5. When a frame is subjected to a horizontal force, frame will sway towards resulted horizontal force sides.



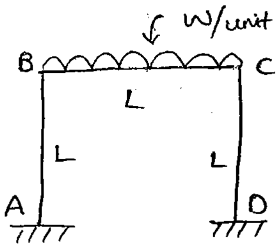
Pure sway:-

Non pure sway:-

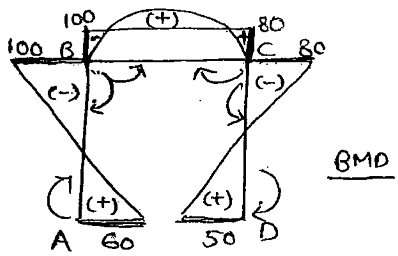


Bending Moment diagram:-

Ex:-1)



EI = constant



$M_{AB} = +60$ kN-m

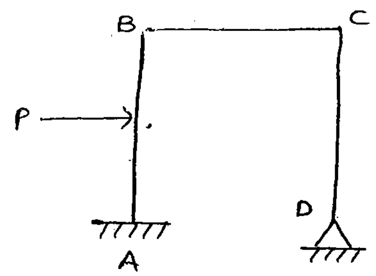
$M_{BA} = +100$ kN-m

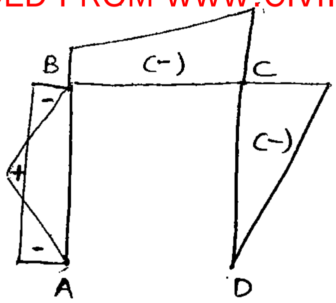
$M_{BC} = -100$ kN-m

$M_{CB} = +80$ kN-m

$M_{CD} = -80$ kN-m, $M_{DC} = +50$ kN-m

Ex:-2)





BMD

P.g NO:-43

4. $D_{CB} = \dfrac{I/2.5}{\frac{I}{2.5} + \frac{I}{2.5} + \frac{I}{5}} = 0.4$

$D_{CD} = \dfrac{I/2.5}{\frac{I}{2.5} + \frac{I}{2.5} + \frac{I}{5}} = 0.4$

$D_{CG} = 1 - 0.4 - 0.4 = 0.2$

6. $D_{BC} = \dfrac{2I/8}{\frac{2I}{8} + \frac{I}{6}} = 0.6$

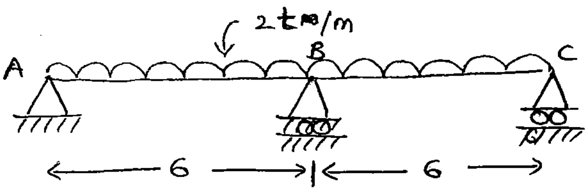
$D_{BA} = 0.4$

7. $D_{BA} = \dfrac{2I/2L}{\frac{2I}{2L} + \frac{I}{L} + \frac{I}{L}} = \dfrac{1}{3}$

$D_{BC} = \dfrac{I/L}{\frac{2I}{2L} + \frac{I}{L} + \frac{I}{L}} = \dfrac{1}{3}$

$D_{BD} = \dfrac{I/L}{\frac{2I}{2L} + \frac{I}{L} + \frac{I}{L}} = \dfrac{1}{3}$

10.

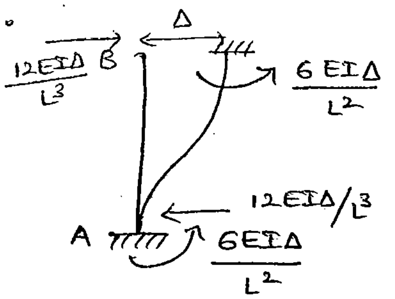


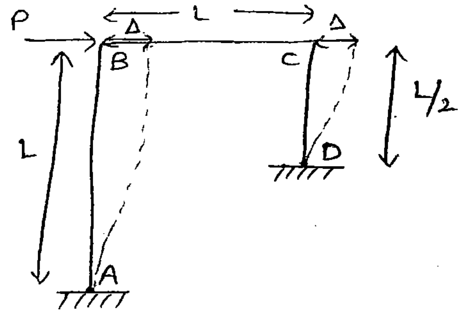
Assume 'B' is fixed

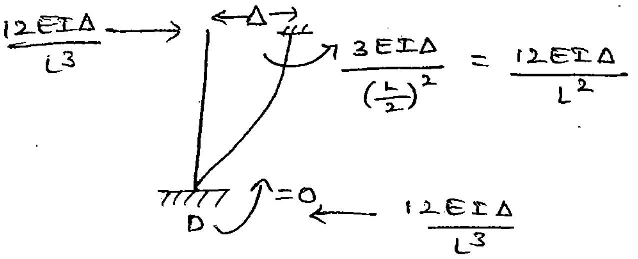
(or) $\dfrac{wl^2}{8} = \dfrac{2(6)^2}{8} = 9$

A		B		C
		0.5	0.5	
AB		BA	BC	CB
-6		+6	-6	+6
+6	(½)→	+3	-3 ←(½)	-6
0		+9	-9	0

14.







$$\frac{M_{BA}}{M_{CD}} = \frac{\frac{6EI\Delta}{L^2}}{\left(\frac{12EI\Delta}{L^2}\right)} = \frac{1}{2} = 1:2$$

Horizontal force required to cause lateral displacement Δ is

$$P = \frac{24EI\Delta}{L^3} = \left(\frac{12EI\Delta}{L^3} + \frac{12EI\Delta}{L^3}\right)$$

P.9 NO:- 45

2.

$$D_{BE} = \frac{\frac{I}{4L}}{\frac{I}{4L} + \frac{I}{4L} + \frac{I}{4L} + \frac{3}{4}\frac{I}{3L}}$$

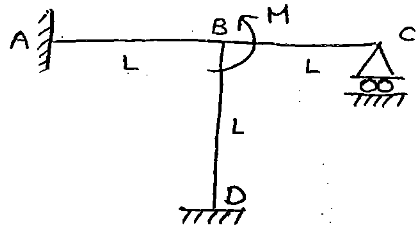
$$= \frac{1}{4}$$

3.

	Member	K	$D.F = \frac{K}{\Sigma K}$
Joint - B	BA	$\frac{1.5I}{6}$	0.5
	BC	$I/4$	0.5
Joint - C	CB	$I/4$	0.4
	CD	$\frac{3}{4}\frac{(2I)}{4}$	0.6

Interior supports B, C must be fixed (assumed)

4. Rotational stiffness $= \dfrac{M}{\theta}$



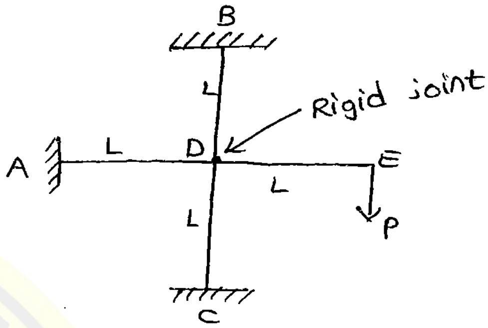
$$\theta = \frac{M}{R\cdot S}$$

$$\theta = \frac{M \times L}{11EI}$$

$$R\cdot S = \frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L}$$

$$= \frac{11EI}{L}$$

5.



$$D_{DB} = \frac{1}{3}$$

$$D_{DA} = \frac{1}{3}$$

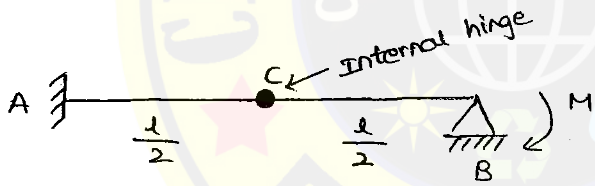
$$D_{DC} = \frac{1}{3}$$

$$D_{DE} = 0$$

$$M_{DA} = \frac{PL}{3}$$

$$M_{AD} = \frac{1}{2}(M_{DA}) = \frac{1}{2}\left(\frac{PL}{3}\right) = \frac{PL}{6}$$

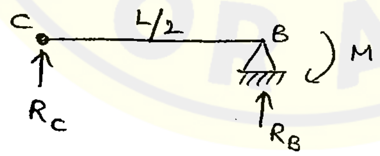
6.

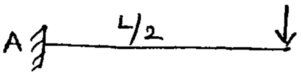


$$\Sigma M_B = 0$$

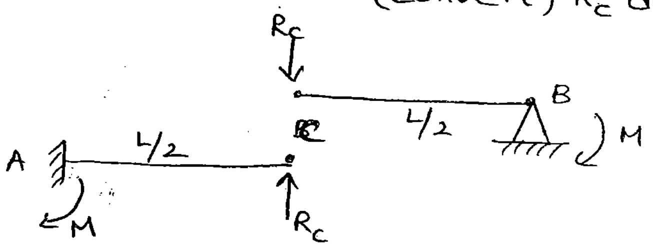
$$R_c \times \frac{L}{2} + M = 0$$

$$R_c = \frac{-2M}{L}$$





(convert) R_c direction

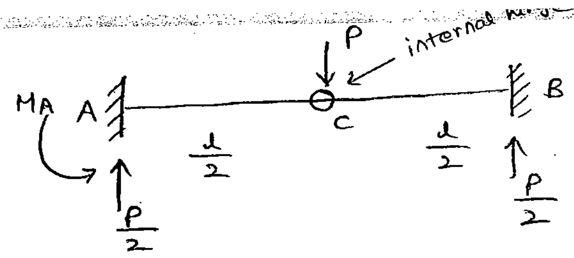


$$\Sigma M_A = 0$$

$$\cancel{R_c}\ \frac{2M}{L} \times \frac{L}{2}$$

$$\Sigma M_A = +M$$

7.

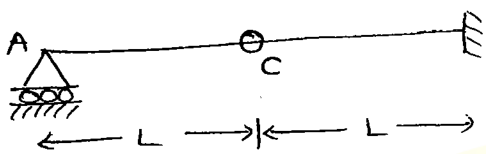


$\Sigma M_c = 0$ (left side)

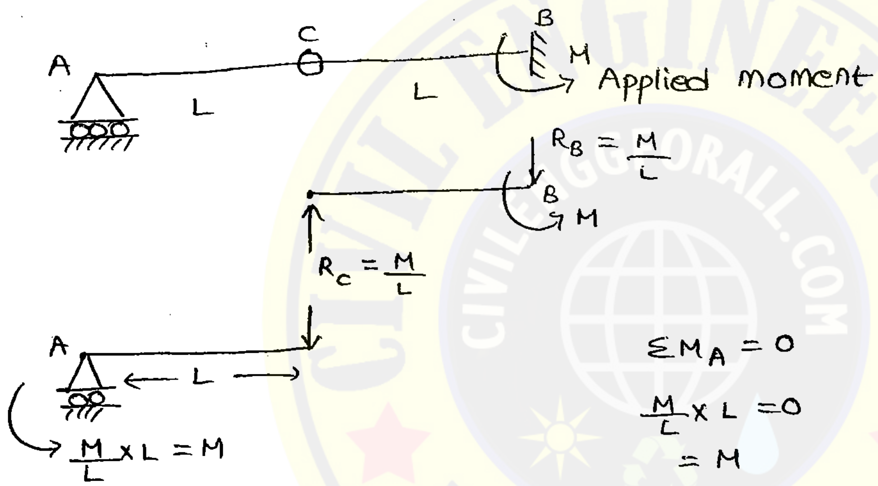
$$-M_A + \frac{P}{2}\left(\frac{l}{2}\right) = 0$$

$$M_A = \frac{Pl}{4}$$

8.



← It is a compound beam



$\Sigma M_A = 0$

$\frac{M}{L} \times L = 0$

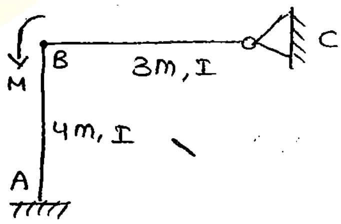
$= M$

$\therefore C \cdot O \cdot F = \frac{M}{M} = 1$

9.

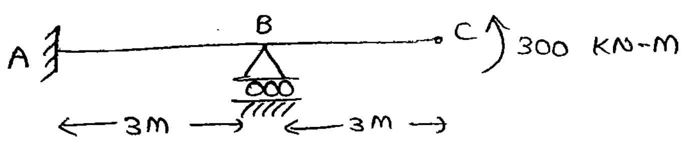
$D_{BA} = \frac{I}{4}$

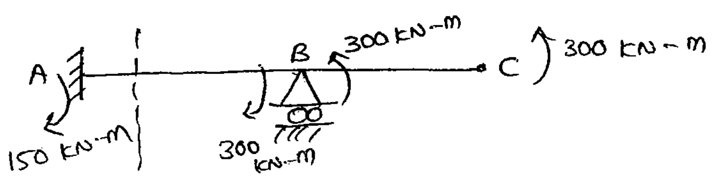
$D_{BC} = \frac{I}{3}$

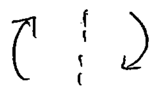


$M_{BA} = \frac{M}{2}$ $\quad M_{BC} = \frac{M}{c}$ ($\frac{1}{2}$ Moment distribute)

10.





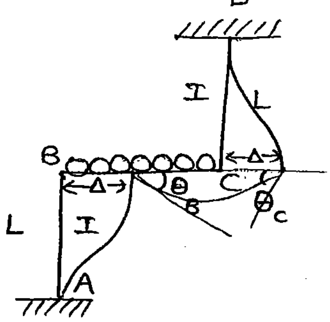


Sagging

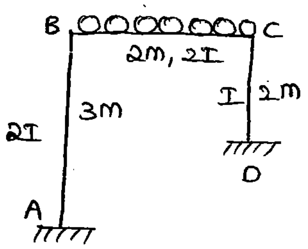
Half moment distribute

$\therefore$ 150 KN-m sagging

12. $\theta_B = -\theta_C \cdot \Delta$ is present



13.



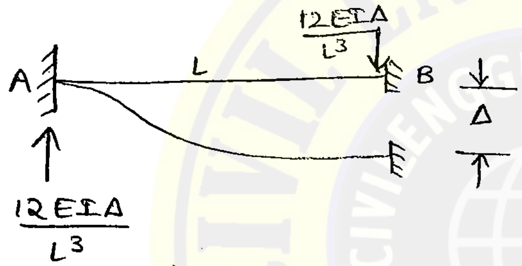
$M^F_{BA} = \frac{6EI\Delta}{L^2} = \frac{6 \times E \times 2I \times \Delta}{3^2} = \frac{4}{3} EI\Delta$

$M^F_{CD} = \frac{6EI\Delta}{2^2} = \frac{3}{2} EI\Delta$

$M^F_{CD} > M^F_{BA}$

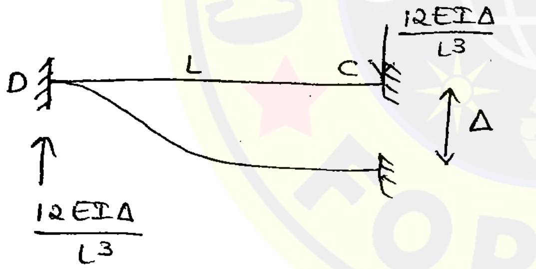
$\therefore$ sway towards left

14.

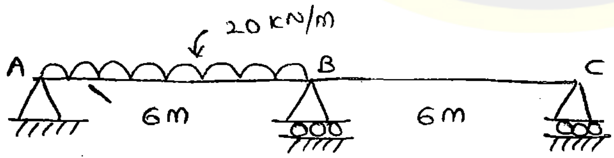


$W = \frac{24EI\Delta}{L^3}$

$\Delta = \frac{WL^3}{24EI}$



15.



$D_{BA} = \frac{I/6}{\frac{I}{6} + \frac{I}{6}} = 0.5$

$D_{BC} = 0.5$

	AB		BA	BC	CB
			0.5	0.5	
F.E.M	−60		+60	0	0
Released. M	+60	$\xrightarrow{(\frac{1}{2})}$	+30		
Total Moment	0		+90	0	0
			−45	−45	
	0		+45	−45	0

~~60×0.5 = 30 (It~~

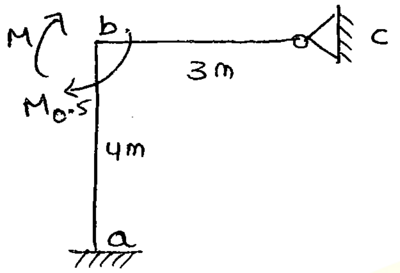
90×0.5 = 45 (It is +ve so convert −ve)

16. $M_{BA} = \dfrac{6EI\Delta}{L^2}$

$M_{CD} = \dfrac{3EI\Delta}{L^2} = \dfrac{3E(0.5I)\Delta}{\left(\frac{L}{2}\right)^2} = \dfrac{6EI\Delta}{L^2}$

$$\boxed{\frac{M_{BA}}{M_{CD}} = 1.0}$$

17.



$D_{ba} = 0.5$

$D_{bc} = 0.5$

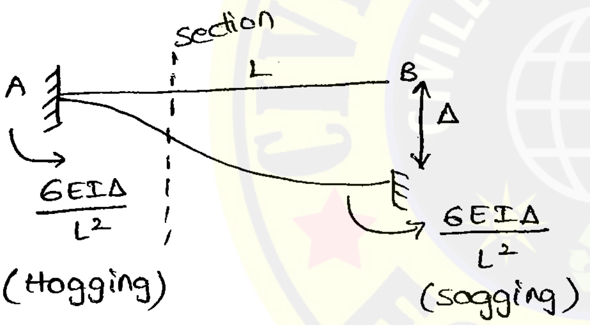
$M_{ba} = \dfrac{M}{2}$ → $\frac{1}{2}$ Moment distribute

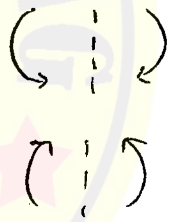
$M_{ab} = \frac{1}{2}(M_{ba}) = \dfrac{M}{4}$

$\boxed{M_{cb} = 0}$ → Hinge, no moment is allowed

P.9 NO:-48

1.

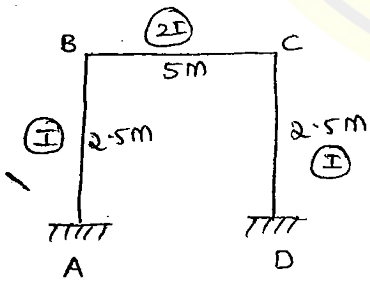




Hogging BM

Sagging BM

2.



$D_{BA} = \dfrac{\frac{I}{2.5}}{\frac{I}{2.5} + \frac{2I}{5}} = 0.5$

$D_{BC} = \dfrac{\frac{2I}{5}}{\frac{I}{2.5} + \frac{2I}{2.5}} = 0.5$

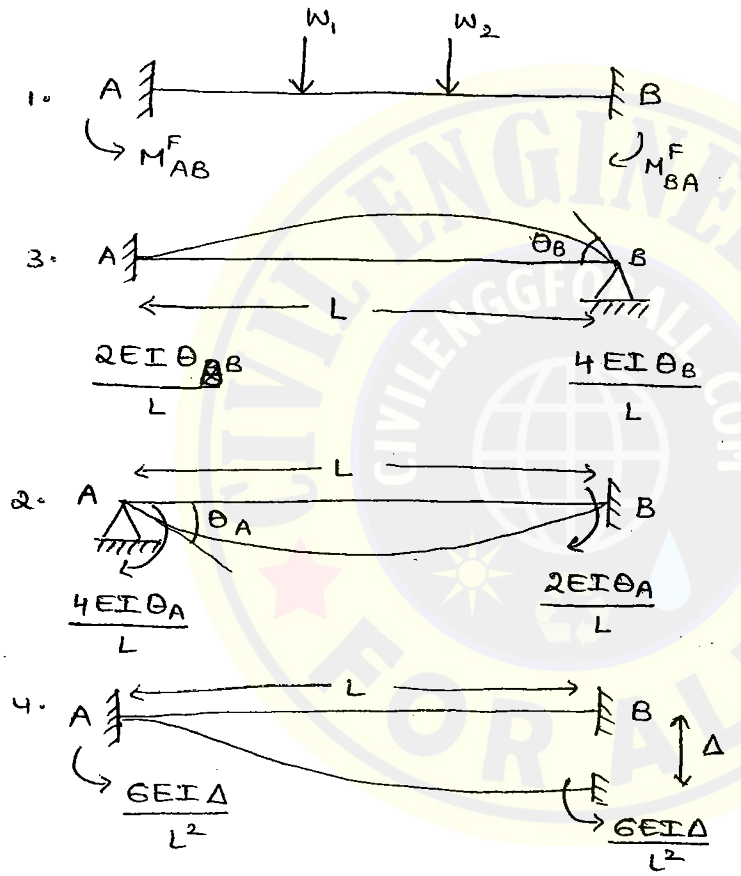
5. $= \dfrac{\frac{3}{4}\frac{I}{L}}{\frac{I}{L}}$

$= \dfrac{3}{4}$

UNIT - 07

SLOPE DEFLECTION METHOD

1. It is proposed by G. A. Manie in 1915.
2. It is implimented by
3. It is used for the analysis of indeterminate beams and Frames
4. It is tedious when compare to moment distribution and Kani's method.
5. It is a displacement (or) equilibrium (or) stiffness coefficient method.



Slope deflection equations :-

For span AB :-

$$M_{AB} = M^F_{AB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2}$$

$$= M^F_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

1. A series of simultaneous equations each expressing the relation between the moments ~~acting~~ at the ends of the members are written in terms of slope and deflection.

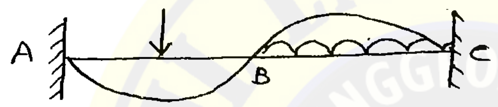
2. The deflection due to shear force is about 1 % to 2 % the deflection due to bending moment and hence the deflection equations.

3. Slope deflection equations gave the relation between bending moment rotation and deflection only.

4. In the analysis of slope and deflection method the no. of condition equations such as equilibrium equations, equal to the degree of freedom. By neglecting the axial deformation.

5. No. of equilibrium equations in slope deflection method to be solved is equal to the total number of joint displacements in the structure.

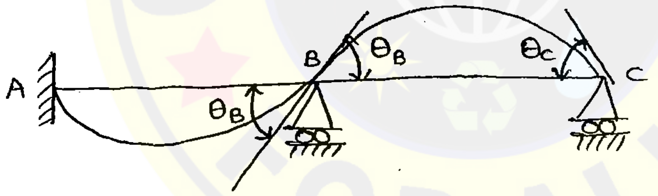
Evaluation of joint displacement :-

1. 

only unknown 'θ_B'

Equilibrium equation

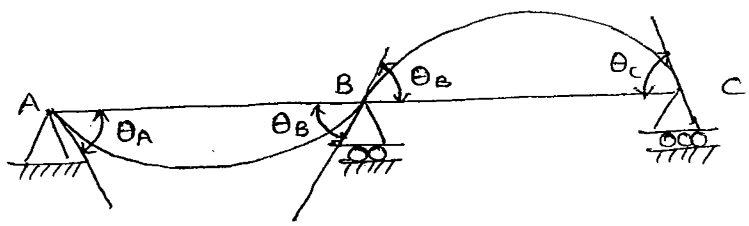
$M_{BA} + M_{BC} = 0$

2. 

$M_{BA} + M_{BC} = 0$

$M_{CB} = 0$

Unknowns are θ_B and θ_c

3. 

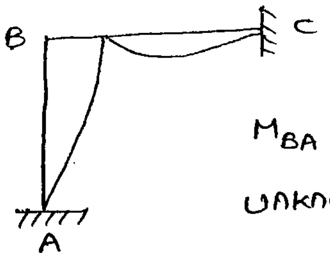
$M_{AB} = 0$

$M_{BA} + M_{BC} = 0$

$M_{CB} = 0$

Unknowns are θ_A, θ_B, θ_c

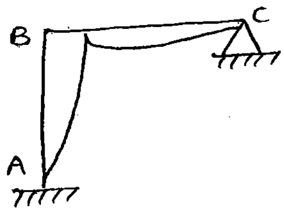
4.



$M_{BA} + M_{BC} = 0$

unknown are θ_B

5.

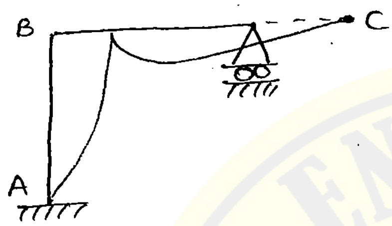


$M_{BA} + M_{BC} = 0$

$M_{CB} = 0$

unknowns are θ_B, θ_C

6.



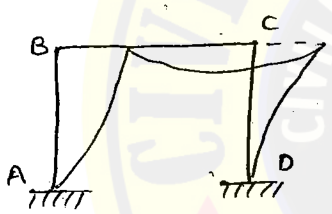
$M_{BA} + M_{BC} = 0$

$M_{CB} = 0$

$\Sigma H = 0$ (shear equation)

unknowns are $\theta_B, \theta_C, \Delta$

7.

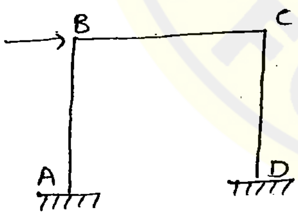


$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

θ_B, θ_C

8.

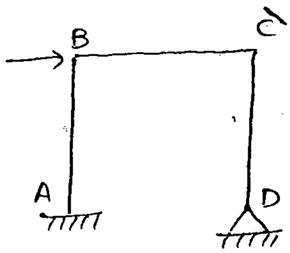


$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$\theta_B, \theta_C, \Delta$

9.



$M_{BA} + M_{BC} = 0$

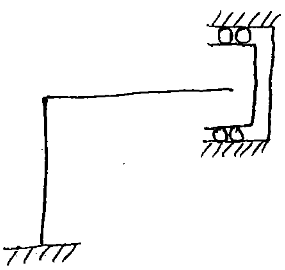
$M_{CB} + M_{CD} = 0$

$M_{DC} = 0$

$\Sigma H = 0$ (shear equation)

$\theta_B, \theta_C, \theta_D, \Delta$

10.



$M_{BA} + M_{BC} = 0$

$\Sigma H = 0$

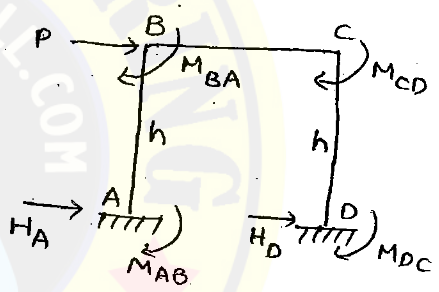
θ_B, Δ

Note:-

1. Evaluate the total no. of joints displacement of a given structure.
2. Treating such span as a fixed beam. Calculate the "Fixed end moments" due to applied load.
3. Write the slope deflection equation for each span interms of end moments, fixed end moments, rotations and displacement
4. Formulate the equilibrium equations for the Individual joints and verify the number of equilibrium equations should be equal to the total no. of joint displacement.
5. Calculate the rotations and displacement from the above equilibrium equations and resubstitute in slope deflection equations to get final moments.

Horizontal shear equations:-

Ex:- $\underline{\Sigma H = 0}$



$H_A + H_D + P = 0$

$\underline{\Sigma M_B = 0}$ (column BA)

$-H_A \times h + M_{AB} + M_{BA} = 0$

$$\boxed{H_A = \frac{M_{AB} + M_{BA}}{h}}$$

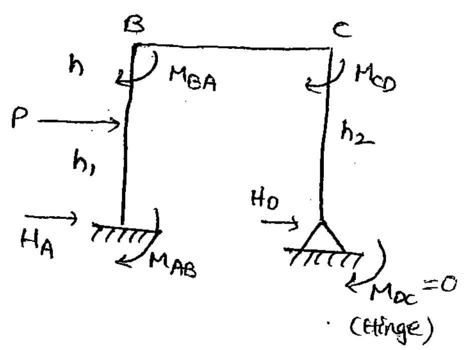
$\underline{\Sigma M_C = 0}$ (column CD)

$-H_D \times h + M_{DC} + M_{CD} = 0$

$$\boxed{H_D = \frac{M_{CD} + M_{DC}}{h}}$$

$$\boxed{\therefore \left(\frac{M_{AB} + M_{BA}}{h}\right) + \left(\frac{M_{CD} + M_{DC}}{h}\right) + P = 0}$$ Horizontal shear equation.

Ex:-2) $\Sigma H = 0 \Rightarrow H_A + H_D - P = 0$



$\underline{\Sigma M_B = 0}$:-

$-H_A \times h_1 - Ph + M_{AB} + M_{BA} = 0$

$$H_A = \frac{M_{AB} + M_{BA} - Ph}{h_1}$$

$\underline{\Sigma M_C = 0}$:-

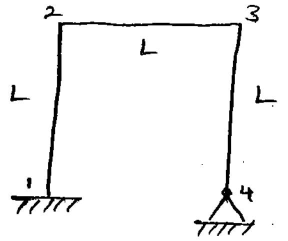
$$H_D = \frac{M_{CD}}{h_2}$$

$$\therefore \boxed{\frac{M_{AB} + M_{BA} - Ph}{h} + \frac{M_{CD}}{h} + P = 0}$$

P·g NO:- 52

4. $M_{21} = 0 + \frac{2EI}{L}\left(2\theta_2 + 0 - \frac{3\delta}{L}\right)$

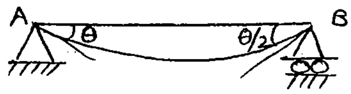
$= \frac{2EI}{L}\left(2\theta_2 - \frac{3\delta}{L}\right)$



P·g NO:- 53

4. $\theta_A = \theta$

$\theta_B = -\theta/2$



Span BA:-

$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$

$= \frac{2EI}{L}\left(2\left(-\frac{\theta}{2}\right) + \theta - \frac{3}{L}\left(\frac{\delta}{2} - \delta\right)\right)$

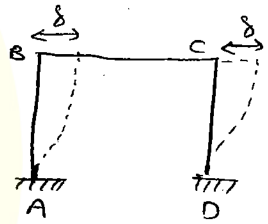
$= \frac{3EI\delta}{L^2}$

5. $M_{BC} = M^F_{BC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\delta}{L}\right)$

$M^F_{BC} = \frac{-15 \times 8^2}{12} = -80$ KN-m

$M_{BC} = -80 + \frac{2EI}{8}\left(2\theta_B + \theta_C - 0\right)$

$= 0.25EI\left(2\theta_B + \theta_C\right) - 80$

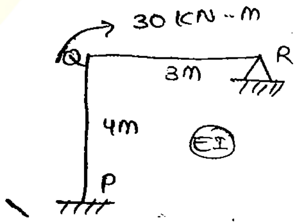


$\delta = 0$ in beam

7. $\theta_Q = \frac{\text{Moment}}{\text{R·S· of joint}}$

$\theta_Q = \frac{30}{\frac{4EI}{4} + \frac{3EI}{3}}$

$\theta_Q = \frac{15}{EI}$



8. Slope deflection equation for R·Q

$M_{RQ} = 0 = \frac{2EI}{3}\left(2\theta_R + \theta_Q\right)$

$\theta_R = \frac{-\theta_Q}{2} = \frac{-15}{2EI} = \frac{-7.5}{EI}$

Kani's method :-

It is proposed by Dr. Gasper kany in 1947. It is used for analysis of indeterminate structure and rigid jointed frames. It is an extension of slope deflection method. No. of equations to be solved is Nill.

It is an efficient method due to simplicity of moment distribution

1. Contribution of rotation moments will be distributed until the desire degree of accuracy is achieved.

$$M_{AB} = M^F_{AB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\delta}{L^2}$$

$$M_{AB} = M^F_{AB} + 2m'_{ab} + m'_{ba} - m\delta_{ab}$$

$$m'_{ab} = \frac{2EI\theta_A}{L}, \quad m'_{ba} = \frac{2EI\theta_B}{L} \longrightarrow \text{Rotation contribution.}$$

$$m\delta_{ab} = \frac{6EI\delta}{L^2} \longrightarrow \text{Displacement contribution.}$$

$$M_{BA} = M^F_{BA} + 2m'_{ba} + \cancel{2}m'_{ab} - m\delta_{ba}$$

$$m'_{ab} = \frac{-1}{2}\frac{K_{AB}}{\Sigma K} \longrightarrow \text{Rotation factor}$$

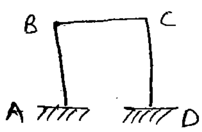
$$m'_{ab} = \frac{-1}{2}\frac{K_{AB}}{\Sigma K}\left[\Sigma M^F_{AB} + \Sigma m'_{ba} + \Sigma m\delta_{ab}\right]$$

$$\text{Rotation factor} = -\frac{1}{2}\left(\frac{K_{AB}}{\Sigma K}\right)$$

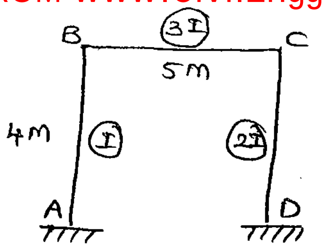
Note :-

1. Sum of the rotation factor of all the members meeting at one joint will be equal to $\left(\frac{-1}{2}\right)$
2. If any one of the end support is fixed take the rotation at that support is zero and the rotation contribution is also be zero.
3. If any end of the member is hinged or pinned it is convinient to assume as fixed and take relative stiffness as $\frac{3}{4}\frac{I}{gL}$
4. The displacement factor for a column of equal heights

$$\delta = \frac{-3}{2}\frac{K_{ab}}{\Sigma K}$$



EX:-) Displacement factor for only columns not beams



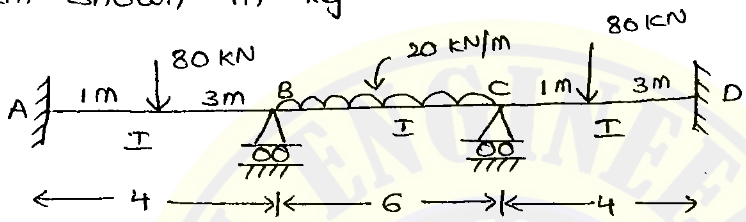
$$\delta = \frac{-3}{2} \frac{K_{AB}}{\Sigma K}$$

$$\delta_{AB} = \frac{-3}{2} \cdot \frac{I/4}{I/4 + 3I/4}$$

$$\delta_{AB} = \frac{-1}{2}$$

$$\delta_{CD} = -1$$

EX:- Calculate the final moments at a supports of a continuou beam shown in fig.



A.
$$M^F_{AB} = \frac{-80 \times 1 \times 3^2}{4^2} = -45 = M^F_{CD}$$

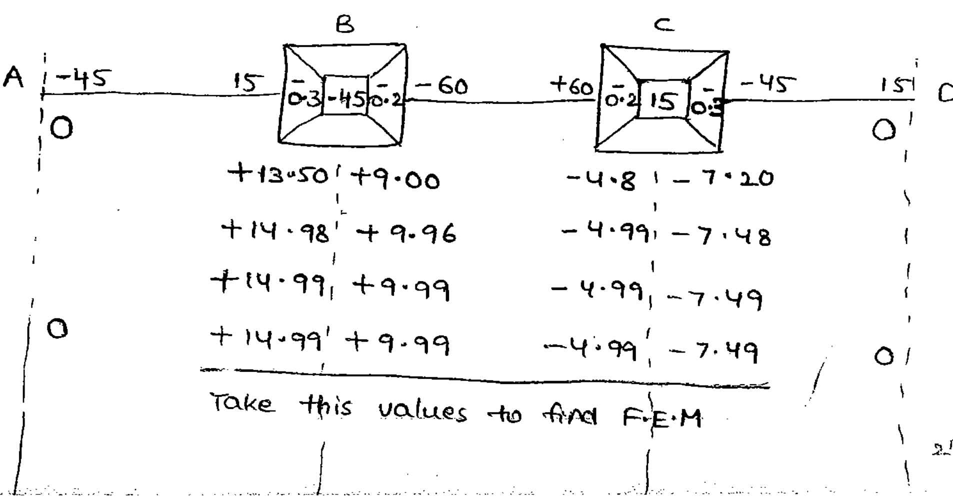
$$M^F_{BA} = \frac{80 \times 3 \times 1^2}{4^2} = 15 = M^F_{DC}$$

$$M^F_{BC} = \frac{-20 \times 6^2}{12} = -60$$

$$M^F_{CB} = +60$$

Joint	Member	K	R·F $\frac{-1}{2}\frac{K}{\Sigma K}$
Joint B	BA	I/4	-0·3
	BC	I/6	-0·2
Joint C	CB	I/6	-0·2
	CD	I/4	-0·3

∴ Assume, first tri at A, B, C, D are fixed so it is zero.



A		B			C		D
-45	15	0·3 -45 0·2	-60	+60	0·2 15 0·3	-45	15
0	+13·50		+9·00	-4·8		-7·20	0
	+14·98		+9·96	-4·99		-7·48	
	+14·99		+9·99	-4·99		-7·49	
0	+14·99		+9·99	-4·99		-7·49	0

Take this values to find F.E.M

∴ +15 − 60 = −45

+60 − 45 = +15

$$m'_{ba} = R \cdot F \left[\Sigma M^F_{AB} + \Sigma M'_{ba} + 0 \right.$$

1st trial

$$= -0{\cdot}3 \left[-45 + \underset{(A)}{0} + \underset{(C)}{0} \right]$$ ($\Sigma M'_{ba}$)

$$= +13{\cdot}50$$

$$= -0{\cdot}2 \left[+9{\cdot}0 + 15 + \underset{(D)}{0} \right]$$

$$= -4{\cdot}8$$

2nd trail

$$= \cancel{-45} \left(-45 + 0 + (-4{\cdot}99) \right)$$

I - trial:-

B

$m'_{ba} = (-45+0+0) \times (-0.3) =$

$m'_{bc} = (-45+0+0)\ (-0.2) =$

C

$m'_{cb} = (15+9.0+0)\ (-0.2) =$

$m'_{cd} = (15+9.0+0)\ (-0.3) =$

Final end moments:-

$$M_{AB} = M^F_{AB} + 2m'_{ab} + m'_{ba}$$

$$= -45 + 2(0) + 14.99$$

$$= -30.00$$

$$M_{BA} = M^F_{BA} + 2m'_{ba} + m'_{ab}$$

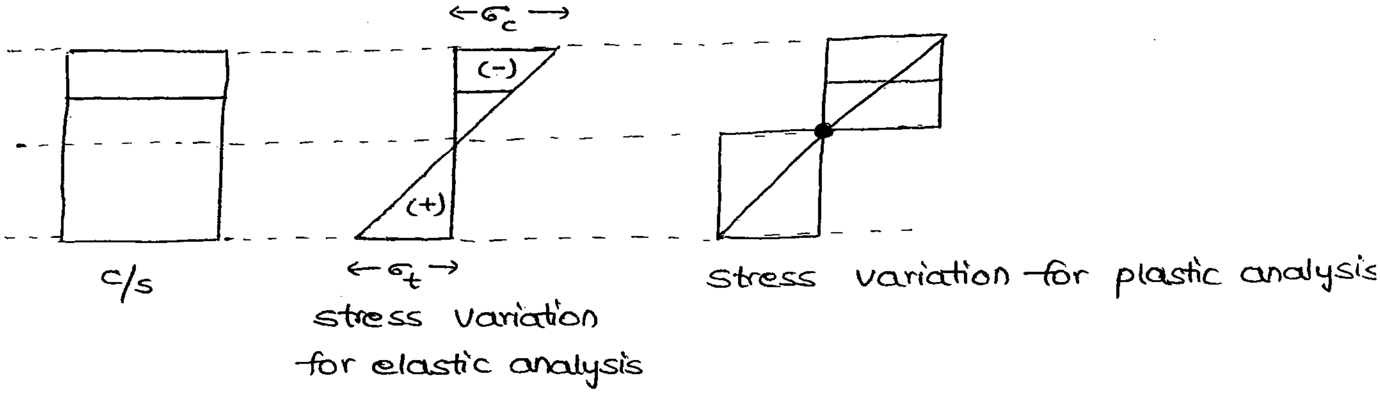
$$= +15 + 2(14.99) + 0$$

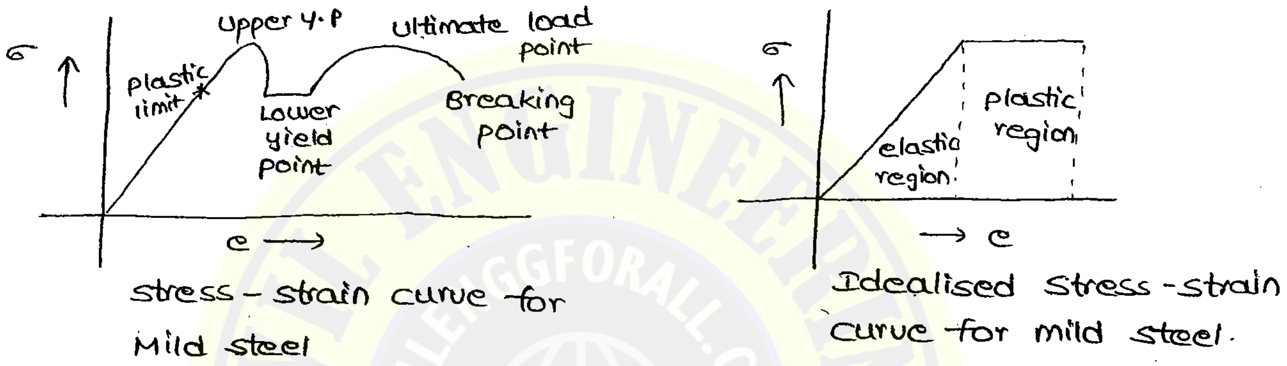
$$= 45$$

A -45	+15 B	-60	+60 C	-45	+15 D	
2x0	2x14.99	2x9.99	2(-4.99)	2(-7.49)	2x0	
14.99	0	-4.99	9.99	0	-7.49	
-30	45	-45	60	-60	7.5	→ F.E.M

UNIT - 8

** PLASTIC THEORY





1. As per Elastic theory the stresses will be considered upto proportional limit and hooke's law is valid. The stress variation along the cross section is linear and also shown in fig.
2. Even if any one of the fibre reaches the maximum stress the beam is said to have failed but infact the inner fibres are under stressed i.e., material is not utilised economically.
3. In plastic theory redistribution of stresses to inner fibres is considered due to plastic zone of materials like structural steel and mild steel. Hence the material is utilised economical
4. Plastic analysis of structures is based on the ultimate load whereas elastic theory is based on working loads.
5. Brittle material did not have a plastic zone.
6. Ductile materials have more plastic deformations is much large than the elastic deformation of structures before fracture.
7. For the purpose plastic design of steel structures the elastic limit and the lower yield point may be assumed numerically equal.

8. The strain at the first yield point of a structural steel is in the range of 10^{-3} to $\frac{1}{10}$ %.

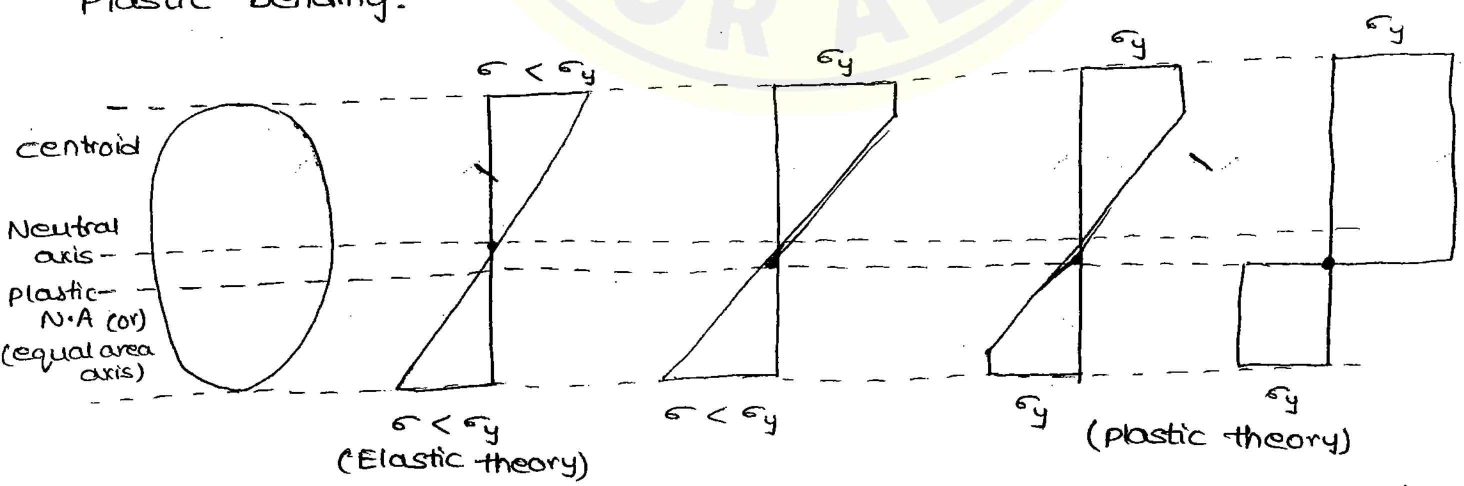
9. Plastic zone means strain increases without increasing the stress. Therefore once extreme fibres reaches maximum stress yielding continues i.e., strain goes on increasing even if inner fibres have same maximum stress.

10. When the stresses are redistributed i.e., all fibres reach their max. stress the variation of a stress is a rectangular stress distribution. Plastic hinge is assumed to formed at infinite rotation and then structures failed.

Validity of plastic theory:-

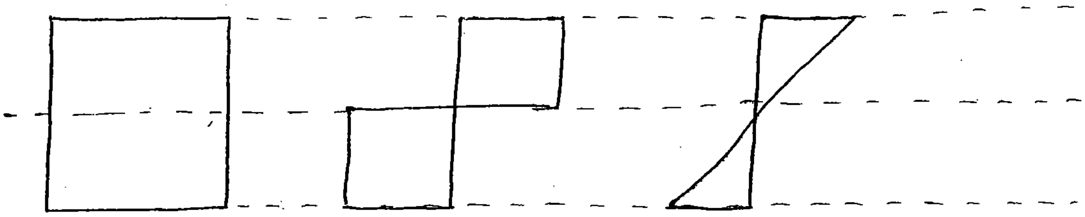
1. It is valid for ductile materials but not for brittle materials.
2. Structures subjected to impacted vibrations shall not be design for plastic theory.
3. In R.C.C. plastic theory can be applied with partial modification compared to steel structures.
4. It cannot be used for brass and plain concrete materials.
5. It is not valid for deep beams.
6. In plastic theory strength is the main criteria and can also be checked for deflection.

Plastic bending:-



1. Plastic hinge is an imaginary hinge developed when all the fibres reached their maximum stress.
2. Bending moment at a plastic hinge is not zero it is a plastic moment (M_p).

3. At the location of plastic hinge curvature is infinite

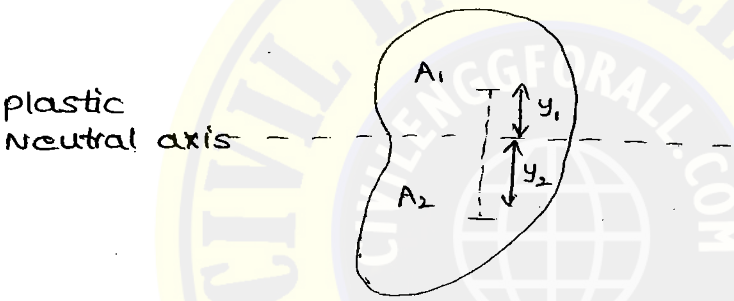


Note:-

1. Both elastic and plastic methods of analysis of indeterminate structu should satisfy the equilibrium condition.

Plastic Modulus (Z_p):-

1. It is the first moment of an area above and below equa area axis



$$Z_p = (A_1\bar{y}_1 + A_2\bar{y}_2)$$

$$A_1 = A_2 = \frac{A}{2}$$

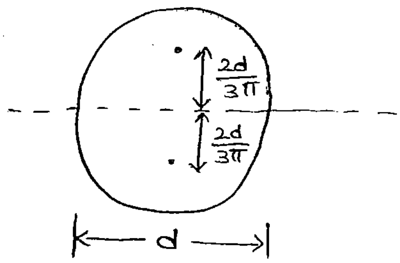
$$Z_p = \frac{A}{2}[\bar{y}_1 + \bar{y}_2]$$

$$\text{Plastic modulus} = \frac{\text{Plastic moment}}{\text{yield stress}}$$

$$\boxed{Z_p = \frac{M_p}{\sigma_y}}$$

Plastic modulus for various cross-section:-

1. Solid circular section:-

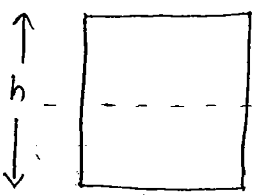


$$Z_p = \frac{A}{2}[\bar{y}_1 + \bar{y}_2]$$

$$= \frac{\pi}{4}\frac{d^2}{2}\left[\frac{2d}{3\pi} + \frac{2d}{3\pi}\right]$$

$$\boxed{Z_p = \frac{d^3}{6}}$$

2. Rectangular section:-



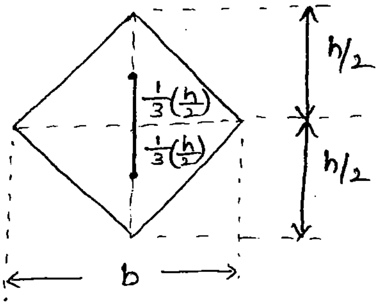
$$Z_p = \frac{bh}{2}\left[\frac{h}{4} + \frac{h}{4}\right]$$

$$\boxed{Z_p = \frac{bh^2}{4}}$$

3. Square section of side 'a'

$$\boxed{Z_p = \frac{a^3}{4}}$$

4. Rhombus (or) diamond section :-



$$Z_p = \frac{bh}{2} \times \frac{1}{2}\left[\frac{h}{6} + \frac{h}{6}\right]$$

$$A = \left(\frac{1}{2} \times b \times \frac{h}{2}\right) \times 2$$

$$= \frac{bh}{2}$$

$$\boxed{Z_p = \frac{bh^2}{12}}$$

Shape factor :-

It is the ratio of fully plastic moment of a section to the yield moment of the section.

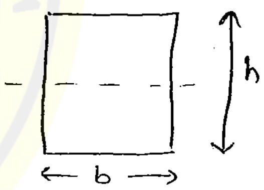
$$\text{Shape factor (S)} = \frac{M_p}{M} = \frac{Z_p}{Z}$$

Z = section modulus.

Shape factors for various cross sections:-

1. Rectangular section :-

$$S = \frac{Z_p}{Z} = \frac{\left(\frac{bh}{4}\right)}{\left(\frac{bh^2}{6}\right)}$$



$$\boxed{S = 1.5}$$

$$I = \frac{bh^3}{12}, \quad y = \frac{h}{2}$$

$$Z = \frac{I}{y} = \frac{bh^2}{6}$$

2. Square section:-

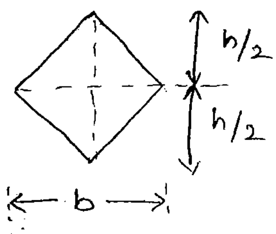
$$\boxed{S = 1.5}$$

3. Solid circular section:-

$$S = \frac{\left(d^3/6\right)}{\left(\frac{\pi d^3}{32}\right)}$$

$$\boxed{S = \frac{16}{3\pi} \text{ (or) } 1.699}$$

4. Diamond section :-



$\cancel{S =}$

$$I = 2\left[\frac{b \times \left(\frac{h}{2}\right)^3}{12}\right]$$

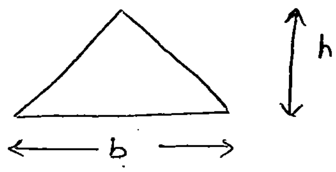
$$I = \frac{bh^3}{48}$$

$$z = \frac{I}{y} = \frac{\frac{bh^3}{48}}{h/2} = \frac{bh^2}{24}$$

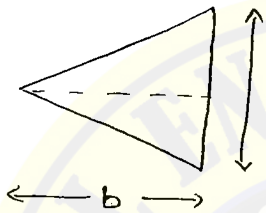
$$S = \frac{z_p}{z} = \frac{bh^2}{12} \times \frac{24}{bh^2} = 2$$

$$\boxed{S = 2}$$

5. Isosceles triangle:-

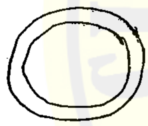


$$\boxed{S = 2.34}$$



$$\boxed{S = 2}$$

6. Tubular section:-



$S = 1.27$

7. Hallow circular section:-

$$S = 1.7\left[\frac{1-K^3}{1-K^4}\right]$$

$$\therefore K = \frac{r_2}{r_1}$$

r_2 = inner radius

r_1 = outer radius

8. Thin rectangular section:-

$$S = \frac{\left(b + \frac{h}{2}\right)}{\left(b + \frac{h}{3}\right)}$$

$$\boxed{S = 1.2}$$

9. I-section:-

$S = 1.12$ about strong axis (XX)

$S = 1.55$ about weak axis (YY)

10. H - section:-

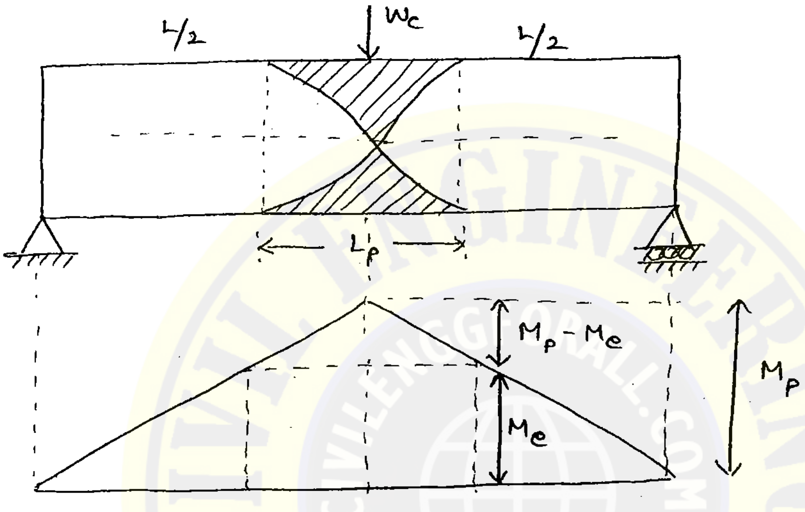
$\therefore S = 1.5$

Note:-

1. Generally for rolled steel sections shape factor ranges between 1.15 to 1.27

Length of plastic zone (L_p):-

It is the length of the beam in which redistribution of stresses occured either partly or fully



BMD

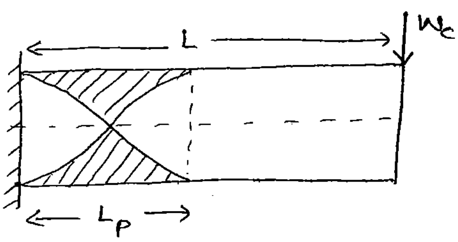
From the similar triangles

$$\frac{L_p}{(M_p - M_e)} = \frac{L}{M_p}$$

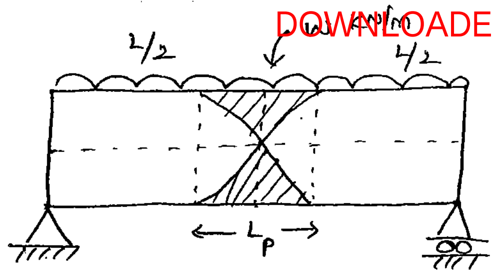
$$L_p = L\left[\frac{M_p - M_e}{M_p}\right]$$

$$\boxed{L_p = L\left[1 - \frac{1}{S}\right]}$$

Cantilever beam subjected to concentrated load at end



$$\boxed{L_p = L\left[1 - \frac{1}{S}\right]}$$



$$\boxed{L_p = L\sqrt{\left(1-\frac{1}{S}\right)}}$$

Note :-

1. If the cross section of simply supported beam with centra concentrated load is rectangle length of the plastic zone

$$\boxed{L_p = \frac{L}{3}}$$

2. If cross section of a simply supported beam with UDL is rectangle the length of the plastic zone

$$\boxed{L_p = \frac{L}{\sqrt{3}}}$$

Load factor :-

Ratio of collapse load (failure load) to working load is a load factor

$$\text{Load factor, } Q = \frac{P_c}{P} = \frac{M_p}{M} = \frac{Z_p \cdot \sigma_y}{Z \cdot \sigma}$$

$$= \left(\frac{\sigma_y}{\sigma}\right)\left(\frac{Z_p}{Z}\right)$$

$$\boxed{\text{Load factor} = (\text{Factor of safety})\ (\text{shape factor})}$$

for gravity load (D.L + L.L)

$$\boxed{\text{Load factor} = \frac{\text{F.S} \times \text{shape factor}}{1 + \%\ \text{of additional stresses}}}$$, for wind loads

Note :-

1. Load factor for structures without wind loads as per IS 800:2007 equals to 1.7
2. For structures with wind loads, Load factor = 1.3

Plastic collapse:-

1. If sufficient no. of plastic hinges are formed structure will be converted into a mechanism and then structure falls.
2. Plastic collapse depends on redandency of the structure
3. If no. of plastic hinges, $N < (D_s + 1)$ it is a partial collapse
4. If no. of plastic hinges, $N = D_s + 1$, it is a fully collapse.
5. If no. of plastic hinges, $N > (D_s + 1)$, it is a over comple collapse.

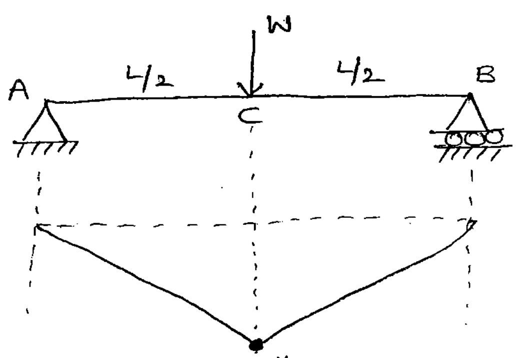
Conditions in plastic analysis:-

1. Equilibrium condition
2. yield criterion
3. Mechanism condition.

Locations of plastic hinges to be formed in the structures:-

1. At the point of maximum bending moment.
2. Under the point load in supported spans but not at the free end.
3. At rigid joints and fixed supports.
4. At a point where the cross section changes.
5. At a point where material changes.
6. If cross sectional area changes from one side to other side plastic hinge will be formed at a weaker side that is where cross sectional area is less and if there is a variable value of plastic moment where the cross sectional area changes lesser value of plastic moment will have to be considered.

Ex:- 1)



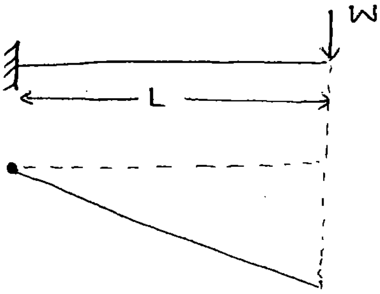
Formation of plastic hinge

$D_s = 0$

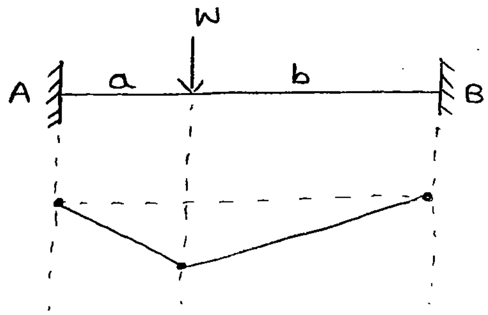
$N = D_s + 1$

$N = 1$

Ex:-2)



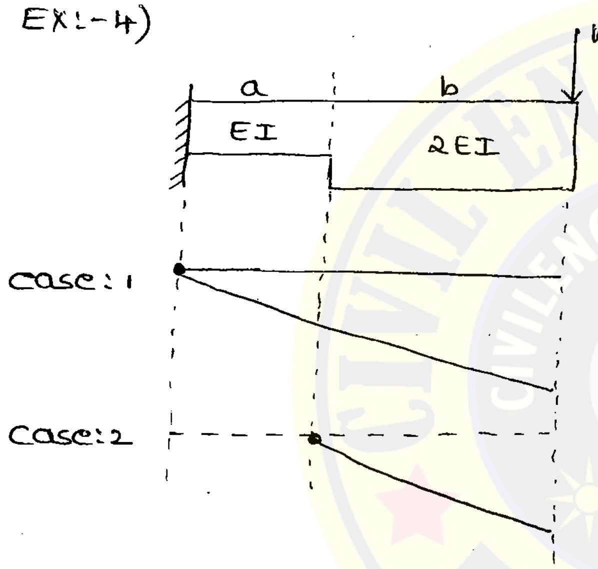
Ex:-3)



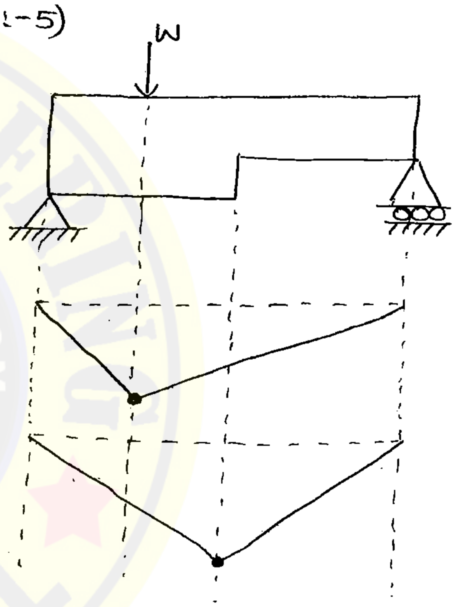
$D_s = 2$

$N = 3$

Ex:-4)

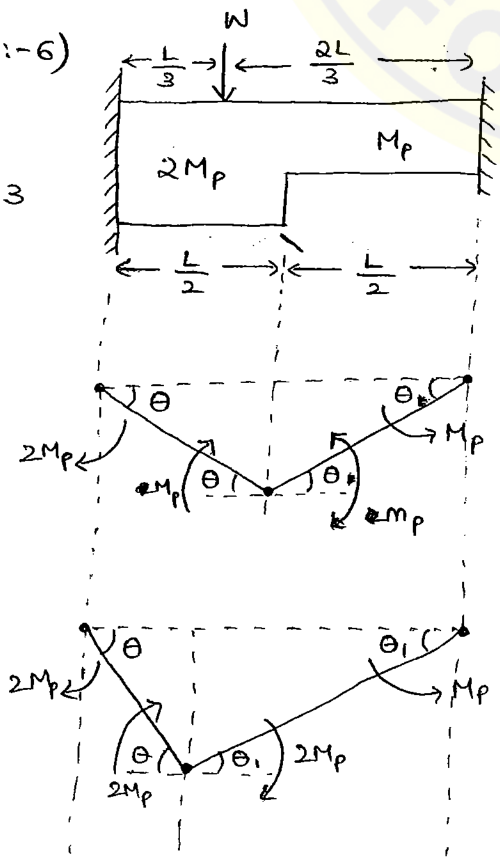


Ex:-5)

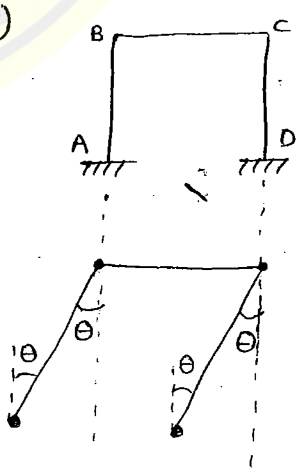


Ex:-6)

N = 3



Ex:-7)



$D_s = 3$

$N = 4$

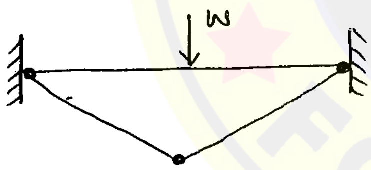
Mechanism :-

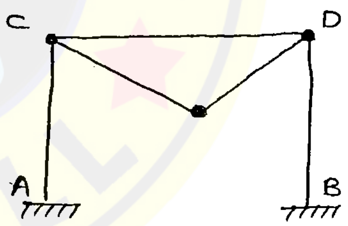
1. When a structure is subjected to system of loads and sufficient no. of plastic hinges are formed to transfer all the moments to the possible hinges, the segment of a beam between the plastic hinges are able to move with increase of the load such a system of arrangement is called as Mechanism.
2. Various failure modes are also called as Mechanisms.

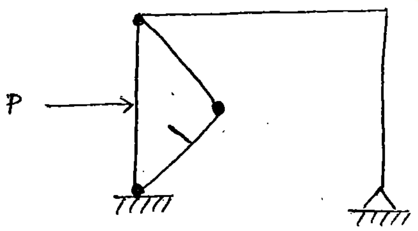
Mechanism

- Independent mechanism
  1. Beam mechanism
  2. Sway mechanism
  3. Gable mechanism
  4. Joint mechanism
- combined mechanism

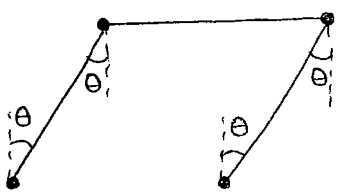
Beam mechanism :-



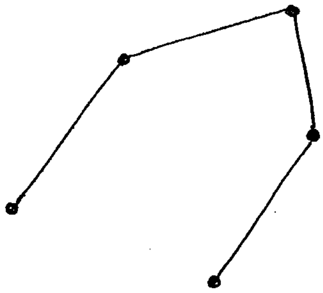




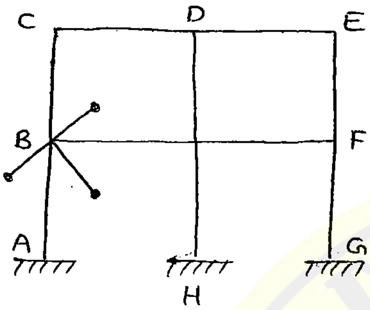
Sway mechanism :-



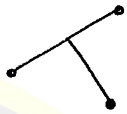
Gable mechanism:-



Joint mechanism:-



Joint B:-



Basic theorems for plastic analysis:-

1. Static method (Lower bound theorem):-

1. It satisfies the equilibrium condition.

2. $W \leq W_c$ and $M \ngtr M_p$ always

W = working load W_c = collapse load

3. It is proposed by "kist".

4. Design is safe side

2. kinematic method (or) Upper bound theorem (or) Virtual work method (or) Mechanism method:-

1. It satisfies the equilibrium criterion it is developed by GVODZER, GREENBERG & PRAGER.

2. Principle of kinematics is used

3. $W \geq W_c$, $M \leq M_p$

4. Design is unsafe

5. principle of Virtual work i.e., external workdone = internal workdone is applied.

$\therefore W_e = W_i$

procedure to calculate collapse load:-

1. Calculate the possible no. of plastic hinges for a given structure

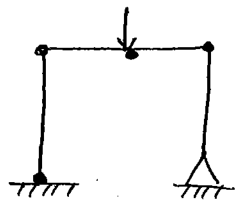
2. calculate the no. of independent mechanisms using

$$n = N - D_s$$

where

N = possible no. of plastic hinges

D_s = Degree of Redendancy.



$$N = D_s + 1$$
$$= 2+1$$
$$= 3$$
$$n = N - D_s$$
$$= 4 - 2$$
$$= 2$$

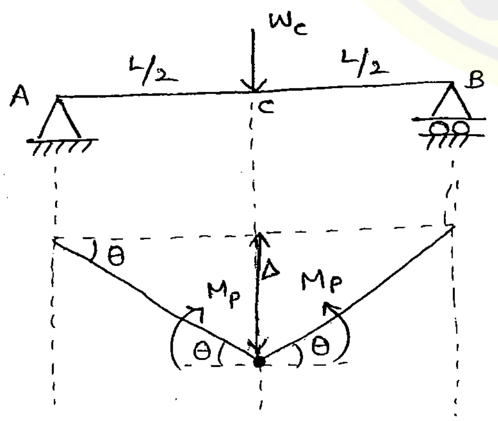
3. Calculate the collapse load for each meachanism by applying, $W_e = W_i$

4. Even if it is required go for combined mechanism and also calculate the collapse load.

5. Finalise the collapse load by taking the lesser values of above calculated collapse load.

Standard cases:-

1.



Static method:-

$$\frac{W_c \cdot l}{4} = M_p$$

$$\boxed{W_c = \frac{4M_p}{l}}$$

Kinematic method:-

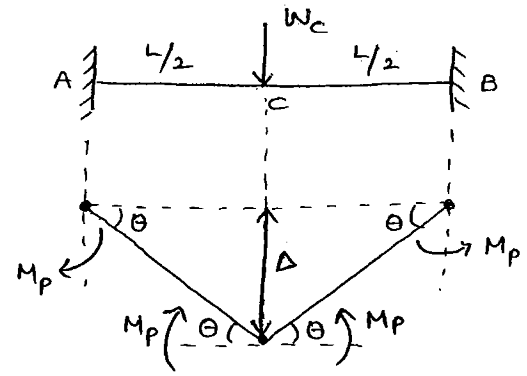
$$W_e = W_i$$

$$W_c \cdot \Delta = M_p\theta + M_p\theta$$

$$W_c \cdot \left(\frac{L}{2}\cdot\theta\right) = 2M_p\theta$$

$$\boxed{W_c = \frac{4M_p}{L}}$$

2.



Static method:-

$$W_c \cdot \frac{L}{4} = 2M_p$$

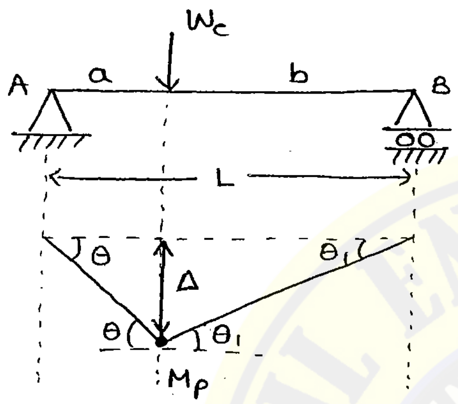
$$\boxed{W_c = \frac{8M_p}{L}}$$

Kinematic method:-

$$W_c \cdot \frac{L}{2} \cdot \theta = 4M_p \cdot \theta$$

$$\boxed{W_c = \frac{8M_p}{L}}$$

3.



Static method:-

$$\frac{W_c \cdot a \cdot b}{L} = M_p$$

$$\boxed{W_c = \frac{M_p L}{ab}}$$

Kinematic method:-

$$W_e = W_i$$

$$\Delta = a \cdot \theta = b \cdot \theta_1$$

$$\theta_1 = \frac{a}{b}\theta$$

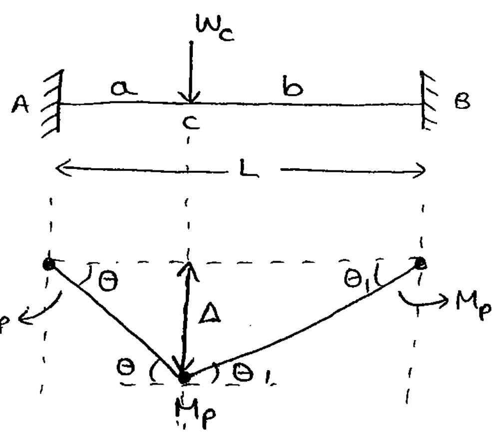
$$W_c \cdot \Delta = M_p \cdot \theta + M_p \theta_1$$

$$W_c(a\theta) = M_p\left(\theta + \frac{a}{b}\theta\right)$$

$$W_c = \frac{M_p\left(\frac{(a+b)\theta}{b}\right)}{a\theta}$$

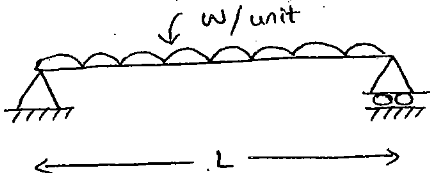
$$\boxed{W_c = \frac{M_p \cdot L}{ab}}$$

4.



$$\frac{W_c \cdot a \cdot b}{L} = 2M_p$$

$$\boxed{W_c = \frac{2M_p L}{ab}}$$

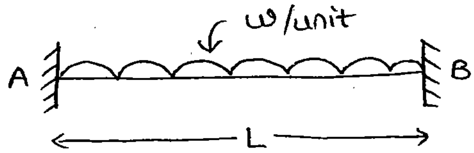
5. 

$$\frac{w_c \cdot L^2}{8} = M_p$$

$$w_c = \frac{8M_p}{L^2}$$

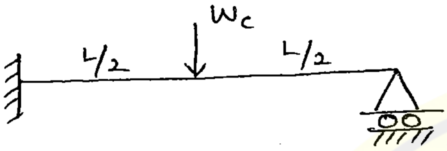
(or)

$$\boxed{W_c = \frac{8M_p}{L}}$$

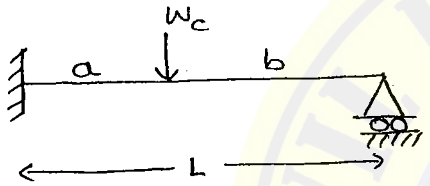
6. 

$$\frac{w_c L^2}{8} = 2M_p$$

$$\boxed{w_c = \frac{16M_p}{L^2}} \quad \text{(or)} \quad \boxed{W_c = \frac{16M_p}{L}}$$

7. 

$$\boxed{W_c = \frac{6M_p}{L}}$$

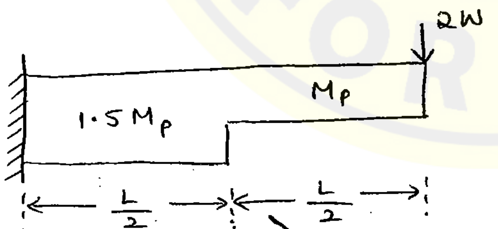
8. 

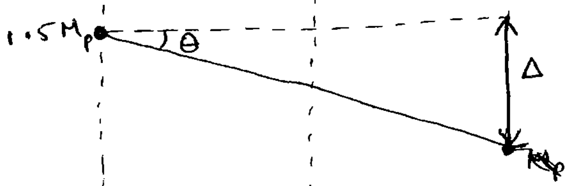
$$\boxed{W_c = \frac{M_p(L+b)}{ab}}$$

9. 

$$\boxed{W_c = \frac{11.656M_p}{L}} \quad \text{(or)} \quad \boxed{w_c = \frac{11.656M_p}{L^2}}$$

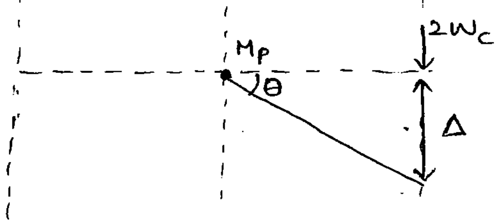
Ex:-) Calculate the collapse load for the beam shown in fig.





$$2W_c \Delta = 1.5M_p \cdot \theta$$

$$W_c = \frac{0.75M_p}{L}$$



$$W_e = W_i$$

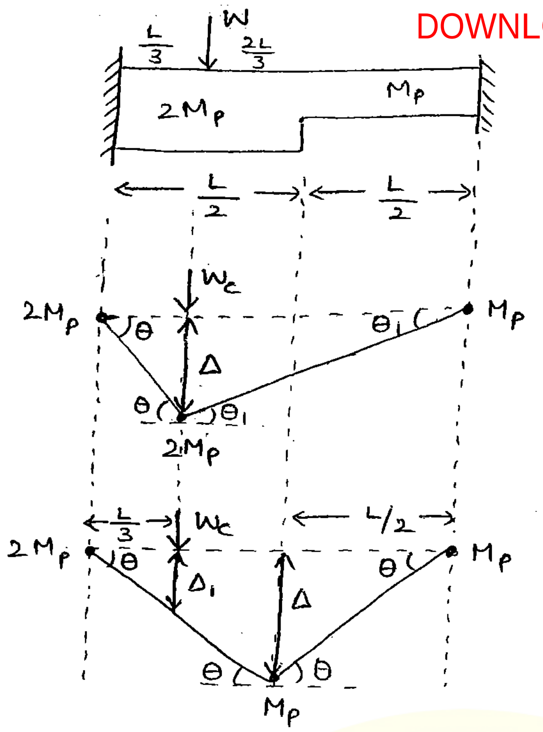
$$2W_c \cdot \Delta = M_p \cdot \theta$$

$$2W_c\left(\frac{L}{2}\theta\right) = M_p \cdot \theta$$

$$W_c = \frac{M_p}{L}$$

$$\boxed{\therefore W_c = \frac{0.75M_p}{L}}$$

Ex:- 2)



Case: 1

Case: 2

Case: 1

$$w_e = w_i$$

$$\Delta = \frac{L}{3}\theta = \frac{2L}{3}\theta_1$$

$$\theta_1 = \frac{1}{2}\theta$$

$$W_c\,\Delta = 2M_p\theta + 2M_p\theta + 2M_p\theta_1 + M_p\theta_1$$

$$W_c\left(\frac{L}{3}\theta\right) = 4M_p\theta + 3M_p\theta_1$$

$$= 4M_p\theta + 3M_p\left(\frac{1}{2}\theta\right)$$

$$W_c = \frac{16.5\,M_p}{L}$$

Case: 2

$$\frac{\Delta}{\Delta_1} = \frac{L/2}{L/3} = \cancel{\frac{2}{3}}\ \frac{3}{2}$$

$$\boxed{\Delta_1 = \frac{2}{3}\Delta}$$

$$w_e = w_i$$

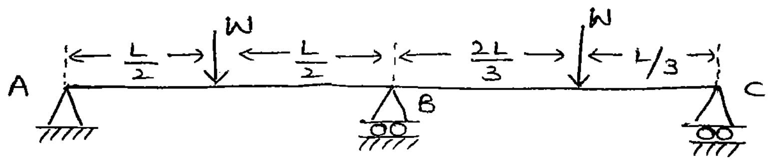
$$W_c \cdot \Delta_1 = 2M_p\theta + M_p\theta + M_p(\theta+\theta)$$

$$W_c\left(\frac{L}{3}\theta\right) = 5M_p\theta$$

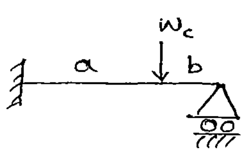
$$W_c = \frac{15\,M_p}{L}$$

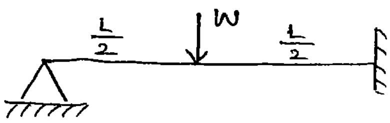
$\therefore$ collapse load, $\boxed{W_c = \frac{15\,M_p}{L}}$

Ex:-)

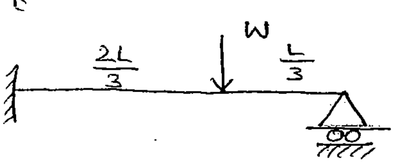


First assume interior member, B is fixed





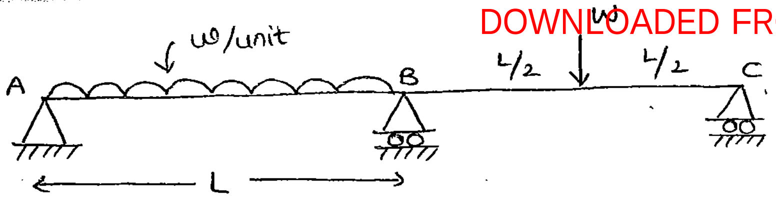
$$W_c = \frac{6M_p}{L}$$

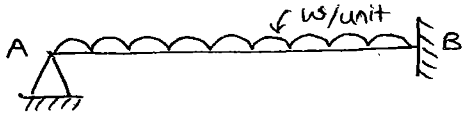


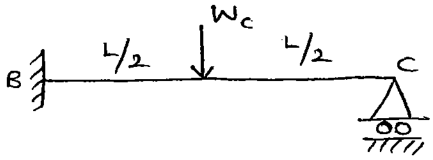
$$W_c = \frac{M_p(L+b)}{ab}$$

$$W_c = \frac{M_p\left(L+\frac{L}{3}\right)}{\left(\frac{2L}{3}\right)\left(\frac{L}{3}\right)} = \frac{6M_p}{L}$$

Ex:-)







$$W_c = \frac{11.656 M_p}{L}$$

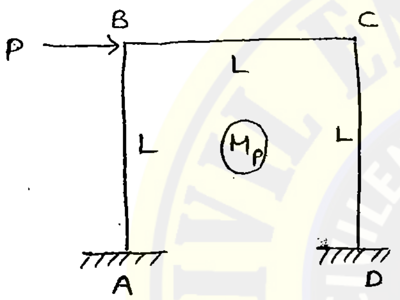
$$w_c = \frac{11.656 M_p}{L^2}$$

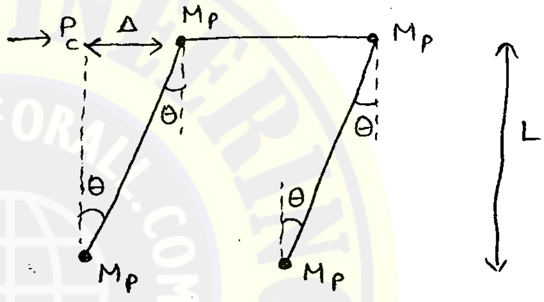
$$W_c = \frac{6M_p}{L}$$

$$\therefore \text{ collapse load, } W_c = \frac{6M_p}{L}$$

Frames:-

Ex:-)





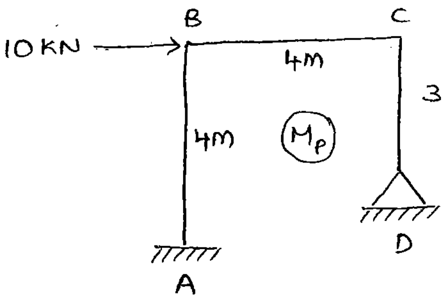
$$W_e = W_i$$

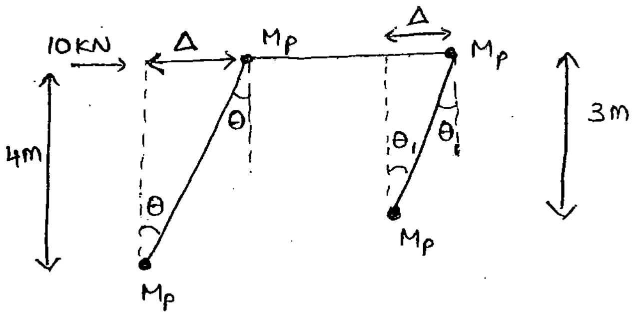
$$P_c \cdot \Delta = 4 M_p \theta$$

$$P_c (L\theta) = 4 M_p \theta$$

$$\boxed{P_c = \frac{4 M_p}{L}}$$

Ex:-2) Plastic moment for the frame shown in fig. is





$$\Delta = 4\theta = 3\theta_1$$

$$\theta_1 = \frac{4}{3}\theta$$

$$W_e = W_i$$

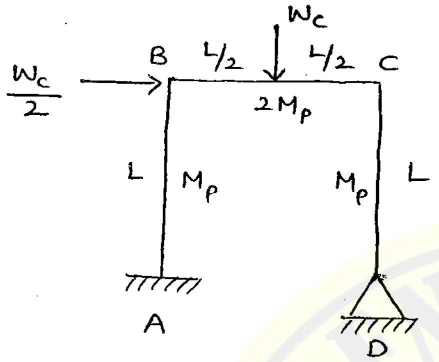
$$W_c \Delta = 2M_p\theta + 2M_p\theta_1$$

$$10(4\theta) = 2M_p\left(\theta + \frac{4}{3}\theta\right)$$

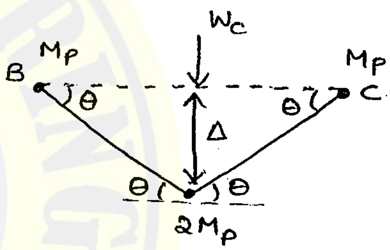
$$40\theta = 2M_p\left(\frac{7\theta}{3}\right)$$

$$\boxed{M_p = 12\ kN\text{-}m}$$

Ex:-)



A. Beam mechanism:-

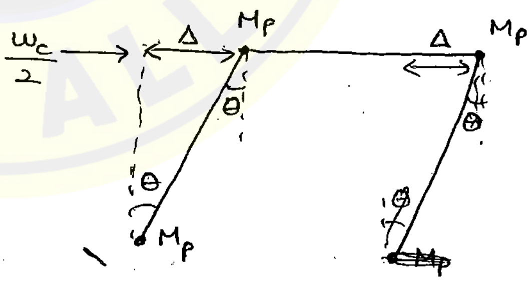


$$W_c \cdot \Delta = M_p\theta + M_p\theta + 2M_p(\theta+\theta)$$

$$W_c\left(\frac{L}{2}\theta\right) = 6M_p\theta$$

$$\boxed{W_c = \frac{12M_p}{L}}$$

Sway mechanism:-

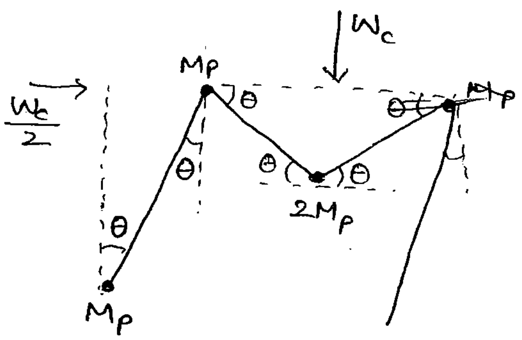


$$W_c \cdot \Delta = M_p\theta + M_p\theta + M_p\theta$$

$$W_c(L\theta) = 3M_p\theta$$

$$\boxed{W_c = \frac{3M_p}{L}}$$

Combined mechanism:-



$$W_{c_1}\Delta + W_{c_2}\Delta_1 = M_p(\theta+\theta) + 2M_p(\theta+\theta) + M_p\theta$$

$$W_c\left(\frac{L}{2}\theta\right) + \frac{W_c}{2}(L\theta) = 7M_p\theta$$

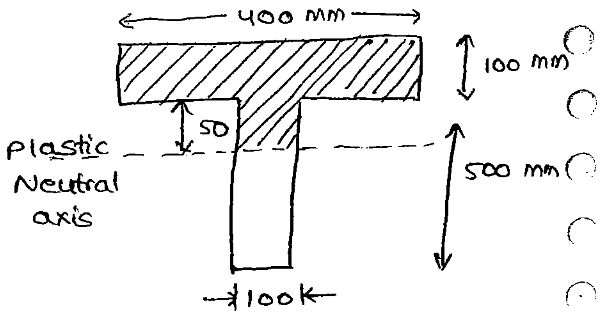
$$\boxed{W_c = \frac{7M_p}{L}}$$

$D_s = 5 - 3 = 2$

$N = 2 + 1 = 3$

P.g NO:- 71

14. Equal area axis:



Top flange:-

$A = 400 \times 100 = 40000 \ mm^2$

Web:-

$A = 500 \times 100 = 50000 \ mm^2$

Total area $= 40000 + 50000$

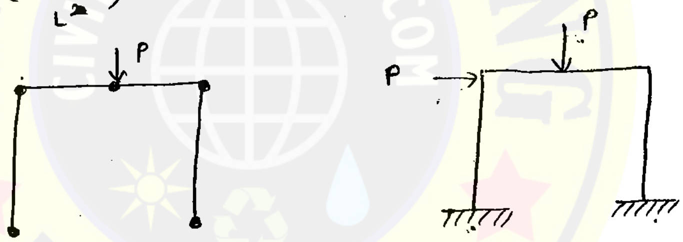
$A = 90000$

$\frac{A}{2} = 45000$

29.

$$\frac{(W_c)_1}{(W_c)_2} = \frac{\frac{11.656 M_p}{L^2}}{\left(\frac{8M_p}{L^2}\right)} = 1.457$$

33.



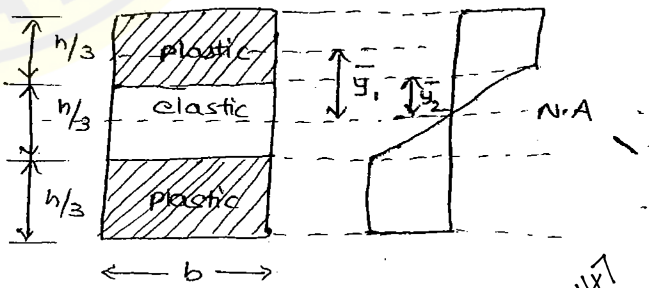
$n = N - D_s$

$N = 5 \ (\bullet)$

$D_s = 6 - 3 = 3$

$n = 5 - 3 = 2$

34. Moment Resistance = (Moment of an area about N.A) × f_y



Moment of an area about N.A =

$$= 2\left[\left(b \times \frac{h}{3}\right) \times (\bar{y}_1) + \left(\frac{1}{2} \times b \times \frac{h}{6}\right) \times \underbrace{\left(\frac{2}{3} \cdot \frac{h}{6}\right)}_{\bar{y}_2}\right]$$

$$= 2\left[\frac{bh}{3}\left(\frac{h}{6} + \frac{h}{6}\right) + \frac{bh}{12}\left(\frac{h}{9}\right)\right]$$

$$= 2\left(\frac{bh^2}{9} + \frac{bh^2}{108}\right)$$

$$= 2\left[\frac{12bh^2 + 1bh^2}{108}\right]$$

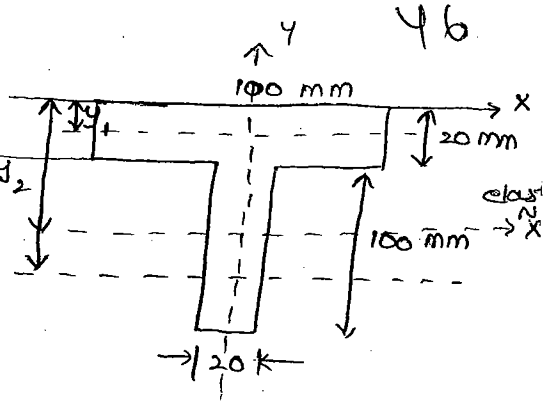
$$= \frac{13}{54} bh^2$$

$$M.R = \left(\frac{13bh^2}{54}\right) f_y = \frac{13 f_y b h^2}{54}$$

2. $$\bar{y}_{\text{from top}} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(100\times 20)(10) + (100\times 20)(70)}{(100\times 20)+(100\times 20)}$$

$= 40$ mm



$\therefore$ Difference $= 40 - 20 = 20$ mm

3. $Z_p = 5\times 10^{-4}\ m^3$ $\quad S = 1.2 \quad M_p = 120$ KN-m

$$M_p = \sigma_y \cdot Z_p$$

$$\sigma_y = \frac{M_p}{Z_p} = \frac{120\times 10^6}{(5\times 10^{-4})10^9}$$

$= 240\ N/mm^2$

4. $S = 1.1$ to 1.2
≈ 1.12

$$L_p = L\left(1 - \frac{1}{S}\right)$$

$$= L\left(1 - \frac{1}{1.12}\right)$$

$$= L\left(\frac{1.12 - 1}{1.12}\right)$$

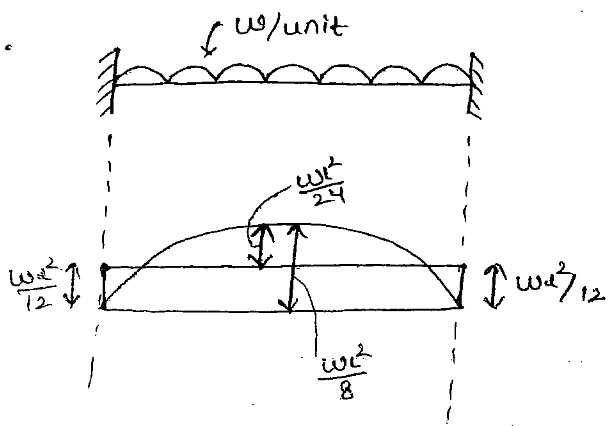
$$L_p = \frac{3L}{28} = 0.107L = \frac{L}{8}$$

5. Load factor (Q) = $\dfrac{\text{Factor of safety} \times \text{shape factor}}{1 + \%\ \text{additional stresses}}$

$$= \frac{1.5\times 1.12}{1+0.2}$$

$= 1.4$

6.



$W_c = \dfrac{16 M_p}{L} \rightarrow$ ① plastic analysis

At the elastic limit, elastic moment is $\frac{1}{2}$ of the end moment

$$\frac{W_c \cdot L}{8} = \frac{3}{2} M_e$$

$W_c = \dfrac{12 M_e}{L} \rightarrow$ ②

$$\frac{①}{②} = \frac{16M_p}{L} \times \frac{L}{12M_e}$$

$$= \frac{4}{3}\left(\frac{M_p}{M_e}\right)$$

$$= \frac{4}{3}(\text{shape factor})$$

$$= \frac{4}{3}(1.5) \qquad \therefore S = 1.5 \text{ for rectangle}$$

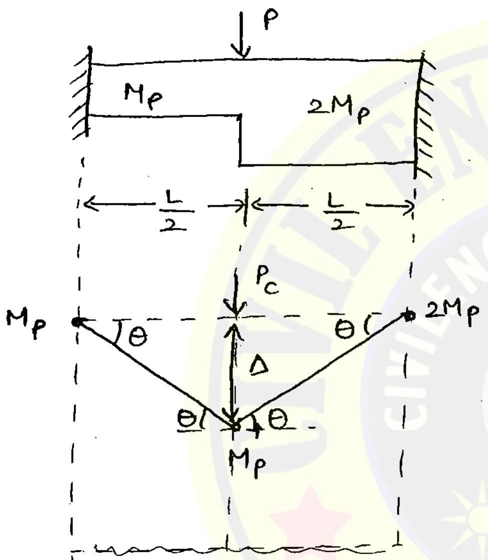
$$= 2$$

Collapse, $W_c = 2W$

$= 2 \times 10$ kN/m

$= 20$ kN/m

8.

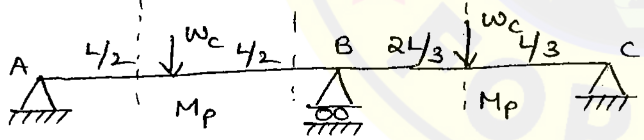


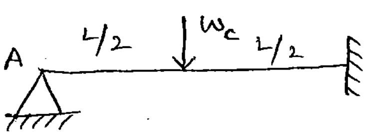
$$W_c \cdot \Delta = M_p\theta + 2M_p + 2M_p$$

$$W_c\left(\frac{L}{2}\theta\right) = \cancel{4}\, 5M_p\theta$$

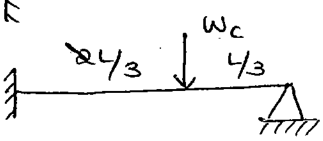
$$W_c = \frac{10M_p}{L}$$

9.





$$W_c = \frac{6M_p}{L}$$

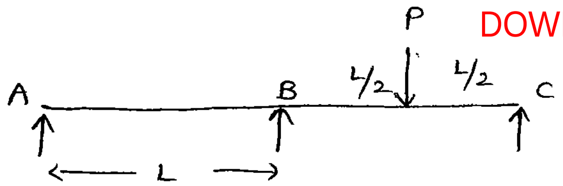


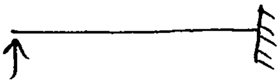
$$W_c = \frac{M_p(\cancel{a}L+b)}{ab}$$

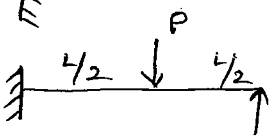
$$= \frac{M_p\left(L+\frac{L}{3}\right)}{\left(\frac{2L}{3}\right)\left(\frac{L}{3}\right)}$$

$$= \frac{6M_p}{L}$$

10.

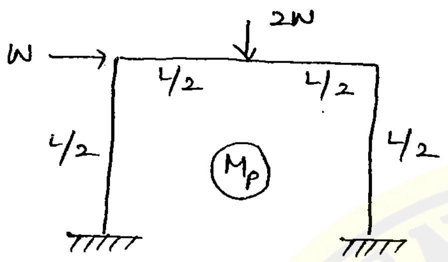




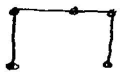


$$\boxed{P_c = \frac{6M_p}{L}}$$

11.



Total = 5



$D_s = 6-3=3$

$N = 4$

Min no. of hinges possible to convert a struct into mechani

No. of possible plastic hinges

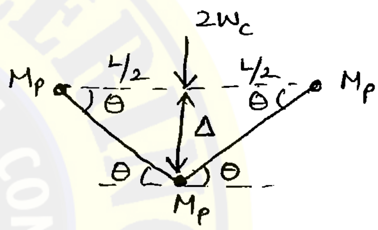
$n = N - D_s$

$= 5 - 3 = 2$

Beam mechanism:-

$$2W_c\left(\frac{L}{2}\theta\right) = 4M_p\theta$$

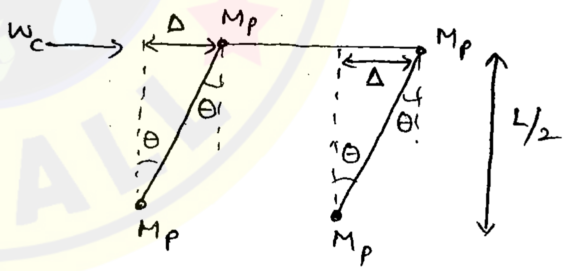
$$W_c = \frac{4M_p}{L}$$



Sway mechanism:-

$$W_c\left(\frac{L}{2}\theta\right) = 4M_p\theta.$$

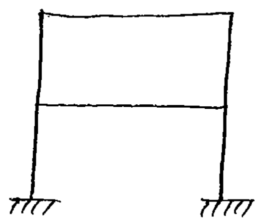
$$W_c = \frac{8M_p}{L}$$

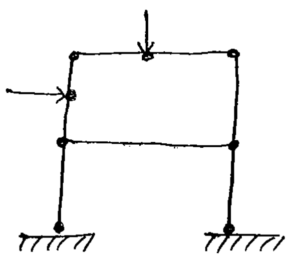


∴ collapse load, $\boxed{W_c = \frac{4M_p}{L}}$

P.9 No:-75

4.





No. of possible plastic hinges = 9

$D_s = 6-3 = 3$

$D_{si} = 3 \times 1 = 3$

$D_s = 3+3 = 6$

$N = D_s + 1 = 6+1 = 7$

MATRIX METHOD

1. Matrix method are used to analyse the statically indeterminate structures of higher

2. In statically indeterminate structures unknown can be either redundant forces or joint displacements. Therefore these redundants can easily be calculated by Matrix method

Flexibility (or) Force (or) compatability method :-

1. In these method the redundants are support reactions (forces or couples)

2. Flexibility is an amount of displacement required to cause a unit force.

3. Flexibility coefficient is an amount of displacement developed in the direction of redundant force due to unit redendent force.

$$F = \frac{\Delta}{P}$$

4. Displacement developed at i^{th} throat due to unit force applied at j^{th} throat.

F_{ij} = Flexibility coefficient.

$$[F] = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$F_{12} = F_{21}$$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

Stiffness method (or) Displacement (or) equilibrium method:-

1. In this method unknowns are joint displacement (deflections or rotations)

2. Stiffness is an amount of force required to cause unit displacement

$$K = \frac{P}{\Delta}$$

3. Stiffness coefficient is an amount of force developed in the direction of redundent displacement due to a unit redundent displacement.

K_{ij} = stiffness coefficient

$$[P] = [K][\Delta]$$

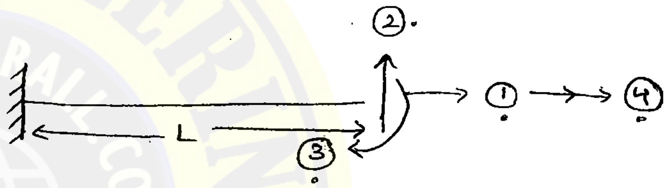
$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

$$K_{12} = K_{21}$$

Note:-

Stiffness is an inverse of flexibility, therefore the product of stiffness and flexibility is equal to 1.

1. Axial deformation:-



$$F = \frac{\Delta}{P}$$

$$\Delta = \frac{PL}{AE}$$

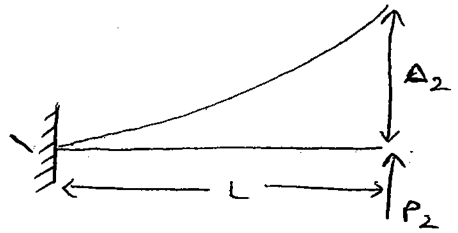
a. Axial flexibility:-

$$\therefore F_{11} = \frac{L}{AE}$$

b. Axial stiffness:-

$$K_{11} = \frac{AE}{L}$$

2. Transverse displacement :-



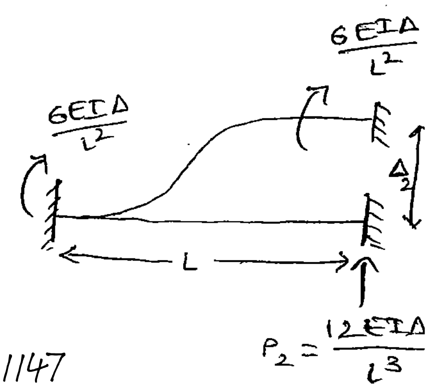
$$\Delta_2 = \frac{P_2 L^3}{3EI}$$

$\Delta = \dfrac{\omega L^3}{3EI}$ for cantilev.

a. Transverse flexibility :-

$$F_{22} = \frac{L^3}{3EI}$$

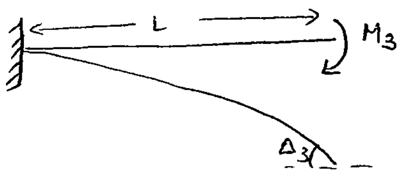
b. Transverse stiffness :-



$K_{22} = \dfrac{12EI}{L^3}$, when far end is fixed

$K_{22} = \dfrac{3EI}{L^3}$, when far end is hinged

3. Flexural displacement :-

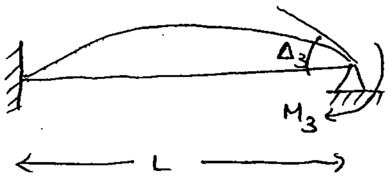


$$\Delta_3 = \frac{M_3 \cdot L}{EI}$$

a. Flexural flexibility :-

$$F_3 = \frac{L}{EI}$$

b. Flexural stiffness :-



$$M_3 = \frac{4EI\Delta_3}{L}$$

$K_{33} = \frac{4EI}{L}$ when far end is fixed

$K_{33} = \frac{3EI}{L}$ when far end is hinged

4. Torsional displacement :-

$$\theta = \frac{TL}{GJ}$$

a. Torsional flexibility :-

$$F_{44} = \frac{L}{GJ}$$

b. Torsional stiffness :-

$$K_{44} = \frac{GJ}{L}$$

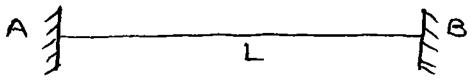
Flexibility method procedure :-

1. Calculate the degree of redendancy for a given structure (D_s)
2. Remove the excess redendents and show a determinate structure
3. Assign the coordinates for the redendants 1, 2, 3, - - - - n
4. Apply the unit force in the directions of coordinates 1 x 1 and calculate the displacements developed in the directions of choosen coordinates.
5. Formulate the flexibility matrix by calculating flexibility coefficients.

Note:-

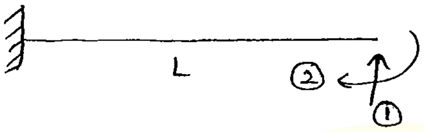
If degree of static indeterminacy of a structure is 'n' then size of flexibility matrix is n x n.

Ex:- Develop the flexibility matrix for a fixed beam of span 'L' by choosing the redendance at any one of the fixed support.

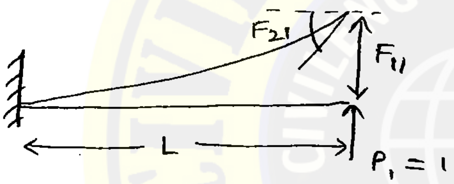


$D_s = 2$

Released structure

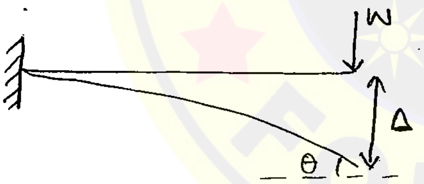


$P_1 = 1$, $P_2 = 0$



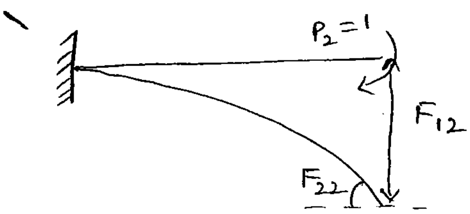
$F_{11} = \frac{L^3}{3EI}$

$F_{21} = \frac{-L^2}{2EI} = F_{12}$

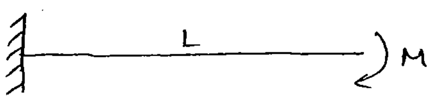


$\Delta = \frac{WL^3}{3EI}$ $\theta = \frac{WL^2}{2EI}$ For cantilever bea

$P_2 = 1$, $P_1 = 0$



$F_{22} = \frac{L}{EI}$



$\theta = \frac{ML}{EI}$ $\Delta = \frac{ML^2}{2EI}$

$$[F] = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \begin{bmatrix} L^3/3EI & \frac{-L^2}{2EI} \\ \frac{-L^2}{2EI} & \frac{L}{EI} \end{bmatrix}$$

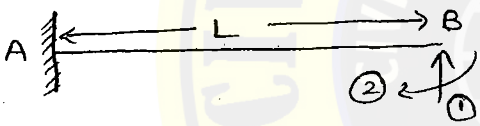
Stiffness method procedure:-

1. Calculate the degree of freedom of a given structure by neglecting axial deformations (D_K)
2. Choose the redundent displacements and assign coordinates 1, 2, 3, ... n
3. Remove all independent displacements to obtain the restrained structure.
4. Apply unit displacement at all the choosen coordinates and calculate the forces or moments developed.
5. Formulate the stiffness matrix by calculating the stiffness coefficients.

Note:-

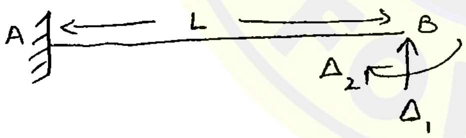
If the value of D_K is 'n', the size of stiffness matrix is $\frac{n}{n}$

Ex:- Develop stiffness matrix for a cantilever beam of span 'L'



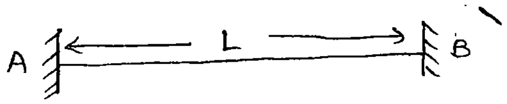
for the choosen displacements shown in fig.

A.

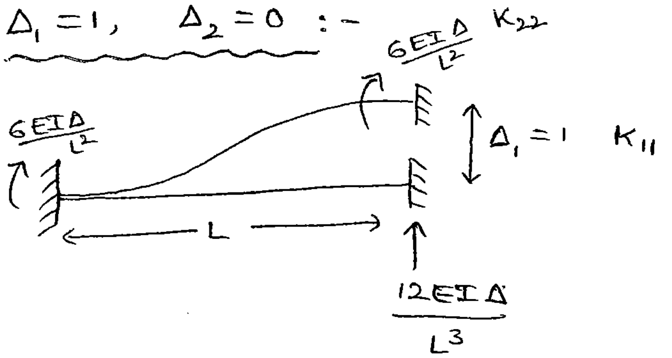


Δ_1 Δ_2 are the choosen displacements

Restrained structure



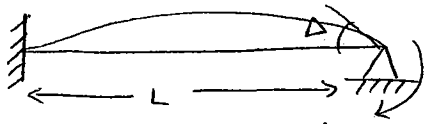
$\Delta_1 = 1$, $\Delta_2 = 0$:-



$$K_{11} = \frac{12EI}{L^3}$$

$$K_{21} = \frac{6EI}{L^2} = K_{12}$$

$\Delta_2 = 1, \quad \Delta_1 = 0$:-



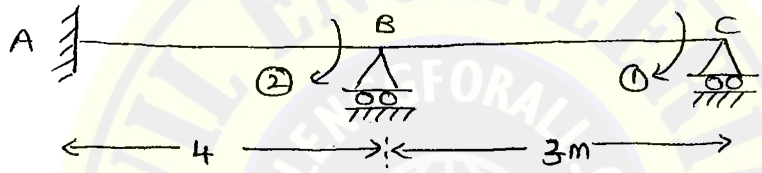
$\frac{4EI}{L} = K_{22}$

$K_{22} = \frac{4EI}{L}$

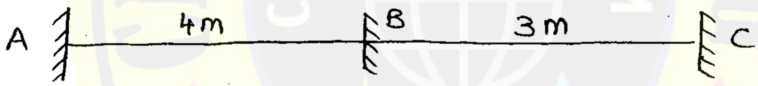
$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 12EI/L^3 & 6EI/L^2 \\ 6EI/L^2 & 4EI/L \end{bmatrix}$$

Ex:- Develop the stiffness matrix of the beam shown in fig.

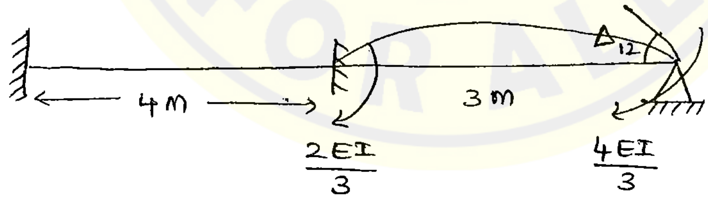


A.



(Restrained structure

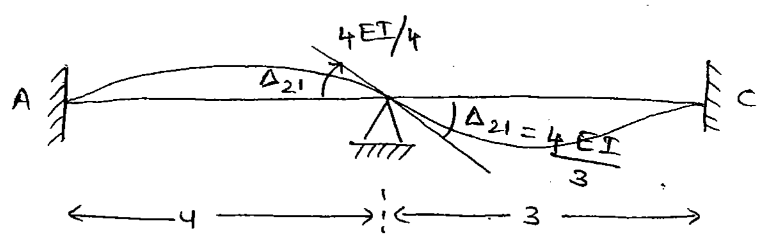
$\Delta_1 = 1, \quad \Delta_2 = 0$:-



$K_{11} = \frac{4EI}{3}$

$K_{21} = \frac{2EI}{3} = K_{12}$

$\Delta_2 = 1, \quad \Delta_1 = 0$:-



$K_{22} = EI + \frac{4EI}{3}$

$= \frac{7EI}{3}$

$$[K] = \begin{bmatrix} 4EI/3 & 2EI/3 \\ 2EI/3 & 7EI/3 \end{bmatrix}$$

$$= \frac{EI}{3}\begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$$

Ex:- What the flexibility matrix for the above problem:-

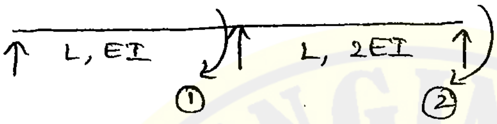
$$[F] = [K]^{-1}$$

$$= \left[\frac{EI}{3} \begin{pmatrix} 4 & 2 \\ 2 & 7 \end{pmatrix} \right]^{-1}$$

$$[F] = \frac{1}{8EI} \begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix}$$

Ex:- Stiffness coefficient K_{21} is [b].

a) $\frac{2EI}{L}$ b) $\frac{4EI}{L}$ c) $\frac{3EI}{L}$ d) $\frac{7EI}{L}$



A.

$K_{11} = \frac{4EI}{L}$

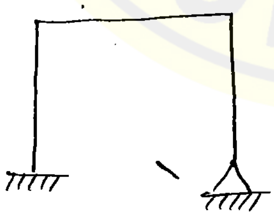
$\frac{4(2EI)}{L}$

$\frac{2(2EI)}{L} = \frac{4EI}{L} = K_{21}$

Ex:- The size of flexibility matrix for the frame shown in figure is [d]

a) 3 X 3 b) 4X4 c) 5 X 5 d) 2X2

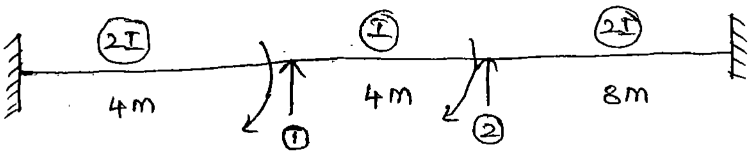
A.



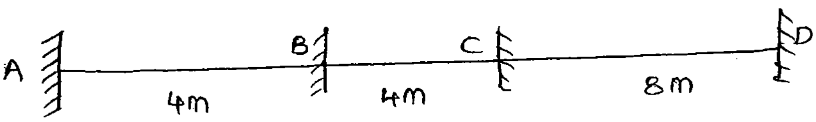
$D_s = 5 - 3 = 2$

$\therefore$ 2X2

Ex:- Develop the stiffness matrix for the beam shown in fig.

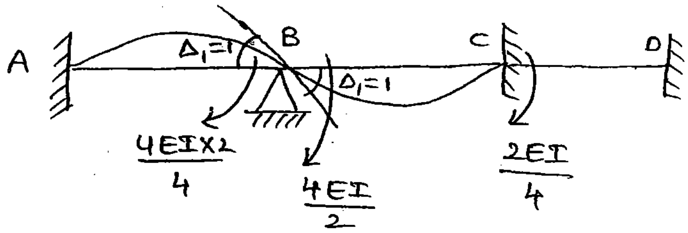


A.



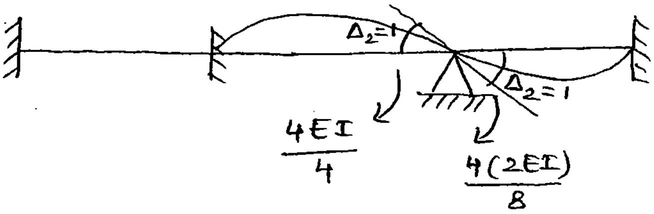
Restrained Structure

$\Delta_2 = 0$, $\Delta_1 = 1$:-



$$K_{11} = \frac{8EI}{4} + \frac{4EI}{4} = 3EI$$

$$K_{21} = \frac{EI}{2} = K_{12}$$



$$K_{22} = \frac{4EI}{4} + \frac{8EI}{8} = 2EI$$

$$[K] = \begin{bmatrix} 3EI & 0.5EI \\ 0.5EI & 2EI \end{bmatrix}$$

$$= EI \begin{bmatrix} 3 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

25-12-2014

UNIT-9

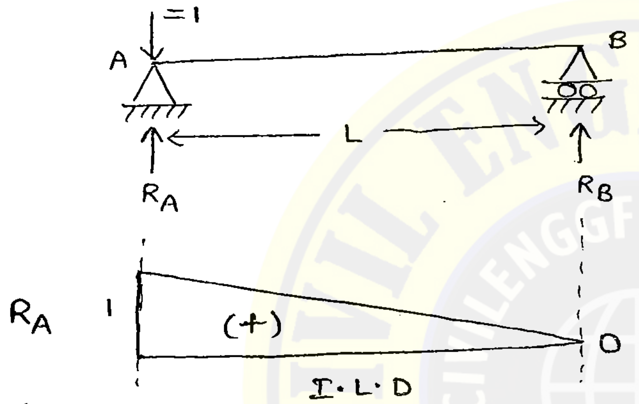
ROLLING LOADS AND INFLUENCE LINES

Influence line:-

It shows the variation of parameters like support reactions, Bending moment, shear force, and slope deflections at a point of a beam when unit load moves over the structure.

Influence Line diagrams for support reactions:-

1. simply supported beam



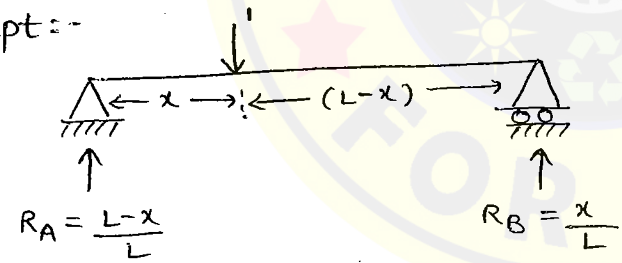
when unit load at 'A'

$R_A = 1$

when unit load at 'B'

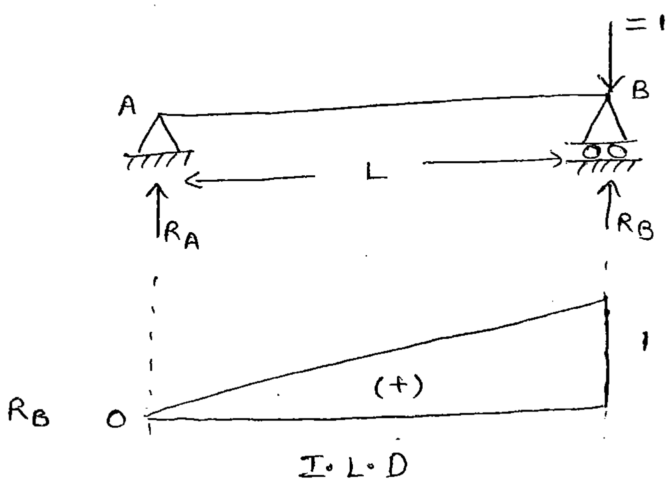
$R_A = 0$

Concept:-



At $x = 0$, $R_A = 1$

$x = L$, $R_A \geq 0$



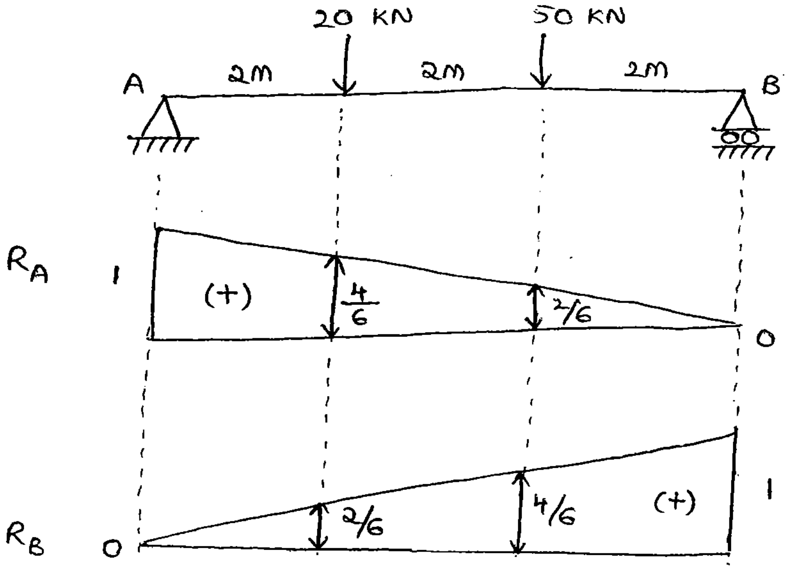
when unit load at B

$R_B = 1$

when unit load at 'A'

$R_B = 0$

Ex:- What is R_A and R_B using influence line diagram?



starting at 'O'

6m ⟶ 1

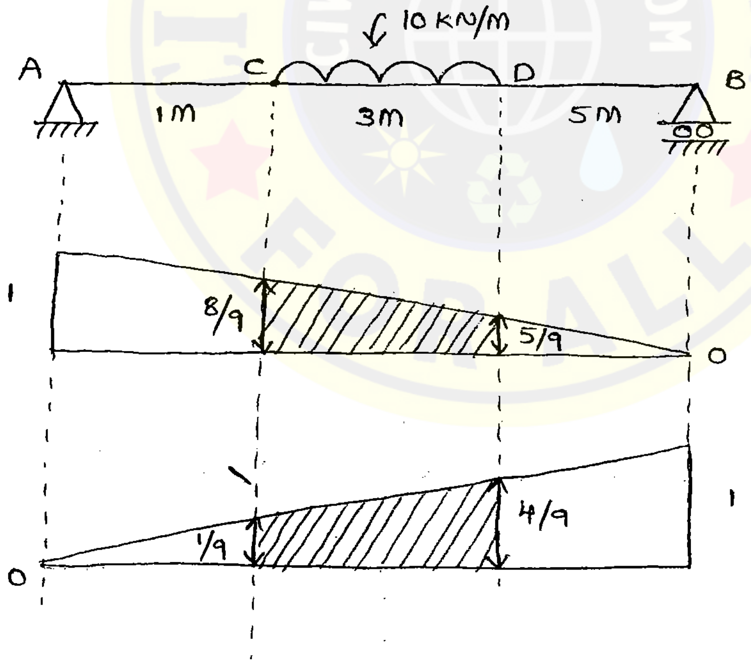
2m ⟶ 2/6

4m ⟶ 4/6

$$R_A = 20 \times \frac{4}{6} + 50 \times \frac{2}{6} = 30 \text{ KN}$$

$$R_B = 20 \times \frac{2}{6} + 50 \times \frac{4}{6} = 40 \text{ KN}$$

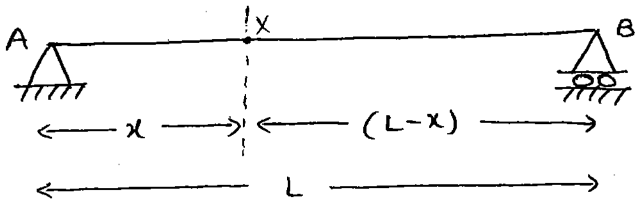
Ex:- What is R_A and R_B using i



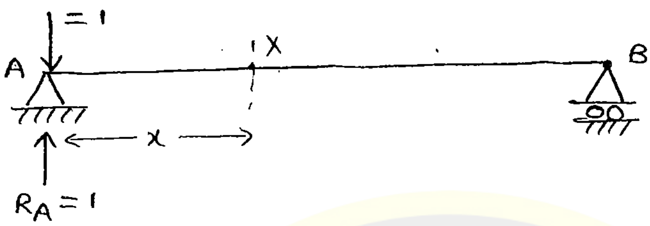
$$R_A = 10\left[\frac{1}{2} \times 3 \times \left(\frac{8}{9} + \frac{5}{9}\right)\right] = 21.67 \text{ KN}$$

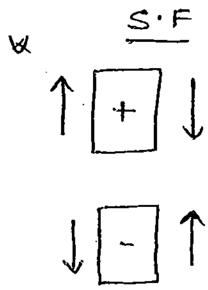
$$R_B = 10\left[\frac{1}{2} \times 3 \times \left(\frac{1}{9} + \frac{4}{9}\right)\right] = 8.33 \text{ KN}$$

Influence line diagram for shear force @ section - x :-



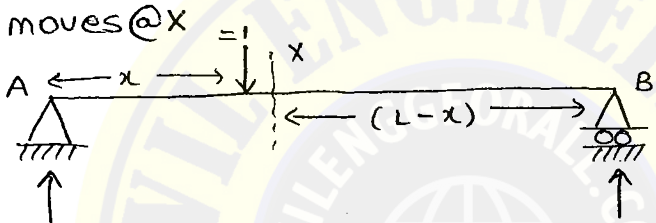
when unit load at 'A'





$$\boxed{V_x = 1 - 1 = 0}$$

when load moves @ X



$R_A = \frac{L-x}{L}$ $\qquad$ $R_B = \frac{x}{L}$

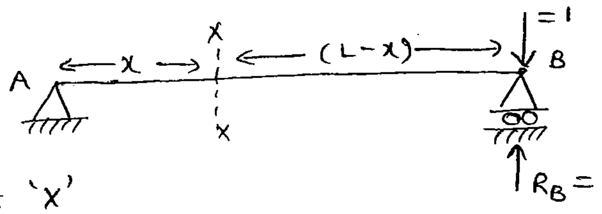
S.F @ 'x'

$$V_x = \left(\frac{L-x}{L}\right) - 1$$

$$= 1 - \frac{x}{L} - 1$$

$$\boxed{V_x = \frac{-x}{L}}$$

When load crosses the section - x . at B :-



$R_B = \frac{x}{L}$

S.F at 'x'

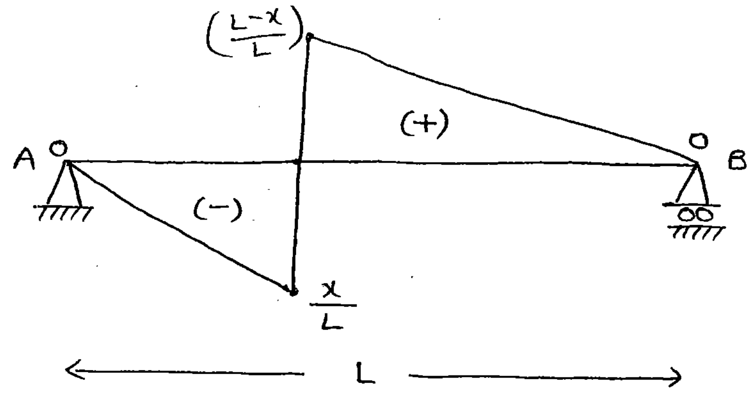
$$V_x = 1 - \frac{x}{L}$$

At $x = L$, $V_x = 0$

At $x = x$, $V_x = \frac{L-x}{L}$

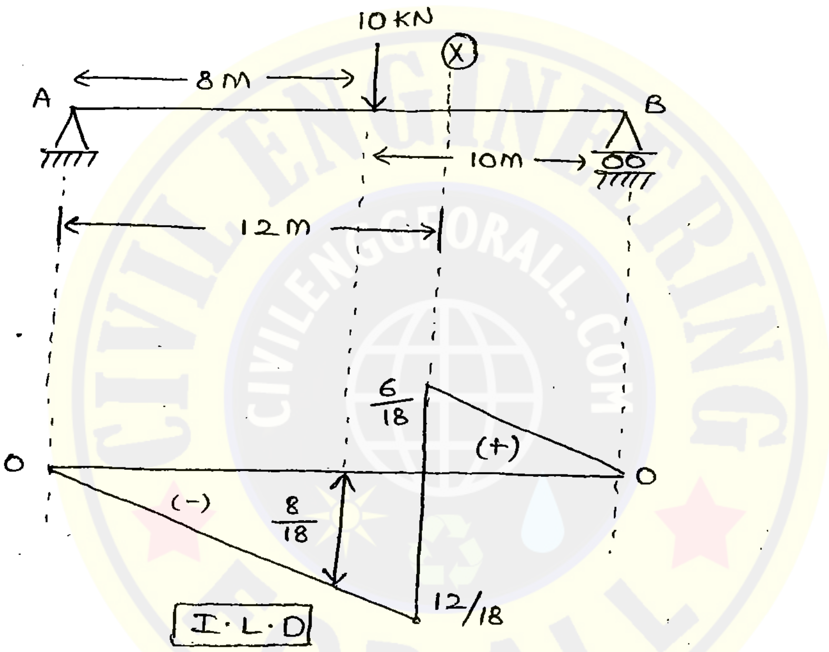
~~when unit load~~

V_x



Influence line diagram

EX:- Calculate the shear Force at section-'x' shown in fig.



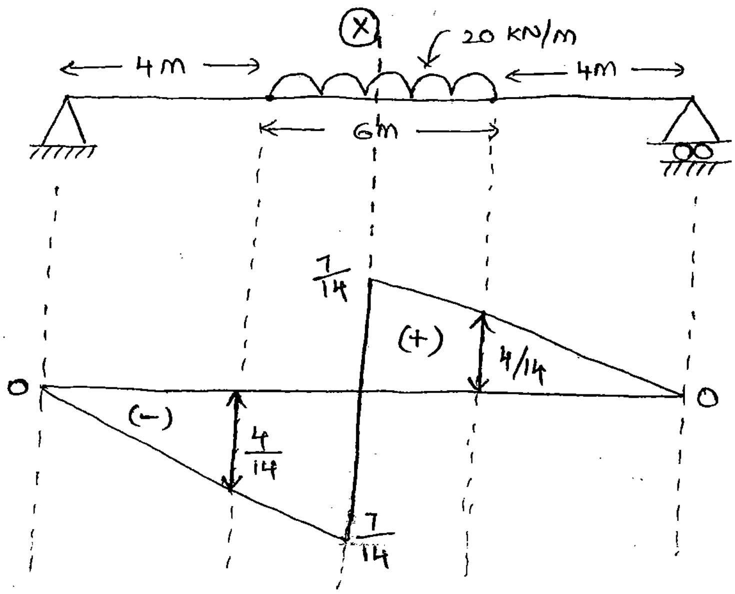
$12\,m \longrightarrow \frac{12}{18}$

$8\,m \longrightarrow\ ?$

$$\frac{8 \times \frac{12}{18}}{12} = \frac{8}{18}$$

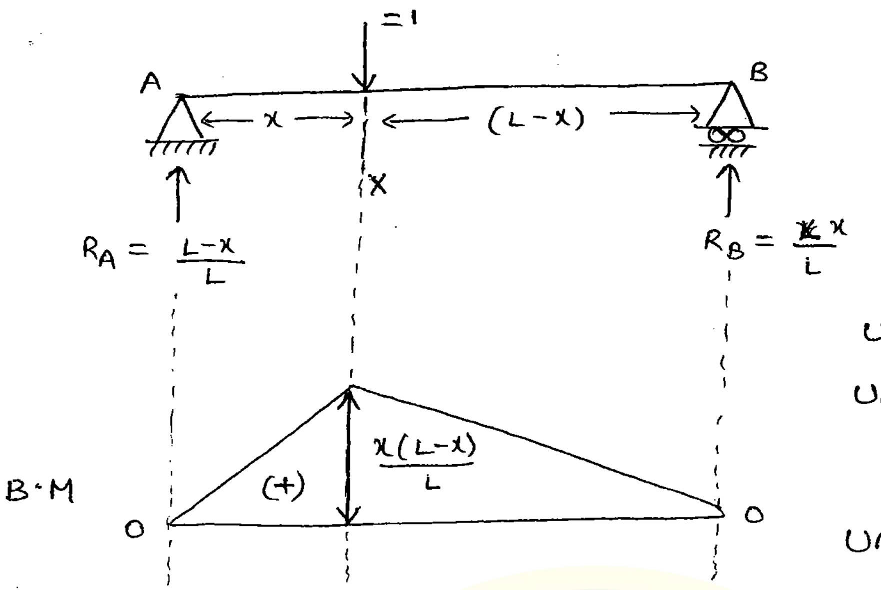
$$V_x = 10\left(\frac{-8}{18}\right) = -4.44\ KN$$

EX:- Calculate the S.F at section - x, shown in fig.



$V_x = 0$ at Section - X

Influence line diagram for B·M at section - x :-



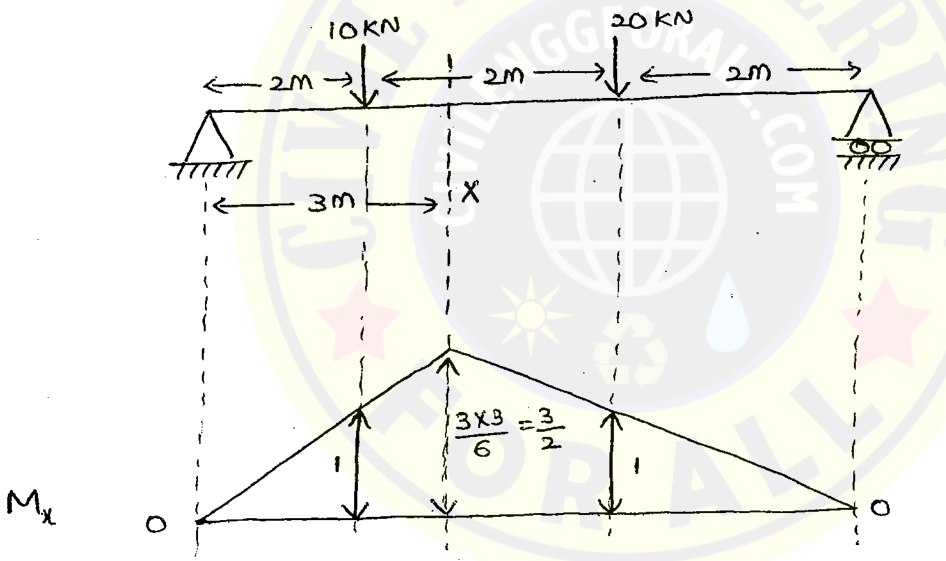
Unit load at A ; $M_x = 0$

Unit load at x ;

$$M_x = \left(\frac{L-x}{L}\right)x$$

Unit load at B ; $M_x = 0$

Ex:- Calculate the B·M at section - 'x' :-



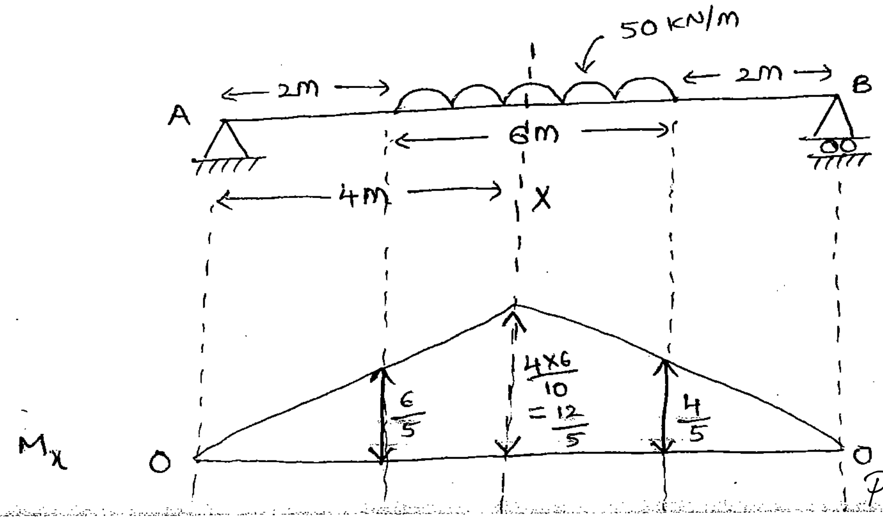
$3m \rightarrow \frac{3}{2}$

$2m \rightarrow$

$$\frac{\frac{3}{2}\times 2}{3} = 1$$

$$M_x = 10\times1 + 20\times1 = 30 \text{ kN-m}$$

Ex:- Calculate the BM at section - x :



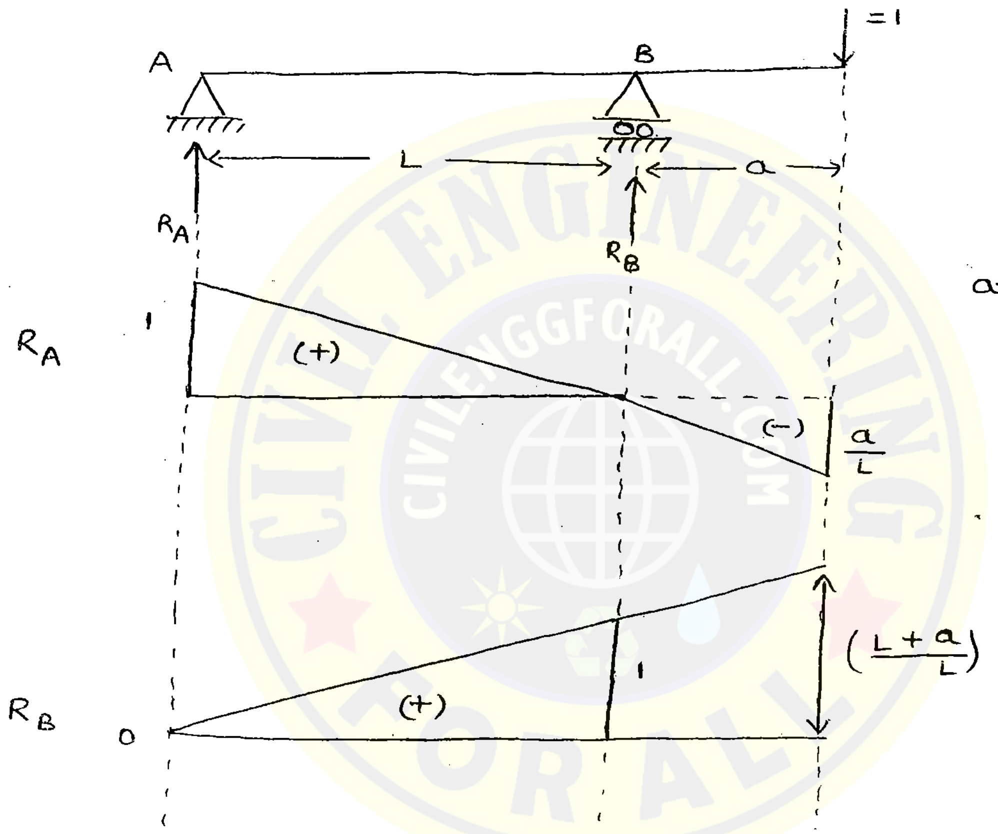
$$M_x = \left[\left(\frac{1}{2}\times 2\times\left[\frac{6}{5}+\frac{12}{5}\right]\right) + \left(\frac{1}{2}\times 4\times\left[\frac{12}{5}+\frac{4}{5}\right]\right)\right]\times 50$$

$$= 500 \text{ kN-m.}$$

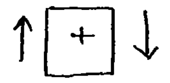
Note:-

1. ILD ordinates for reaction and SF as <u>No units</u>
2. ILD ordinates for B·M as <u>Linear unit</u>

Over hanging Beams:-



S·F



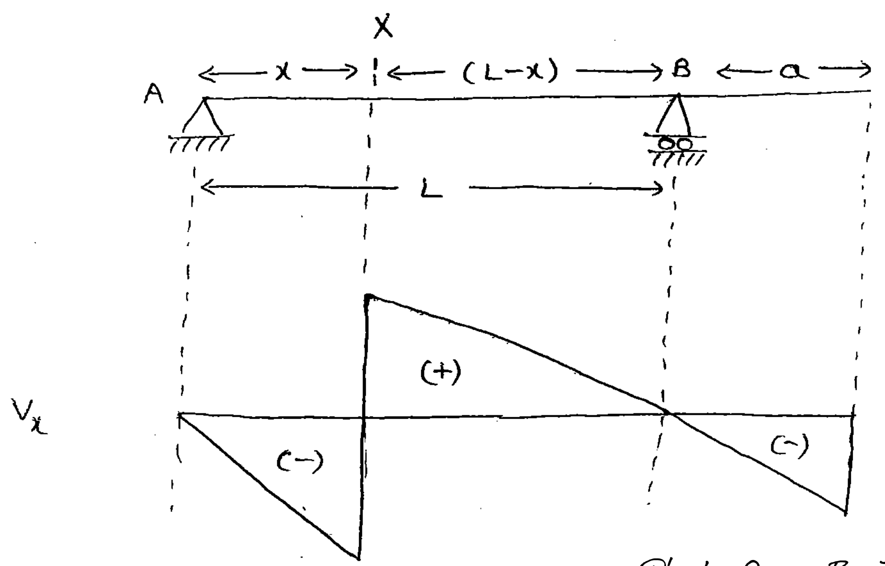
Assume unit load at free end

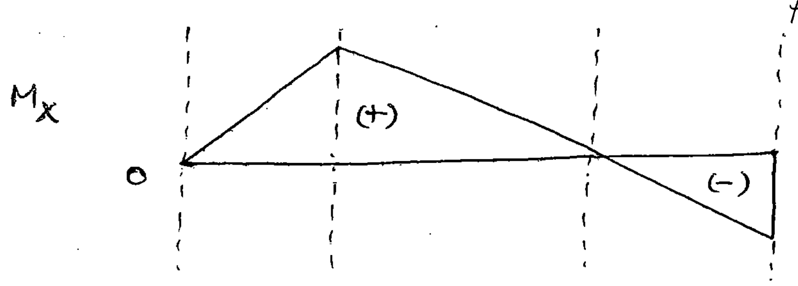
$\Sigma M_B = 0$

$R_A \times L + 1\times a = 0$

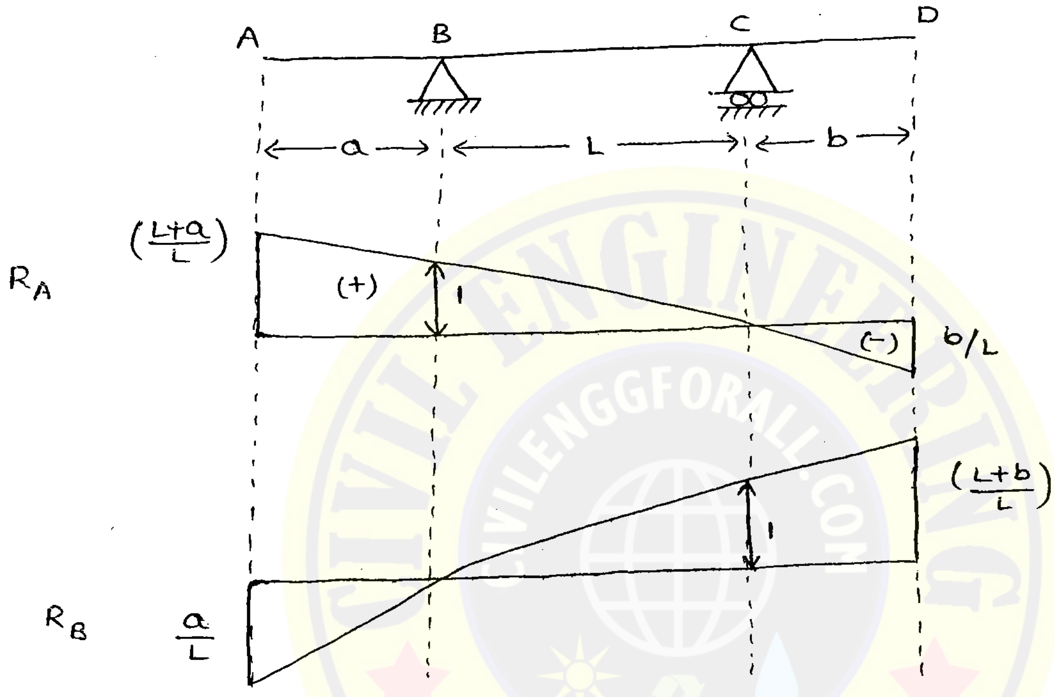
$R_A = \frac{-a}{L}$

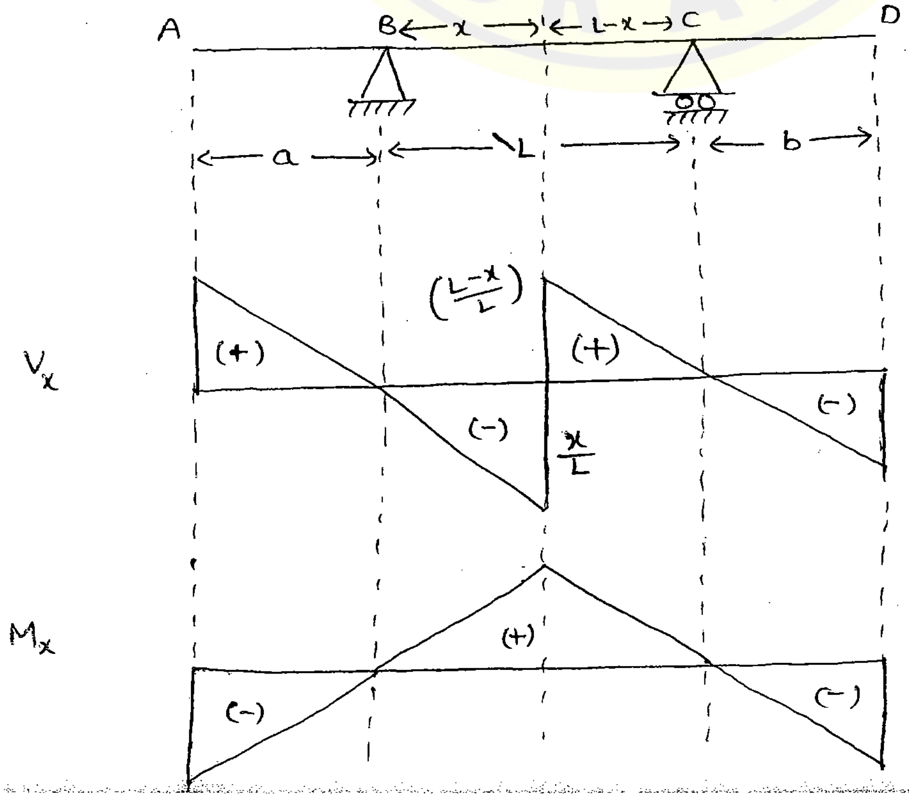
Ex:- Calculate the S·F at section − X



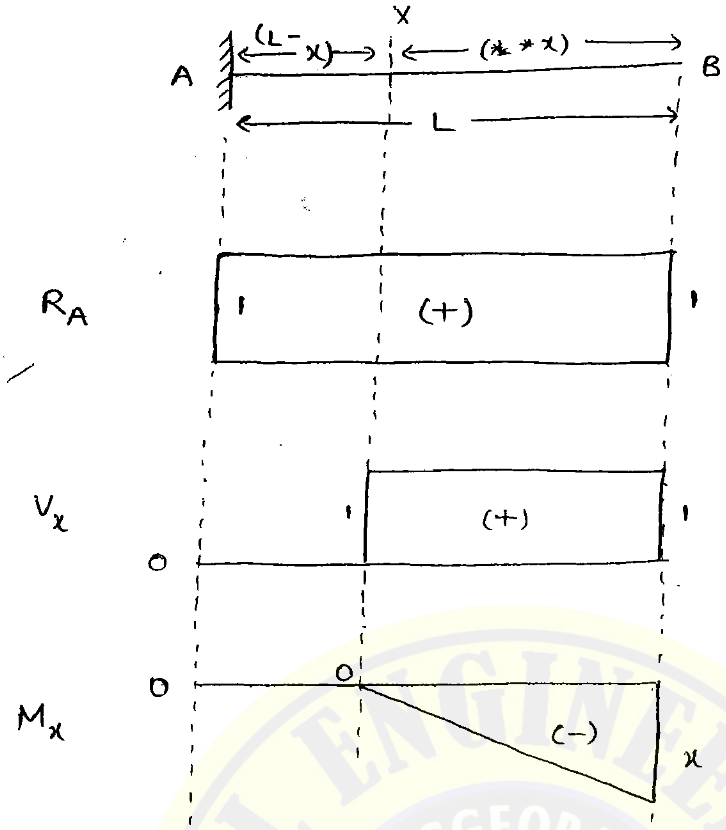


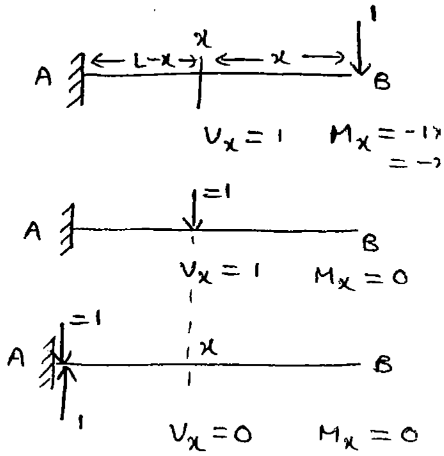
Ex:- Influence line diagram for over hanging beams on both sides.



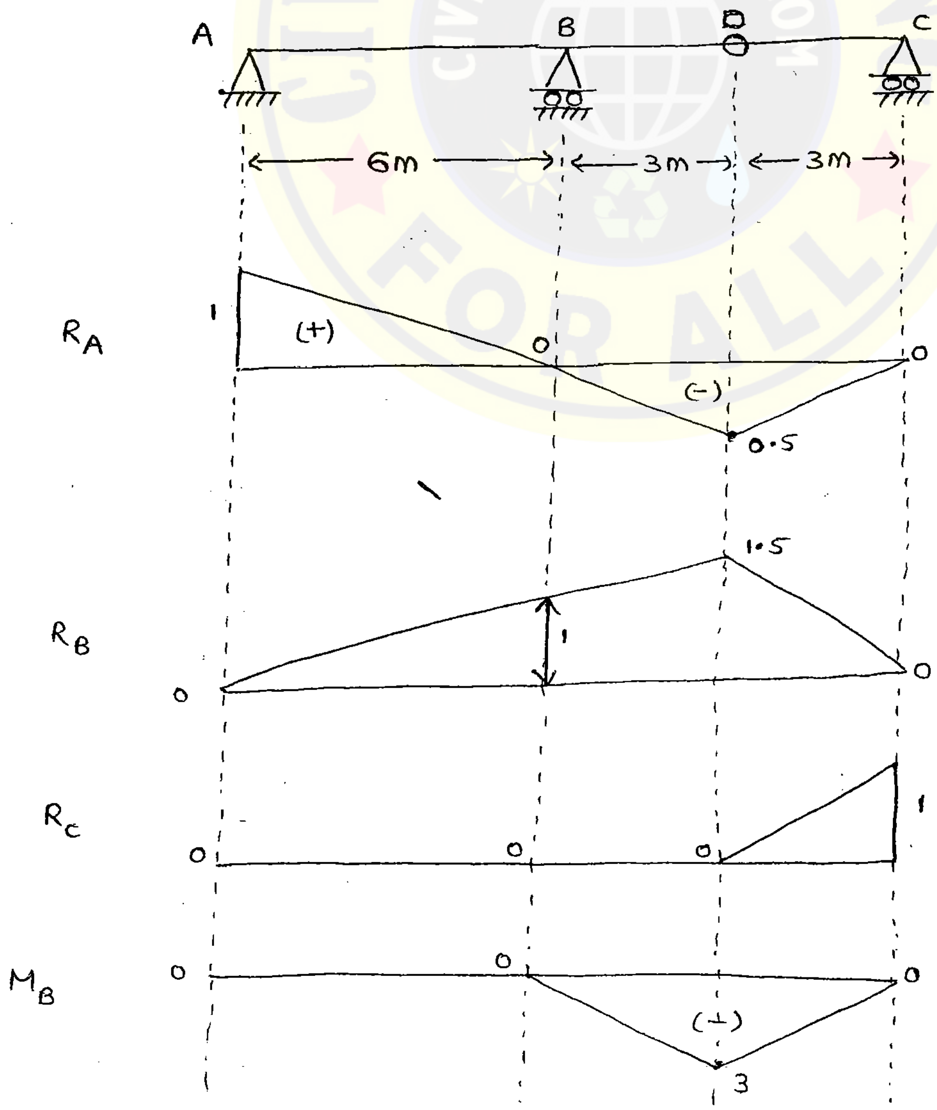


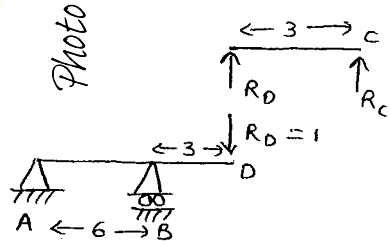
Cantilever beam:-





Ex:-





when unit load is kept in $R_D = 1$

$R_A + R_B = 1$

∴ $\Sigma M_B = 0$

$6R_A + 3 = 0$

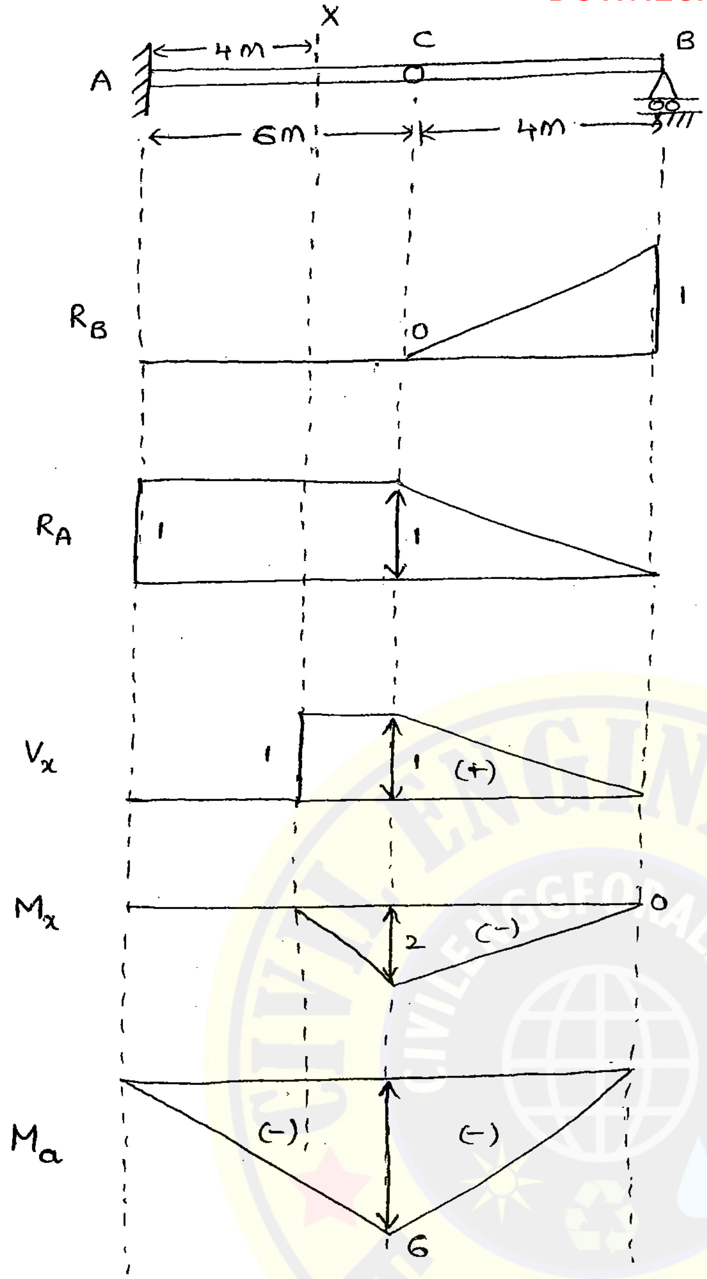
$R_A = -0.5$ kept this under the applied load.

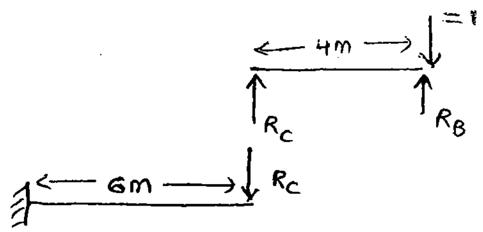
$R_B = 1 + 0.5 = 1.5$

when keep unit load at C, $R_c = 1$ then B.M at B = 0

when load kept at D=1 then B.M at B is (-1×3)

Ex:-)





$R_B = 1, \quad R_C = 0$

when load kept at 'c' then

$R_A = 1 \quad R_C = 1$

when unit load at 'B' then

B·M at A = 0

when unit load at 'c' then

$M_A = -6$

when unit load at A then

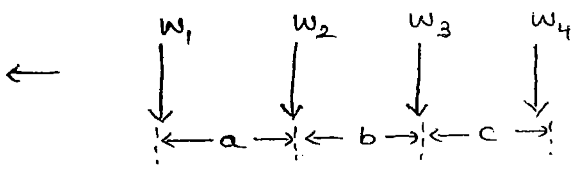
$M_A = 0$

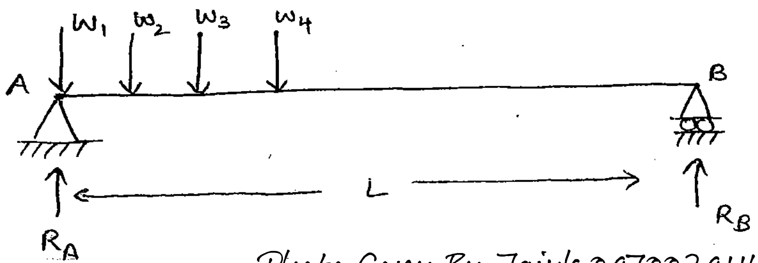
(where unit load is placed, that point only "calculated value" is taken)

Rolling Loads:-

1. Maximum end shear:-

a. System of wheel loads:-





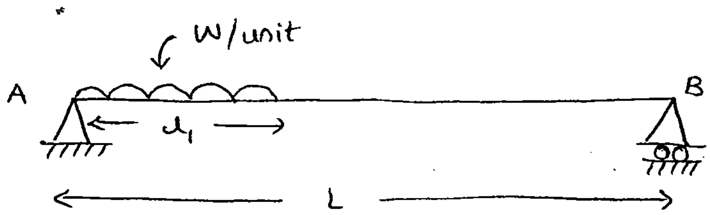
$(R_A)_1 > (R_A)_2$

$$\frac{\text{Load Rolled off}}{\text{Succeding wheel loads}} > \frac{\text{sum of remaining loads}}{\text{span}}$$

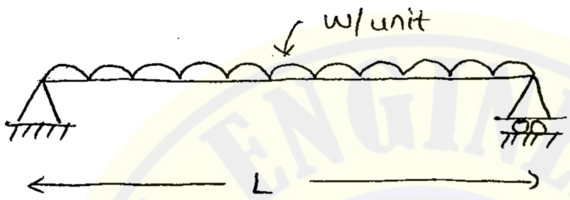
$(R_A)_1$ = support reaction at A when w_1 is placed at A

$(R_A)_2$ = support reaction at A when w_2 is placed at A (w_1 is rolled off)

b) U.D.L shorter than the span

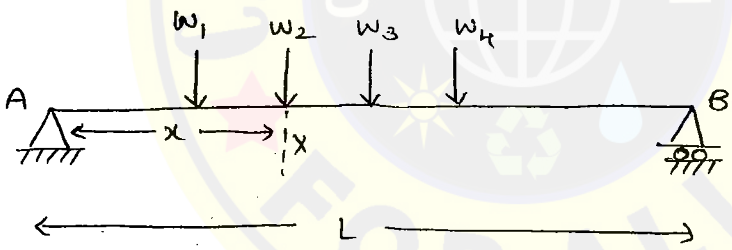


c) U.D.L longer than the span



2. Maximum shear force at a given section X:-

a. System of wheel loads



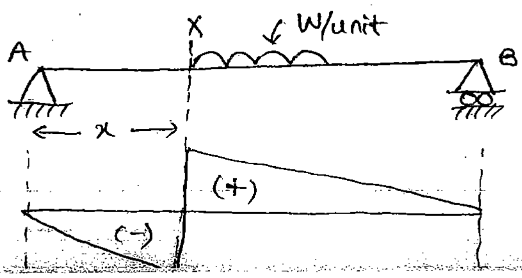
$(V_x)_1 > (V_x)_2$ if $\dfrac{\text{Load Rolled off}}{\text{Succeeding wheel span}} > \dfrac{\text{sum of all loads}}{\text{span}}$

$(V_x)_1$ = shear force at section-x, when w_2 is placed at section

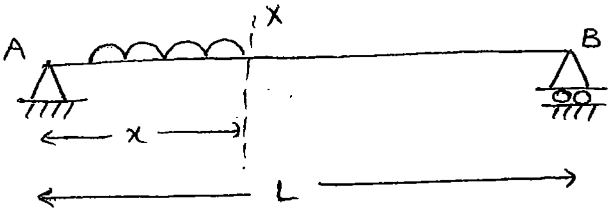
$(V_x)_2$ = shear force at section-x when w_3 is placed at section-x

b. UDL shorter than span :-

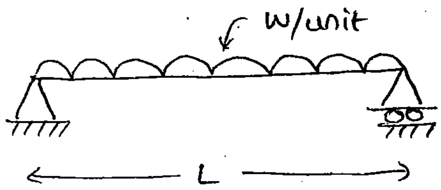
i. For maximum +ve shear at section-x :-



ii) For max -ve shear :-

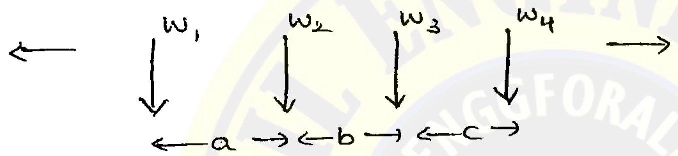


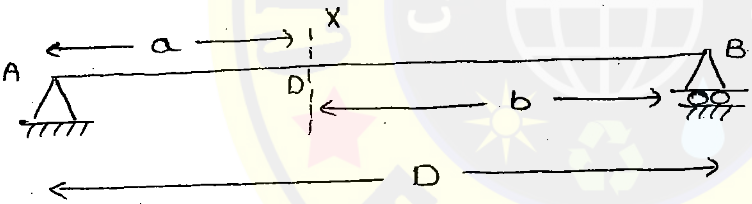
c) UDL longer than the span:-



3. Maximum B.M at a section - x :-

a. System of wheel loads :-

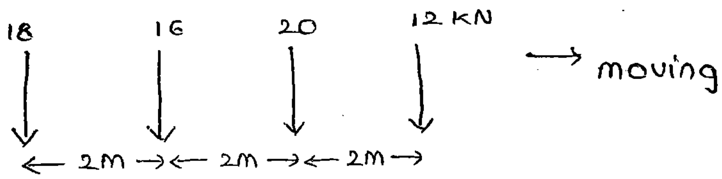


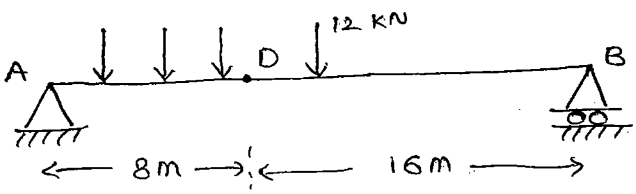


1. Average load on AD = Average load on BD

i.e., Max BM at section occurs if avg. load on AD - avg. load on BD changes its sign

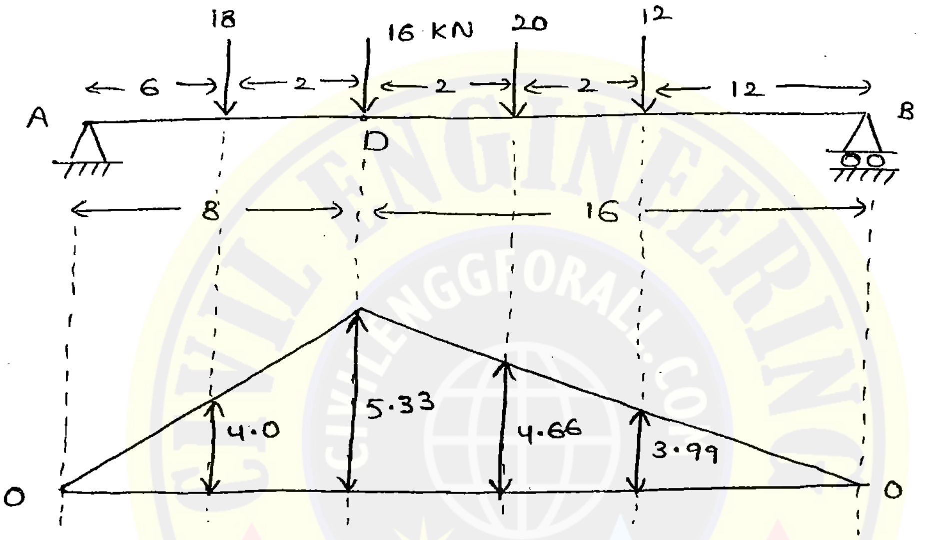
Ex :-





Load Rolled off at section - D (i)	Avg. Load on AD (ii)	Avg. load on BD (iii)	Remarks (iv)
12 kN	$\frac{54}{8} = \frac{(18+16+20)}{8}$	$\frac{12}{16}$	ii > iii
20 kN	$\frac{24}{8} = \frac{(18+16)}{8}$	$\frac{32}{16} = \frac{12+20}{16}$	ii > iii
$\boxed{16\text{ kN}}$	$\frac{18}{8}$	$\frac{48}{16}$	ii < iii

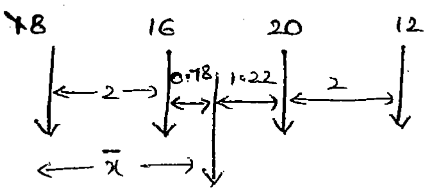
Max. BM at the section



$$\frac{8\times16}{24} = 5.33$$

$$M_D = 18\times4 + (16\times5.833) + 20\times4.66 + 12(3.99)$$
$$= 298.36 \text{ kN-m}$$

(or)



$R = (18+16+20+12) = 66$ kN

$$\bar{x} = \frac{16\times2 + 20\times4 + 12\times6 + 18\times0}{66}$$

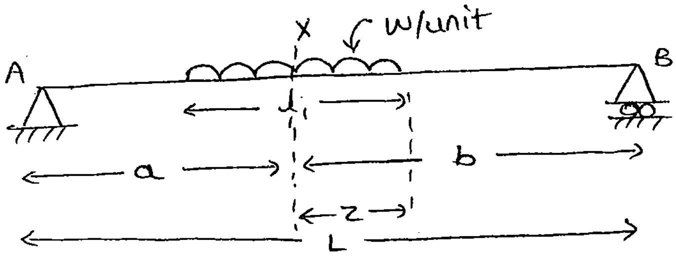
$\bar{x} = 2.78$ m

Note:-

which load is nearer to $\bar{x}$ that load is Maximum load

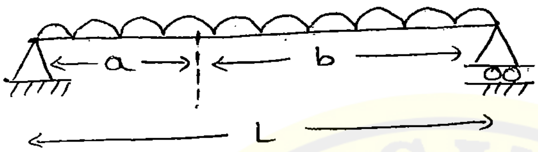
$\therefore$ 16 kN

b) UDL shorter than the span :-



$$\boxed{z = \frac{L_1}{L} \times b}$$

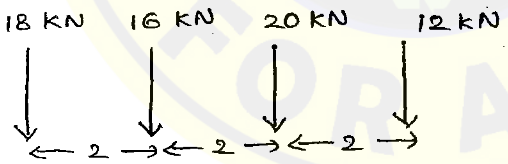
c) UDL longer than the span :-



4. Maximum B.M under a choosen load :-

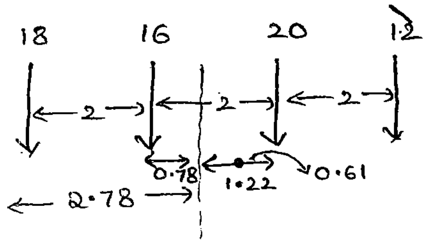
1. Calculate the magnitude and position of a resultant for a given wheel loading system
2. Place the choosen load and the resultant at equi distance are either side of the centre of the beam.
3. Draw ILD for a BM under a choosen load and calculate the maximum BM under a choosen load.

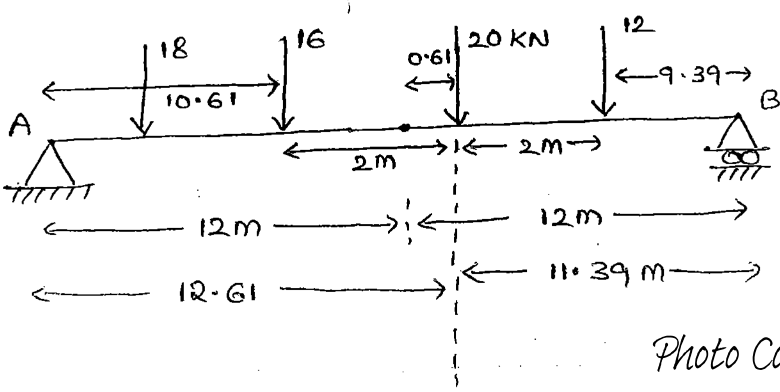
Ex:-

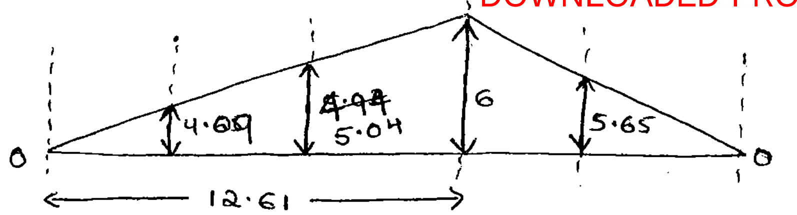


What is the max. BM under a 20 kN load ?

A)







$\frac{12.61 \times 11.39}{24} = 6$

$12.61 \longrightarrow 6$

$10.61 \longrightarrow ?$

$\frac{63.66}{12.61} = 5.0$

$8.61 \longrightarrow ?$

$\frac{8.61 \times 6}{12.61} = 4.0$

$$M_{max} = 18 \times 4.65 + 16 \times \overset{5.04}{\cancel{4.94}} + 20 \times 6 + 12 \times 5.65$$

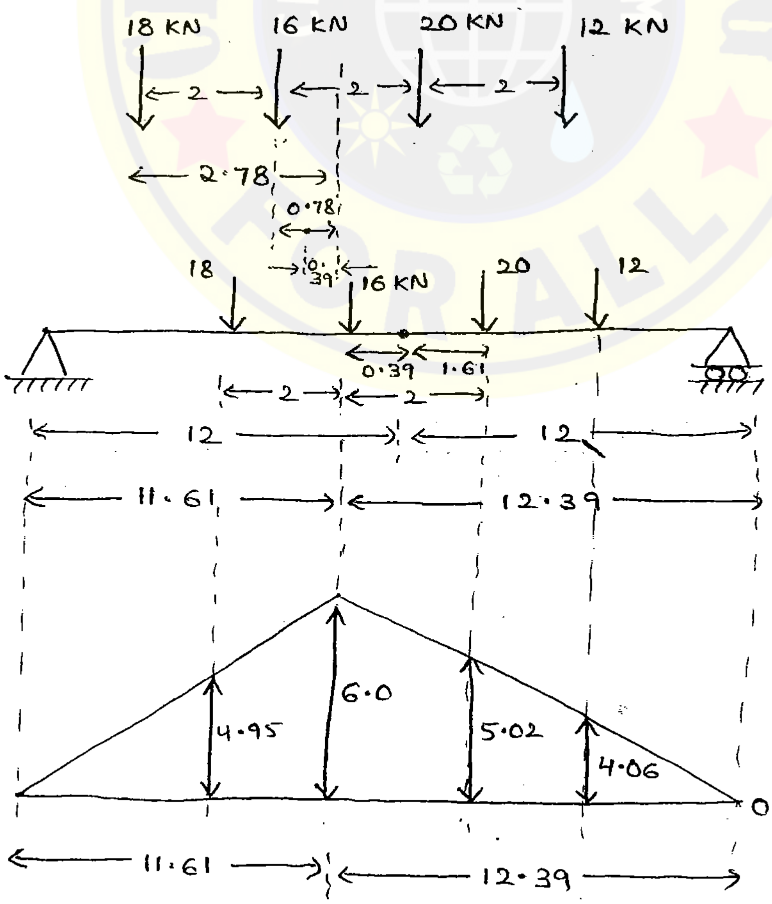
$$= \cancel{343}\ \cancel{352\ KN-m}$$

$$= 333.7\ KN-m$$

Absolute max. BM :-

1. In this method neither section is given nor choosen load is mention.
2. By inspection select the load by calculating the resulta of the wheel load system.
3. Take section will be the centre of the beam to calcula absolute maximum B.M
4. Draw influence line diagram under a choosen load to get the absolute maximum B.M

Ex:-



Absolute Max BM = $18 \times 4.95 + 16 \times 6 + 20 \times 5.02 + 12 \times 4.06$

$= 334.2\ KN-m$

Muller Breslau's principle:-

1. It is a very convinient method for both qualitative and quantitative. Determination of ILD's for various functions in a structure.
2. It is possible to sketch the shape of ILD's and thus determine accurately for design purposes the loading pattern of the structure in order to produce maximum effect.

Statement:-

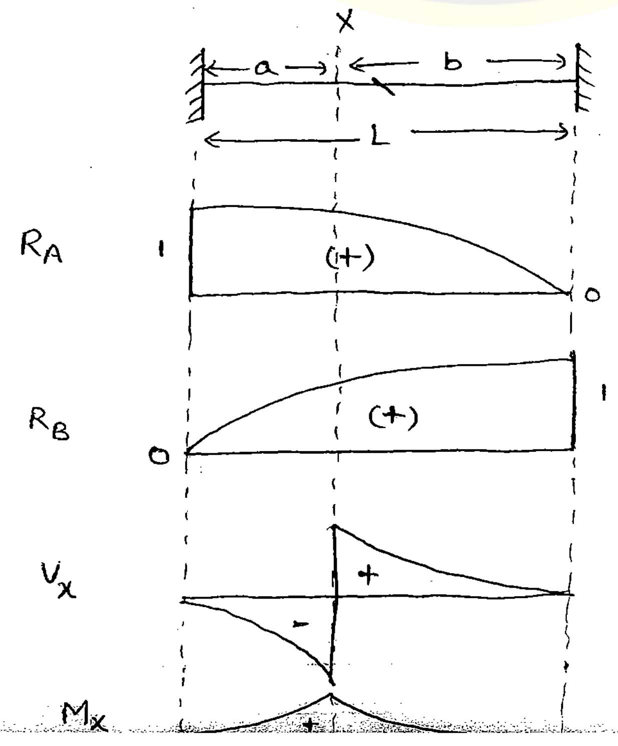
It states that the ordinates of the influence line for any functions such as reaction, shear, BM, torsion, fibre stresses etc in an element of a structure are proportional to those of the deflection curve drawn for the structure obtain by removing the constraints corresponds to that element from the structure and introducing in its place a corresponding deformation a primary structure which remains.

Applications:-

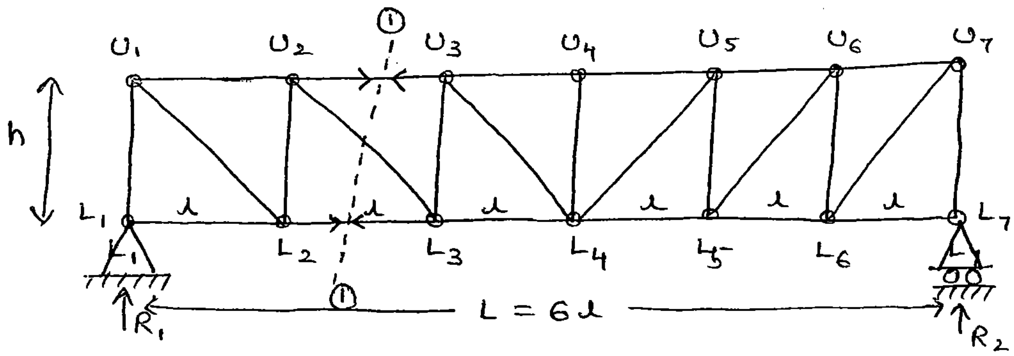
1. It can be used for drawing ILD's for a determinate and indeterminate structures like beams, frames and articulated structures assuming that structures follows Hooke's law.

Examples:-

1. Fixed Beams



Influence line diagram for the trusses :-



(upper chord mem be

(Lower chord mem

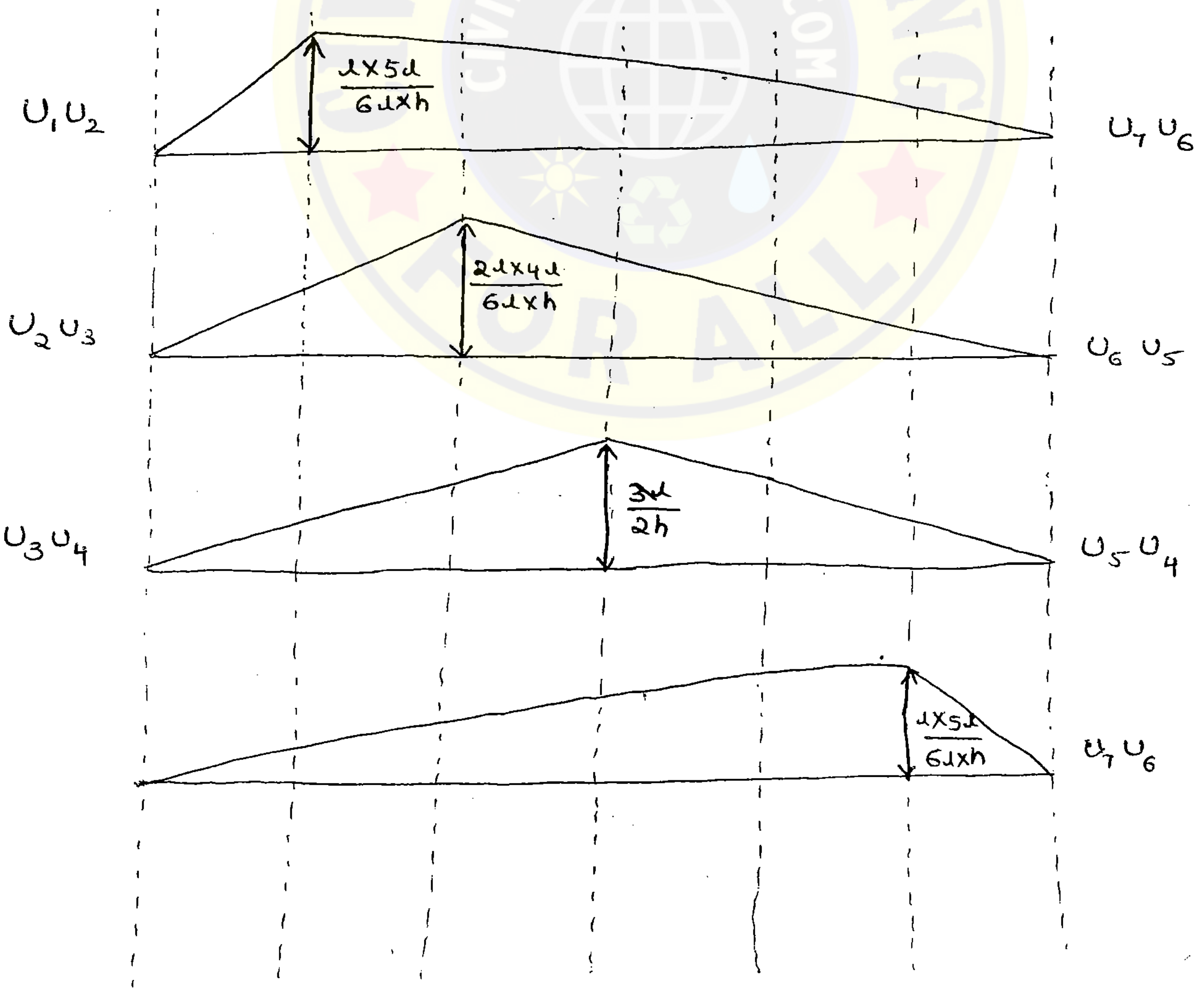
To calculate the force in top chord member $U_2 U_3$:-

$$\Sigma M_{L_3} = 0$$

$$-R_2 \times L_3 L_7 - F_{U_3 U_2} \times h = 0$$

$$F_{U_3 U_2} = \frac{-R_2 \times L_3 L_7}{h}$$

$$\boxed{F_{U_3 U_2} = -\frac{M}{h}}$$



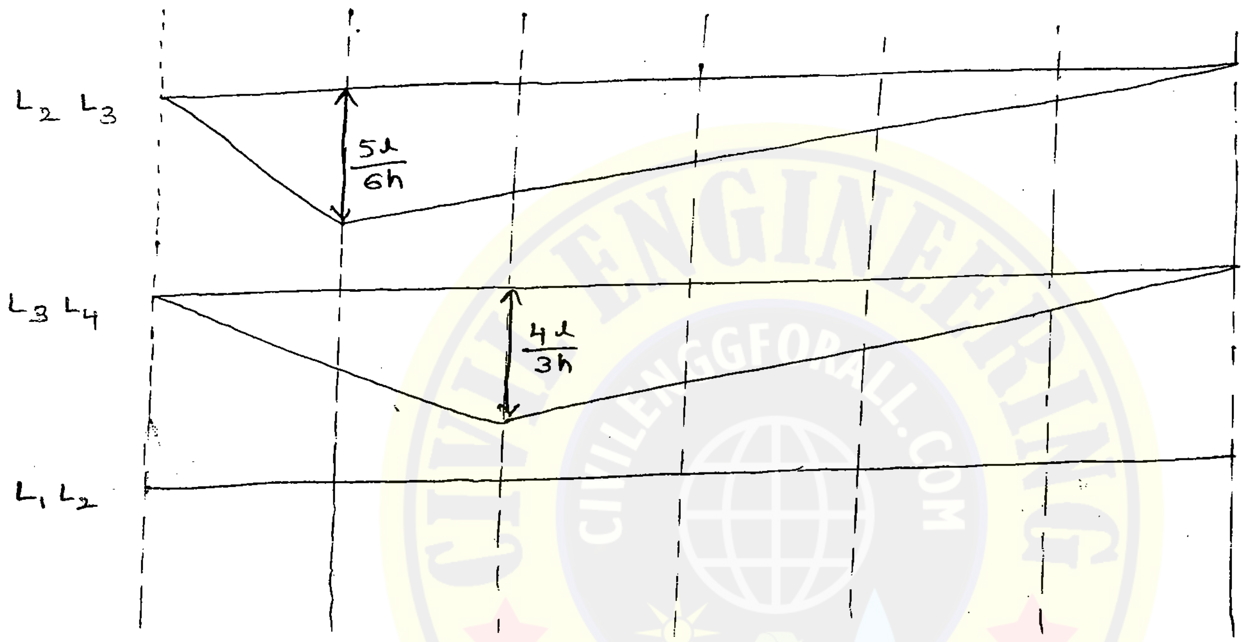
To calculate the force in bottom chord member L_2L_3 :-

$$\Sigma M_{U_2} = 0$$

$$-F_{L_2L_3} \times h + R_1 \times L_1L_2 = 0$$

$$F_{L_2L_3} = \frac{R_1 \, L_1L_2}{h}$$

$$\boxed{F_{L_2L_3} = \frac{M}{h}}$$



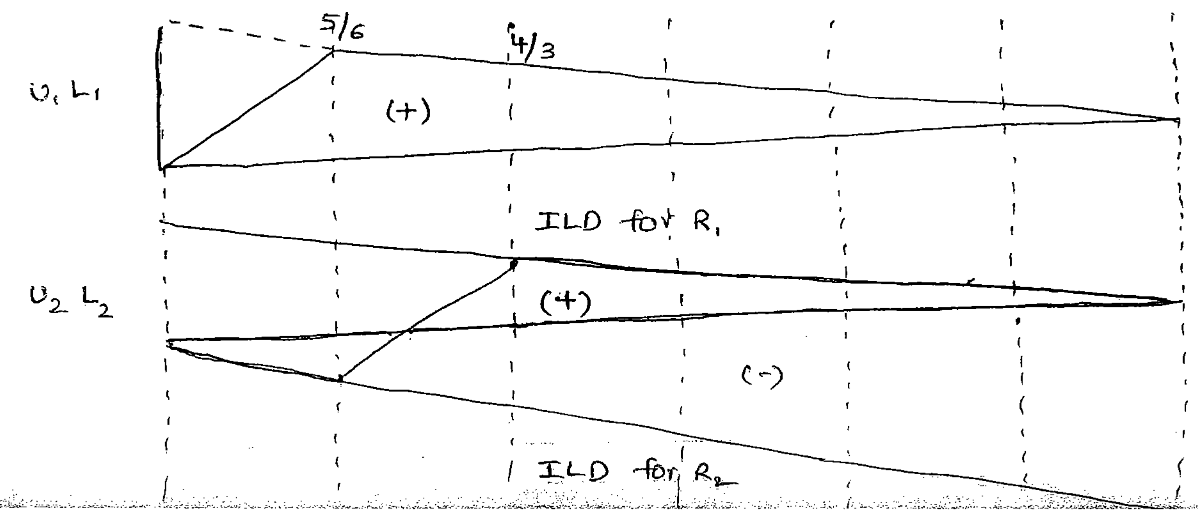
Verticle members:-

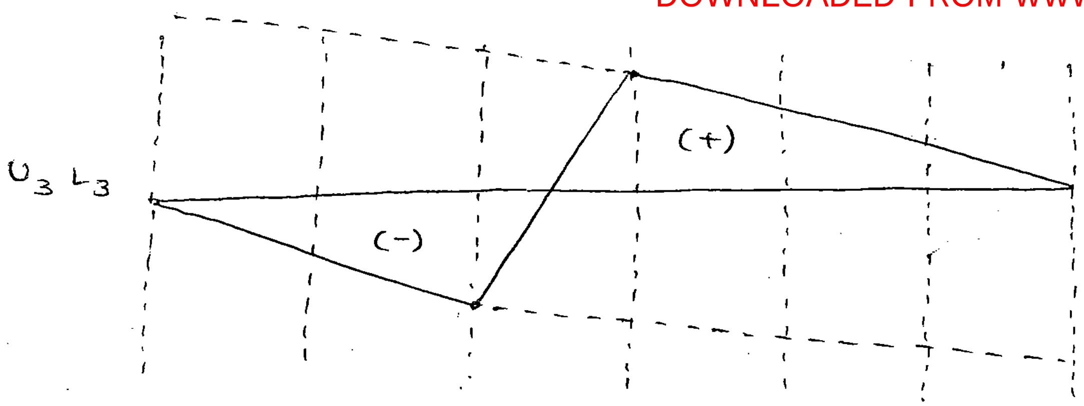
$$F_{U_1L_1} = F_{L_1U_1}$$

when unit load is at UL

$$F_{L_1U_1} = 1\,(R_1)$$

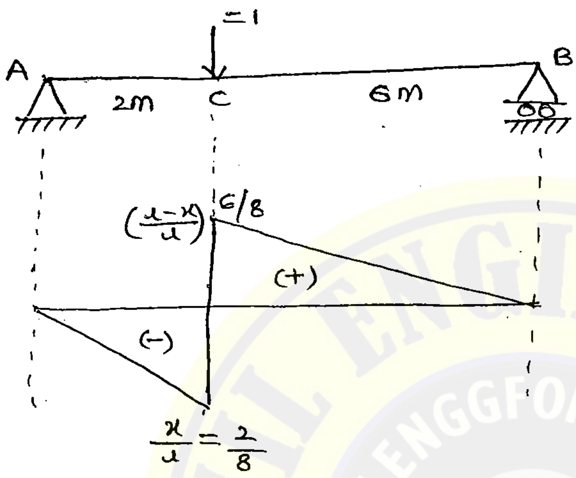
when unit load is at U_2 variation of $F_{U_1L_1}$ is linear between U_1 and U_2



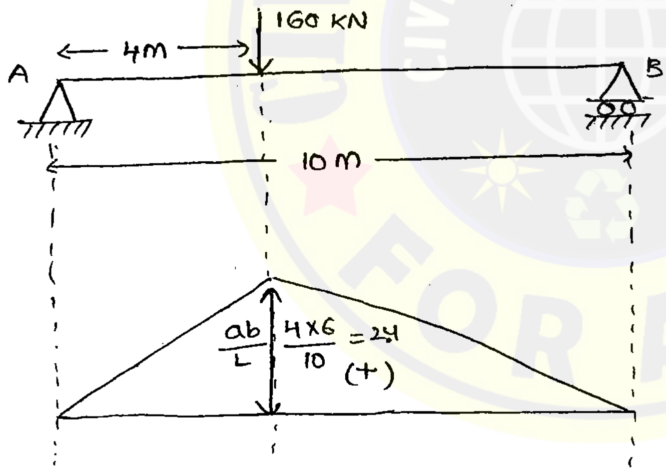


P.g No:- 86

6.

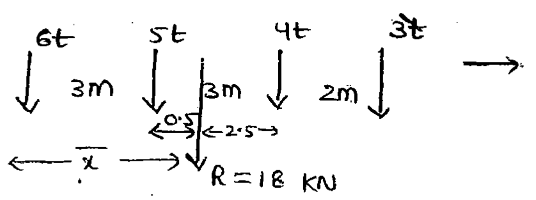


7.



$M_x = 160 \times 2.4 = 384$ kN-m

8.



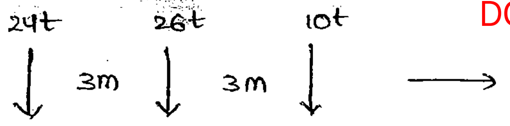
$R = (6+5+4+3) = 18$ kN

$18\,\bar{x} = 5 \times 3 + 4 \times 6 + 3 \times 8$

$\bar{x} = 3.5$ m

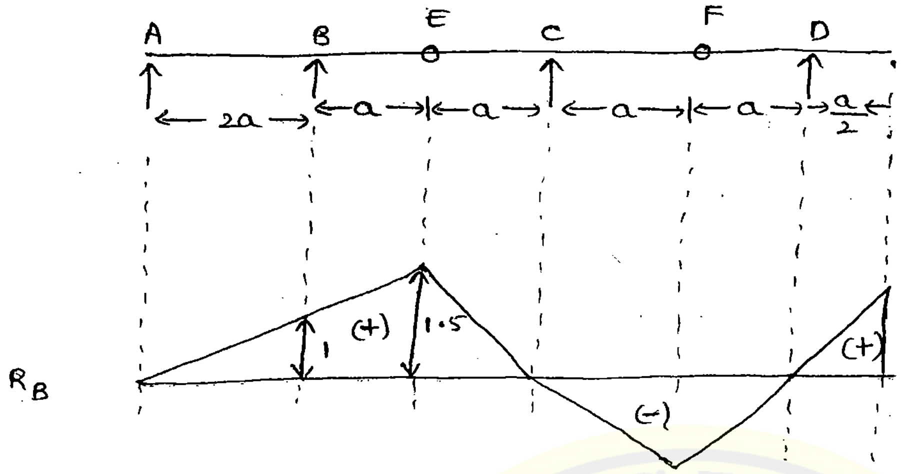
Nearest value is "5t"

9.



Maximum B.M at 26 t passes the section

13.

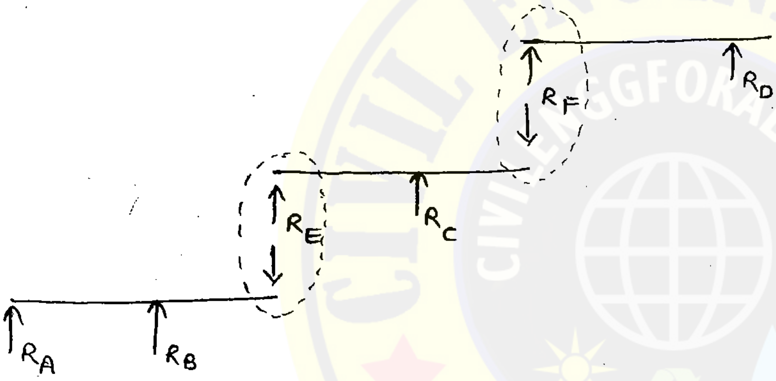


when unit load at A,
$R_B = 0$

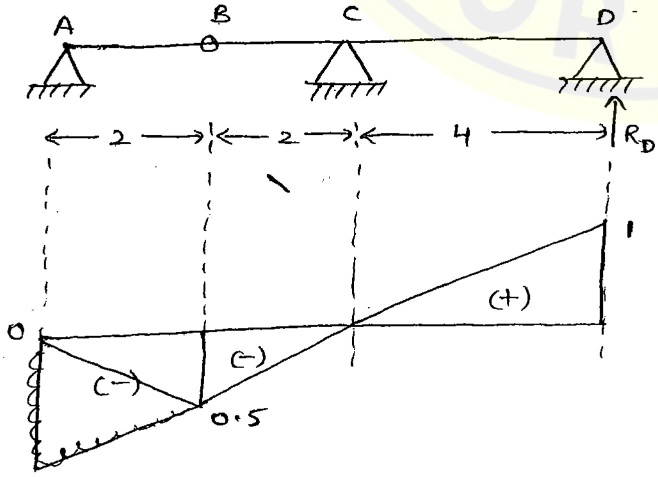
when unit load at B,
$R_B = 1$

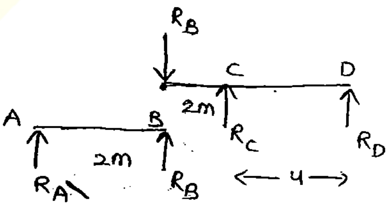
when unit load at C,
$R_B = 0$

when unit load at E
$R_B = 1.5$



14.





when unit load is at D
$R_D = 1$

when unit load is at C
$R_D = 0$

when unit load is at B
$R_D = -0.5$

$\Sigma M_c = 0$

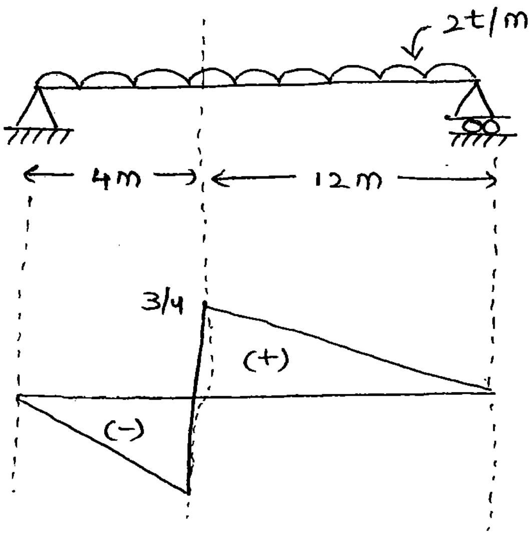
$-1 \times 2 - R_D \times 4 = 0$

$R_D = -0.5$

when load at A,
$R_A = 1, \quad R_B = 0, \quad R_D = 0$

P.9 NO-87

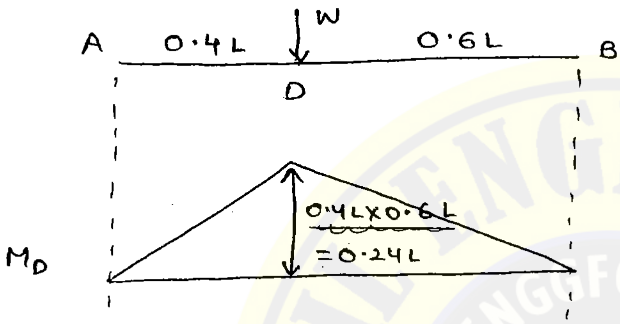
1.



$V_x = \left\{ -\left(\frac{1}{2} \times 4 \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 12 \times \frac{3}{4}\right) \right\} 2$

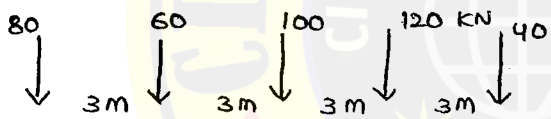
$= 8t$

2.



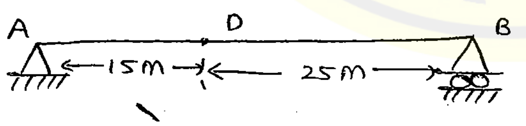
$M_D = 0.24\,WL$

4.



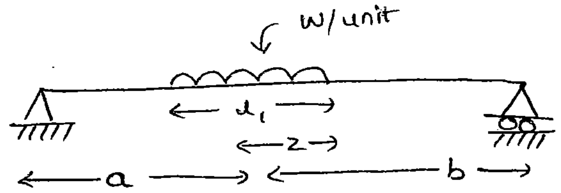
$400\,\bar{x} = (60 \times 3) + (100 \times 6) + (120 \times 9) + (40 \times 12)$

$\bar{x} = 6.01\,m$

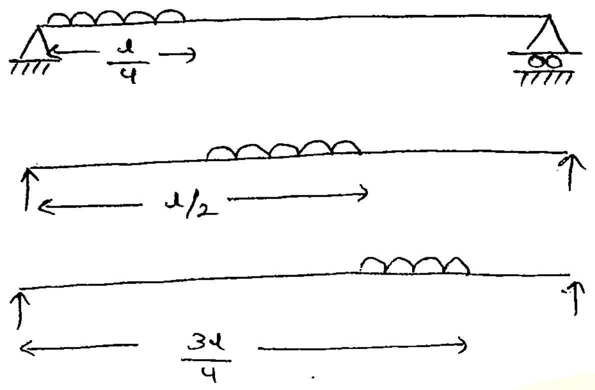


Load Rolled Off (i)	Avg. load on AD (ii)	Avg load on BD (iii)	Remarks (iv)
40 KN	$\frac{260}{15}$	$\frac{40}{25}$	(ii) > (iii)
120 KN	$\frac{240}{15}$	$\frac{160}{25}$	(ii) > (iii)
100 KN	$\frac{140}{15}$	$\frac{260}{25}$	(ii) < (iii)

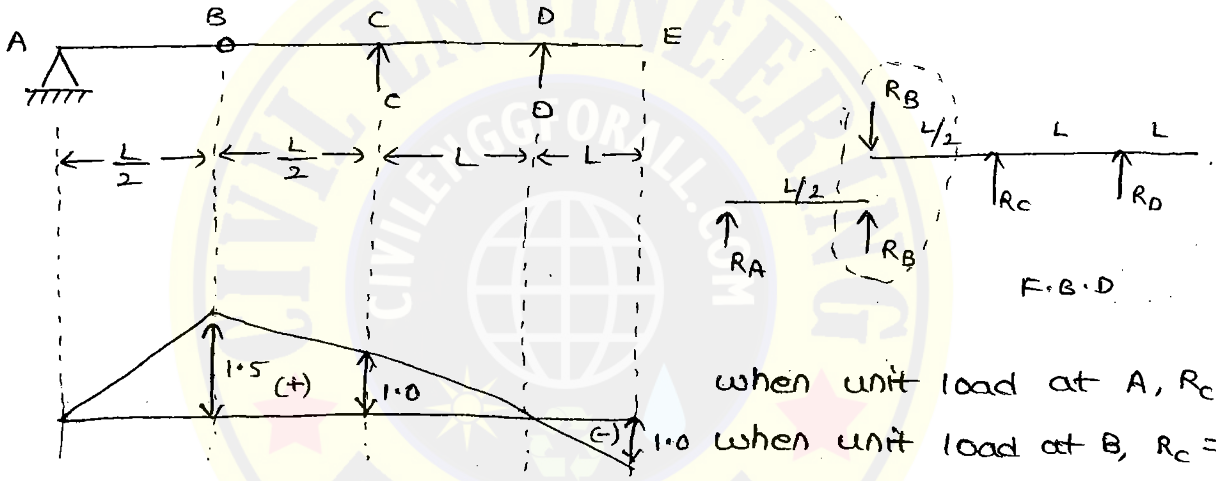
5.



$$z = \frac{d_1}{L} \times b$$



6.



when unit load at A, $R_C = 0$

when unit load at B, $R_C = 1.5$

$\Sigma M_D = 0$

$-1 \times \frac{3L}{2} + R_C \times L = 0$

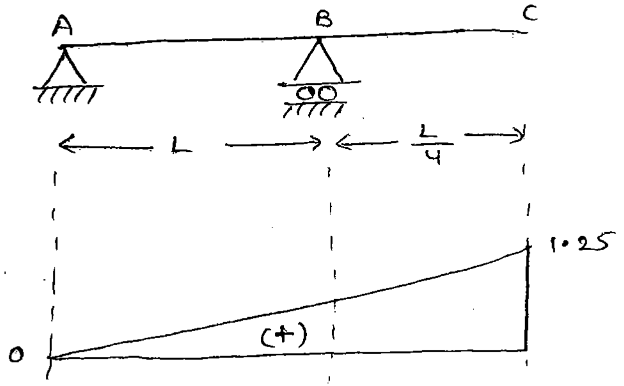
$R_C = \frac{3L}{2L} = 1.5$

when unit load at C, $R_C = 1$

when unit load at D, $R_C = 0$.

P.9 No:- 88

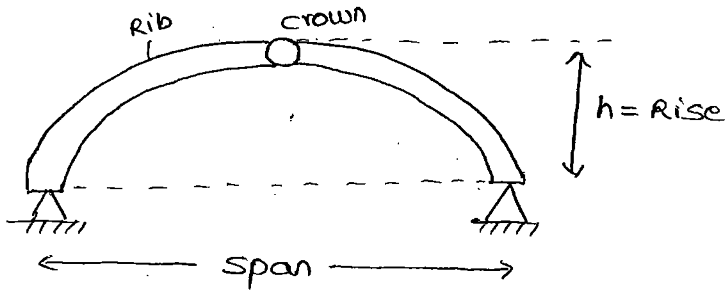
2.



UNIT - 10

ARCHES

Arch is a curved beam in which horizontal movement is ~~only~~ wholly or partly can be prevented hence horizontal thrus will be induced at the supports.



Rise = mid height of the arch from crown to the base of the arch

1. Arch is economical for long span since the B·M are very les
2. Arches are predominently subjected to axial thrust (or) axial ~~thr~~ force (or) normal thrust.
3. Granite material is best suited for arches

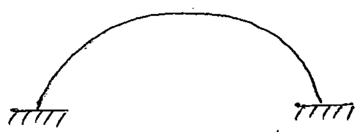
ARCHES

- semi circular (udl's)
- parabolic
- segmental (point load)

Classification of Arches:-

1. Fixed Arch :-

Hinge less arch



$D_s = 6 - 3 = 3$

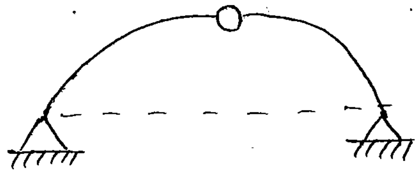
2. single hinge arch:-

$D_s = 5 - 3 = 2$

3. Two hinge arch:-

$D_s = 4-3 = 1$

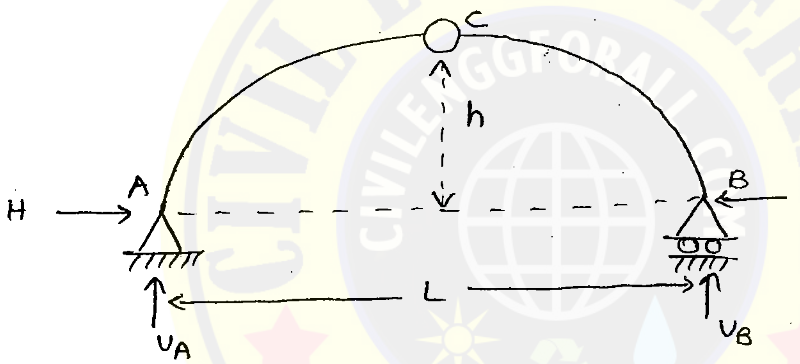
4. Three hinge arch:-



$D_s = (4-3)-1 = 0$

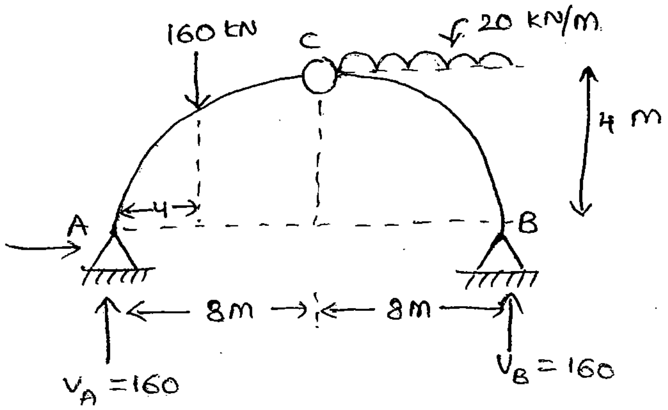
This is statically determinate and the remaining arches are statically indeterminate.

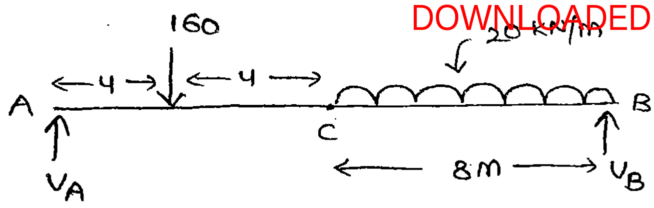
a. Supports are at the same level:-



1. verticle reactions can be calculated by treating this as a simply supported beam subjected to given loading when supports are at the same level.
2. Horizontal reactions at the support should be same if it is subjected verticle loading only.
3. After calculating the verticle reactions V_A and V_B use $\Sigma M_c = 0$ to calculate horizontal thrust.

EX:-





$V_A + V_B = 160 + (20 \times 8) = 320\ KN \rightarrow (1)$

$\Sigma M_A = 0$

$160 \times 4 + 20 \times 8 \times 12 - V_B \times 16 = 0 \rightarrow (2)$

$V_A = 160\ KN$

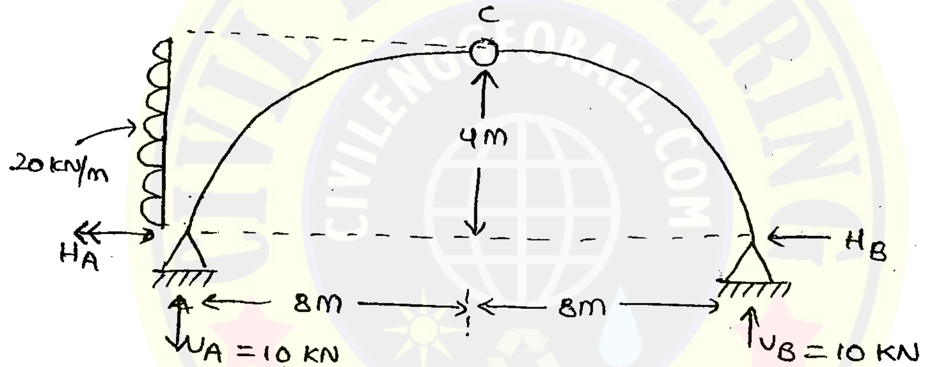
$V_B = 160\ KN$

$\Sigma M_c = 0$ (Left part of Hinge at c)

$-H \times 4 + 160 \times 8 - 160 \times 4 = 0$

$H = 160\ KN$

Ex:- 2)



$V_A + V_B = 0$

$\Sigma M_A = 0$

$-V_B \times 16 + 20 \times 4 \times 2 = 0$

$V_B = 10\ KN\ (\uparrow)$

$V_A = -10\ KN\ (\downarrow)$

$\Sigma M_c = 0$ (Right part of Hinge at c)

$+H_B \times 4 - 10 \times 8 = 0$

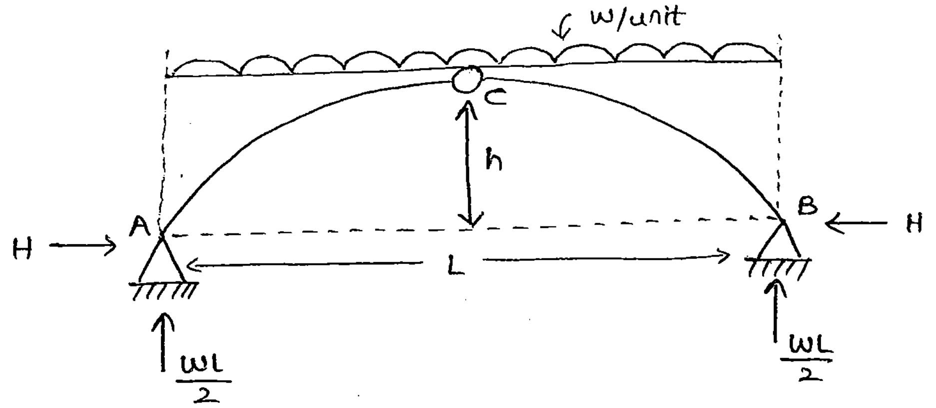
$H_B = +20\ KN$

$-(H_A + H_B) = -80$

$H_A = +60\ KN$

Symmetrical three hinged parabolic arches:-

EX:-3

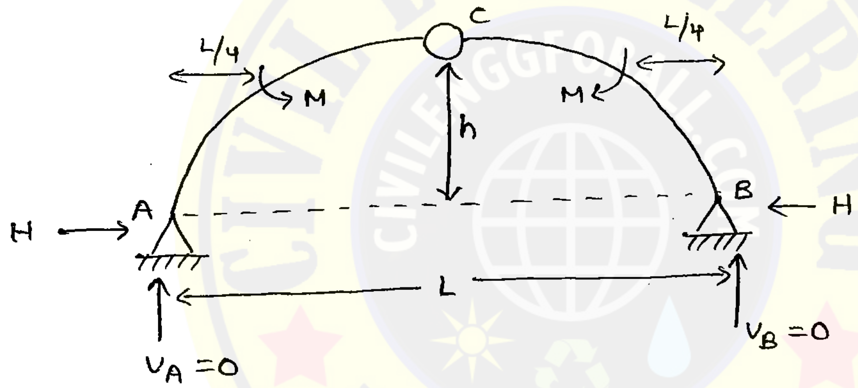


$\Sigma M_c = 0$

$$\frac{WL}{2} \times \frac{L}{2} - H \times h - \left(\frac{WL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$\boxed{H = \frac{WL^2}{8h}}$$

EX:-4)



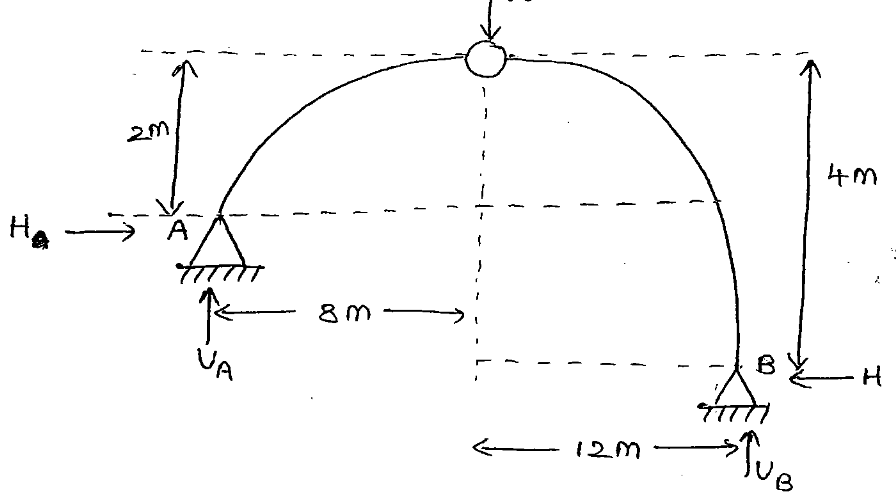
$\Sigma M_c = 0$ (Left part of Hinge at C)

$$-H \times h - M = 0$$

$$\boxed{H = \frac{-M}{h}}$$

Three hinge arch with supports at different levels:-

1. Since the supports are at the different levels arch cannot be treated as a simply supported beam.
2. Applying $\Sigma y = 0$ for an arch subjected to external loading
3. Take the moments about internal hinge one time to the left side of the internal hinge and second time and the right side of the hinge and make them equals to zero.



$$V_A + V_B = W$$

$\Sigma M_c = 0$ (left side)

$$-H_A \times 2 + V_A \times 8 = 0 \longrightarrow (1)$$

$\Sigma M_c = 0$ (Right side)

$$H \times 4 - V_B \times 12 = 0 \longrightarrow (2)$$

solve (1) & (2)

$$-2H + 8V_A = 0 \longrightarrow (1) \times 2$$

$$4H - 12V_B = 0$$

$$= -4H + 16V_A = 0$$

$$16V_A - 12V_B = 0$$

$$4V_A - 3V_B = 0$$

$$V_A = \frac{3V_B}{4}$$

$$\frac{3V_B}{4} + V_B = W$$

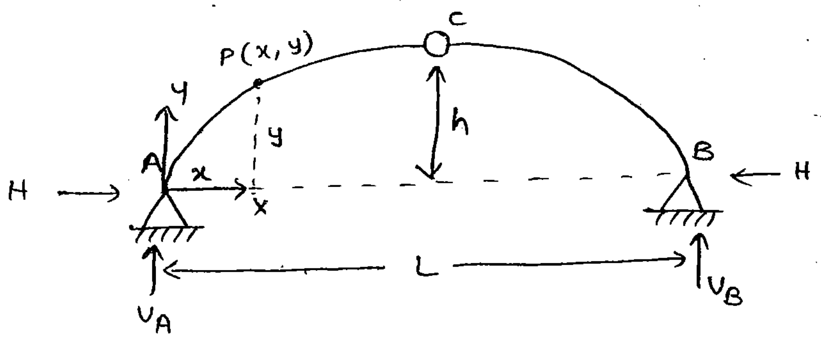
$$\frac{7V_B}{4} = W$$

$$V_B = \frac{4W}{7}$$

$$V_A = \frac{3\left(\frac{4W}{7}\right)}{4} = \frac{12W}{7 \times 4} = \frac{3W}{7}$$

$$\therefore H = \frac{12W}{7}$$

Three hinged parabolic arches:-



Equation of parabola w.r.t support 'A'

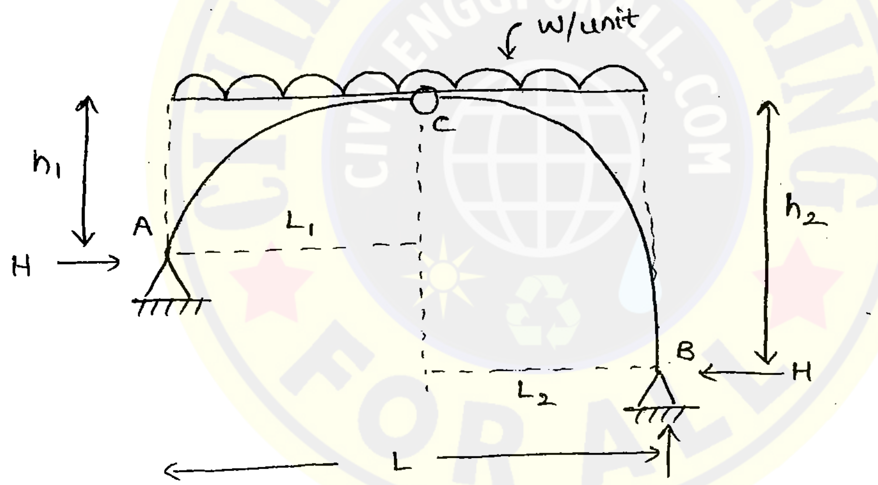
$$y = \frac{4h}{L^2} x(L-x)$$

w.r.t crown 'c'

$$\frac{x^2}{y} = \text{constant}$$

$$\frac{x}{\sqrt{y}} = \text{constant}$$

Ex:-



$$\frac{L_1}{\sqrt{h_1}} = \frac{L_2}{\sqrt{h_2}} = \frac{L_1 + L_2}{(\sqrt{h_1} + \sqrt{h_2})} = \frac{L}{(\sqrt{h_1} + \sqrt{h_2})}$$

$$L_1 = \frac{L\sqrt{h_1}}{(\sqrt{h_1} + \sqrt{h_2})}$$

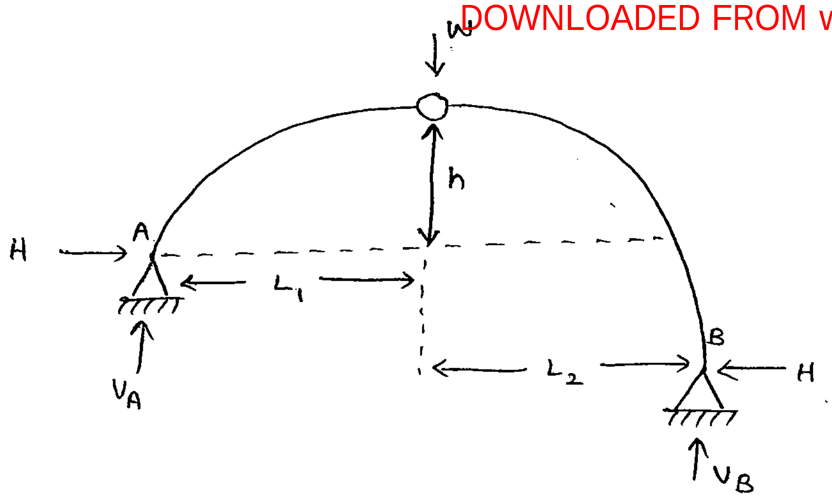
$$L_2 = \frac{L\sqrt{h_2}}{(\sqrt{h_1} + \sqrt{h_2})}$$

$$V_A = \frac{WL_1}{2} + H \cdot \frac{h_1}{L_1}$$

$$H = \frac{WL^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$

$$V_B = \frac{WL_2}{2} + H \cdot \frac{h_2}{L_2}$$

Ex :-



$$V_A = H \cdot \frac{h_1}{L_1}$$

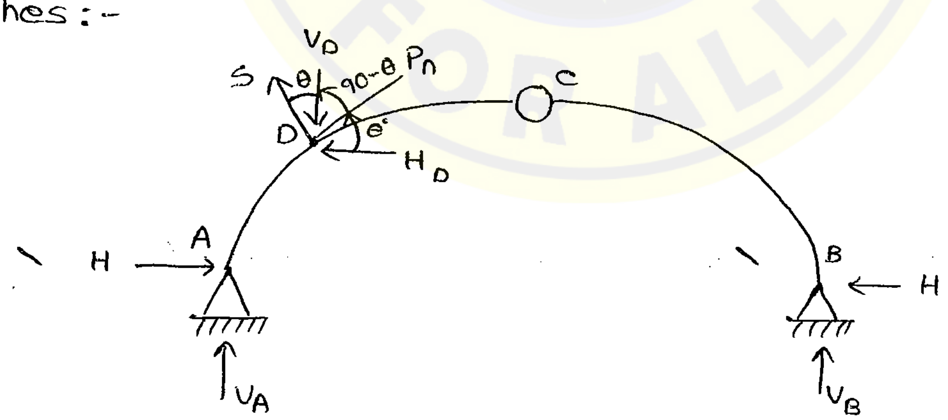
$$V_B = H \cdot \frac{h_2}{L_2}$$

$$H = \frac{WL}{(\sqrt{h_1} + \sqrt{h_2})^2}$$

Note :-

1. BM at any section of a three arches is equal to zero when arch is subjected to UDL.
2. SF at any section is also zero.
3. It has only axial thrust.

Normal thrust and Radial shear in two hinged / three hinge arches :-



Normal thrust :-

1. It is a force acts on a direction of tangent at a point.

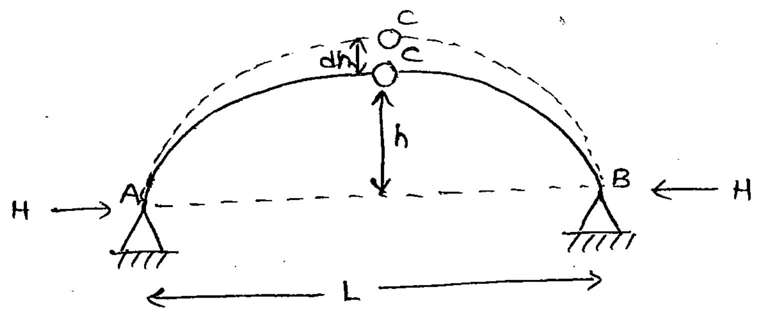
$$\boxed{P_n = H_D \cos\theta + V_D \sin\theta}$$

2. Radial shear :-

1. It is a force acts along perpendicular to the direction of tangent

$$\boxed{S = H_D \sin\theta - V_D \cos\theta}$$

Temperature effect on three hinged arches :-



When it is raised by $t^\circ C$, there is no thermal stresses

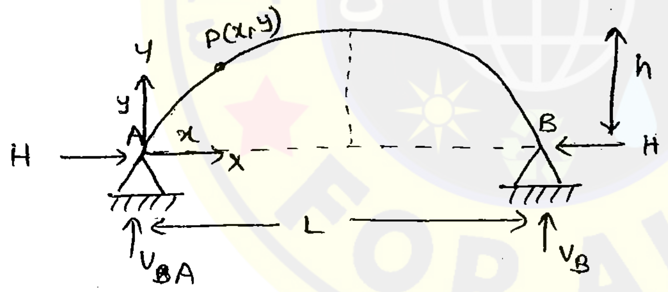
$$dh = \left[\frac{L^2+4h^2}{4h}\right] \alpha T$$

$$\frac{D_H}{H} = \frac{-dh}{h}$$

Note:-

1. If temperature increases horizontal thrust decreases

~~Temperature effect on~~ two hinged arch:-



1. It is a statically indeterminate to 1°
2. verticle reactions V_A and V_B can be calculated by taking moments about either hinge if both ends are at the same level.
3. Horizontal reaction can be calculated from the condition that the horizontal displacement of either hinge w.r.t other is zero

$$M_x = M - Hy$$

M = Beam moment

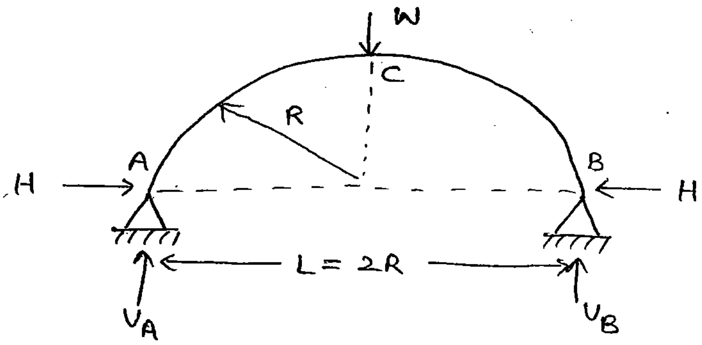
$H \cdot y$ = 'H' moment

$$H = \frac{\int M y \cdot ds}{\int y^2 \cdot ds}$$

Standard cases:-

1. semi circular two-hinged Arches:-

(a)

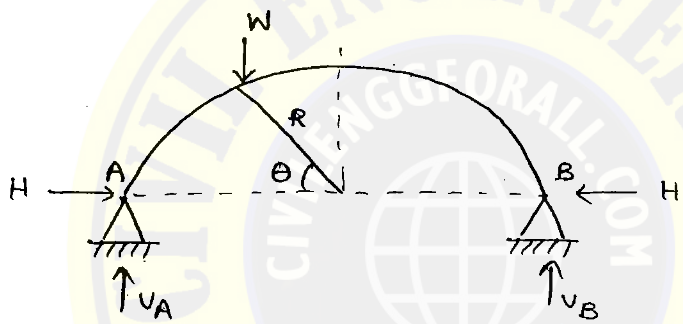


$$\boxed{H = \frac{W}{\pi}}$$

Note:-

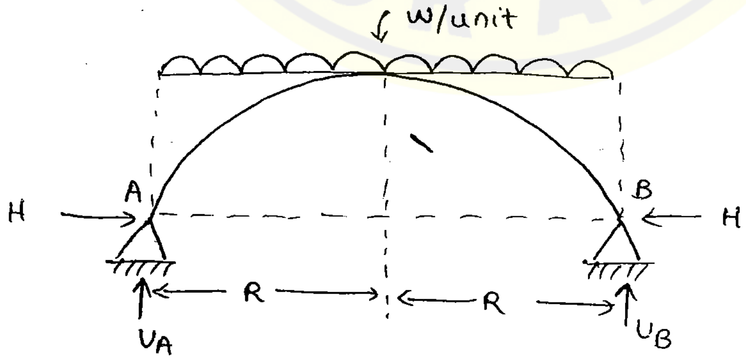
Horizontal reaction is independent of the radius of the arch.

(b)



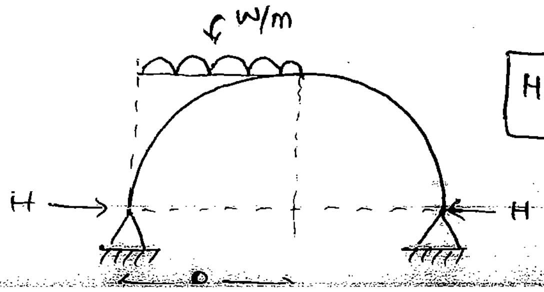
$$\boxed{H = \frac{W}{\pi} \sin^2\theta}$$

(c)

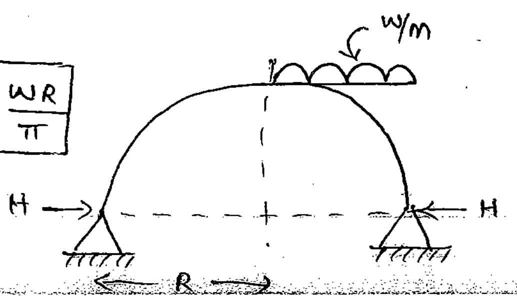


$$\boxed{H = \frac{4}{3} \frac{wR}{\pi}}$$

(d)

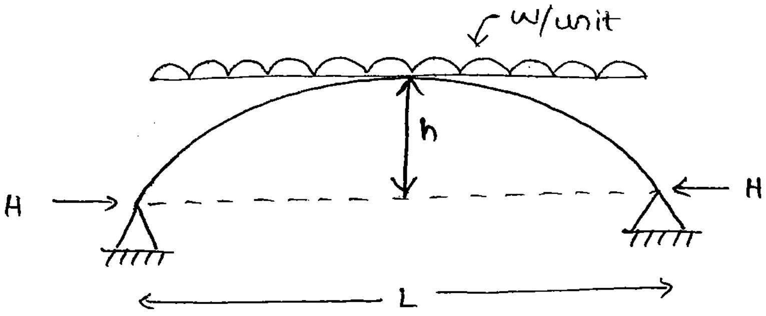


$$\boxed{H = \frac{2}{3} \frac{wR}{\pi}}$$



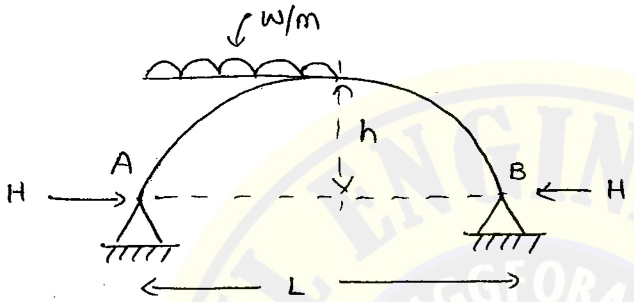
2. Two hinged parabolic arches :-

(a)



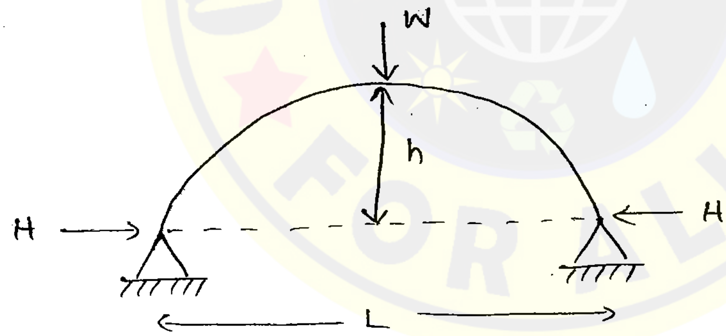
$$\boxed{H = \frac{wL^2}{8h}}$$

(b)



$$\boxed{H = \frac{wL^2}{16h}}$$

(c)



$$\boxed{H = \frac{25}{128}\frac{WL}{h}}$$

Temperature effect on two hinged arches :-

1. Since it is a statically indeterminate structure thermal structure will be produce due to yielding of the hinge supports.
2. Max BM due to raise of temperature $M = H \cdot h$. therefore Max. stress due to temperature rise $\boxed{\sigma = \frac{M}{Z}}$

1. For two hinged semi circular arch subjected to temperature rise 't' horizontal reaction 'H'

$$H = \frac{4EI\alpha T}{\pi R^2}$$

2. For two hinged parabolic arches under temperature rise the horizontal reaction

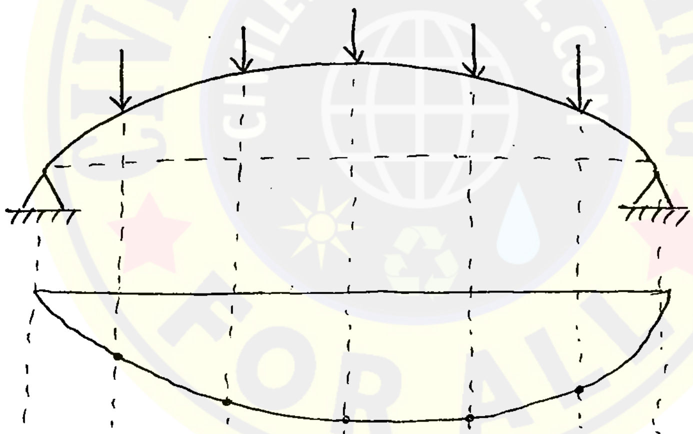
$$H = \frac{15EI\alpha T}{8h^2}$$

Note:-

1. In two hinged arches temperature rises horizontal reactio 'H' increases.

Linear Arch (or) theoretical arch (or) pressure line:-

1. Linear arch is the one which represents the thrust line
2. For a system of point loads it is a funicular polygon.



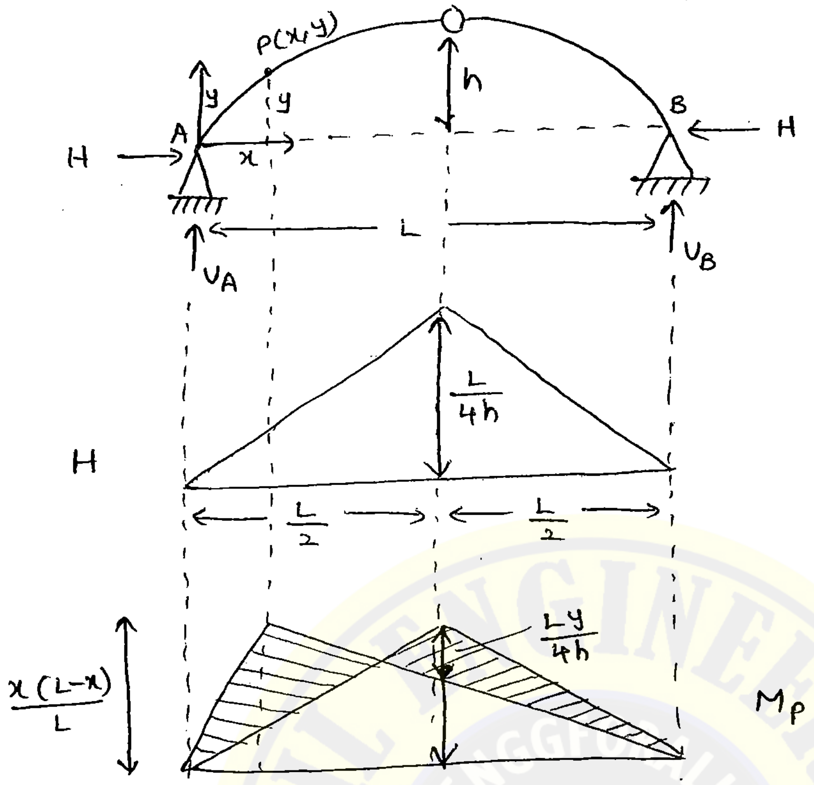
3. Linear arch for a UDL is a parabolic shape
4. Eddys theorem can be used for calculating the BM in a structure for a given loading

Eddy's theorem :-

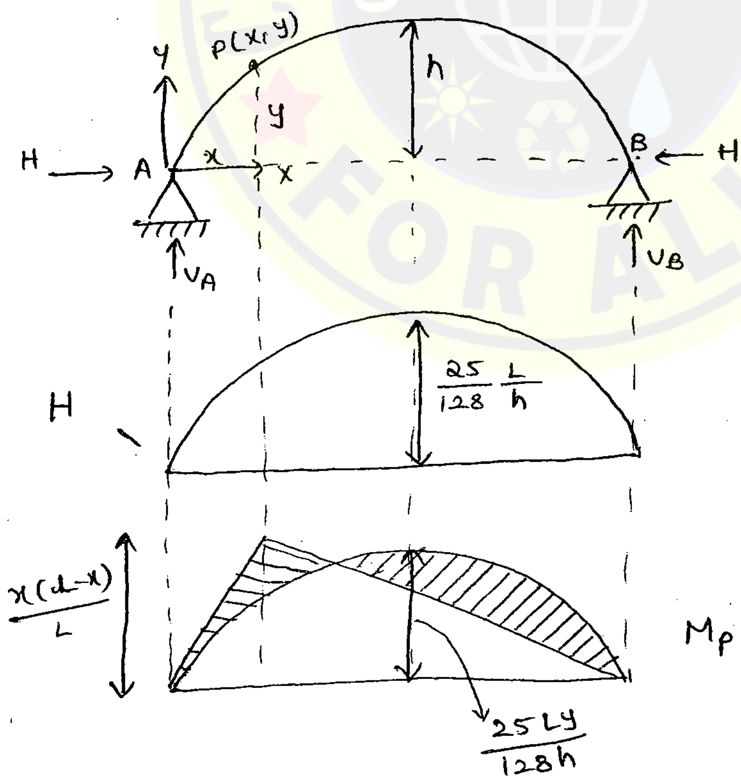
1. The BM at any section of an arch is proportional to th ordinate i.e., intersect between given arch and Linear arc

ILD's for arches :-

1. Three hinged arch :

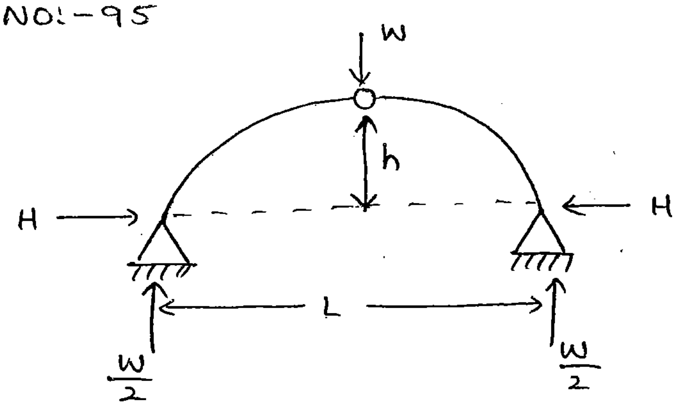


2. Two hinged arch :-



P.9 NO:-95

8.



(Left) $\Sigma M_c = 0$

$$-H \times h + \frac{W}{2} \times \frac{L}{2} = 0$$

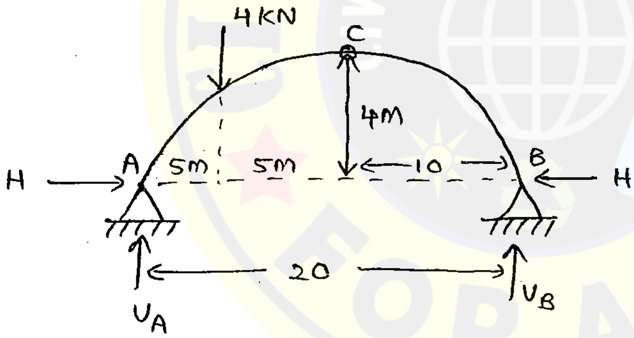
$$H = \frac{WL}{4h}$$

$$\frac{L}{\cancel{H}h} = \frac{4H}{W}$$

$$= \frac{4W}{W}$$

$$\frac{L}{h} = 4$$

10.



$$V_A + V_B = +4$$

$\Sigma M_c = 0$ (Left)

$$-H \times 4 + 10 V_A - 20 = 0$$

$$10 V_A = 20 + 4H$$

$$V_A = \frac{20 + 4H}{10} =$$

$$\cancel{V_B} = \frac{-60 - \cancel{4H}}{10}$$

$$30 = 20 + 4H$$

$$4H = 10$$

$$H = 2.5 \text{ kN}$$

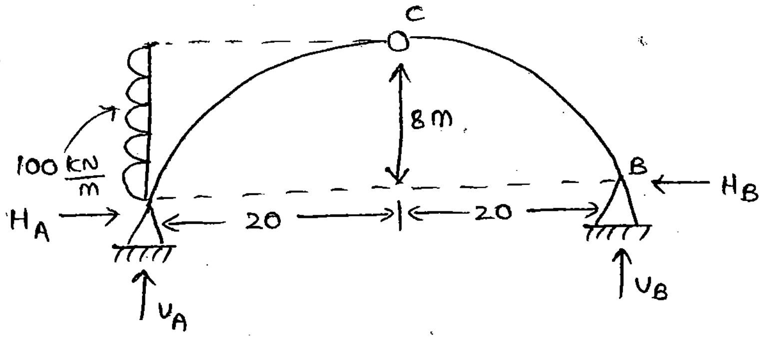
$\Sigma M_B = 0$

$$20 V_A - 60 = 0$$

$$V_A = 3 \text{ kN}$$

$$V_B = 1 \text{ kN}$$

1.



$\Sigma M_A = 0$

$(100 \times 8) 4 - 40 V_B = 0$

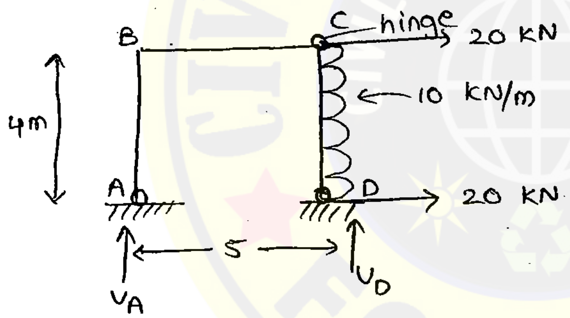
$V_B = 80$ kN

$\Sigma M_c = 0$ (Right)

$-80(20) + 8 H_B = 0$

$\boxed{H_B = 200 \text{ kN}}$

3.



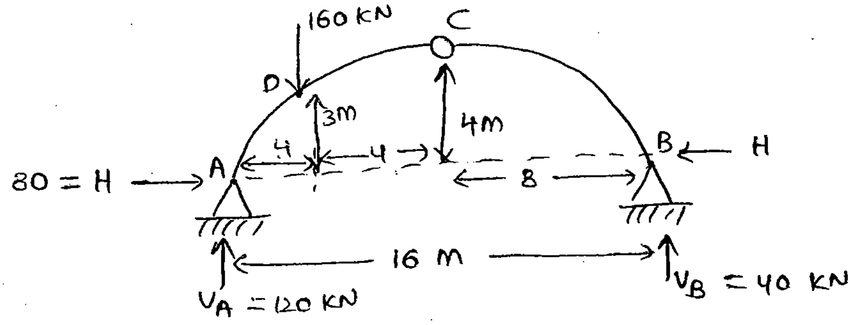
$\Sigma M_D = 0$

$5 V_A + 80 \cancel{-(4 \times 10 \times 2)} = 0$

$5 V_A = 80$

$V_A = 16$ kN

5.



$y = \frac{4h}{L^2} x(L-x)$

$= \frac{4(4)}{16^2} (4)(16-4)$

$y = 3m$

$V_A + V_B = 160$

$\Sigma M_B = 0$

$16 V_A - (160 \times 12) = 0 \Rightarrow V_A = 120$ KN, $V_B = 40$ KN