UNIT - 1

A₃ - civil

SIMPLE STRESS AND STRAINS

The main aim of strength of material is to access various properties of material and use them according to ťt requirement.

Strength:-

Ability of a material to resist external load against failure.

Note:-

1. The primary design parameter is strength.
2. All the designs of Engineering are strength based design only.

Stiffness:-

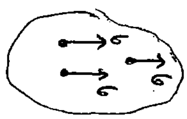
Ability to resist deformation is stiffness. stiffness is the secondary design parameter.

$\uparrow$ Deflection : $\downarrow$ stiffness
$\uparrow$ flexible] reciprocal to each other.

Assumptions:-

1. Material is solid and continuous. No cracks in the material and no voids in the material.
2. Material are homogeneous and Isotropic.

$\underbrace{\text{Homo}}_{\text{same}}$ $\underbrace{\text{geneous}}_{\text{origin}}$

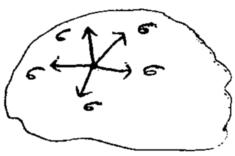


@ any point in one direction

EX:- Wood, Iron

All the base materials without any mixture is called Homogeneous.

$\underbrace{\text{Iso}}_{\text{same}}$ $\underbrace{\text{tropic}}_{\text{directional property}}$



@ one point in any direction

EX:- Fine grained material like silver, copper, Iron, brass

Note :-

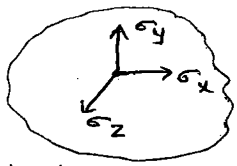
Wood is a homogeneous but not Isotropic. Brass is a Isotropic but not homogeneous.

All homogeneous material need not be isotropic and vice versa but few homogeneous material are also Isotropic.

Ortho tropic :-

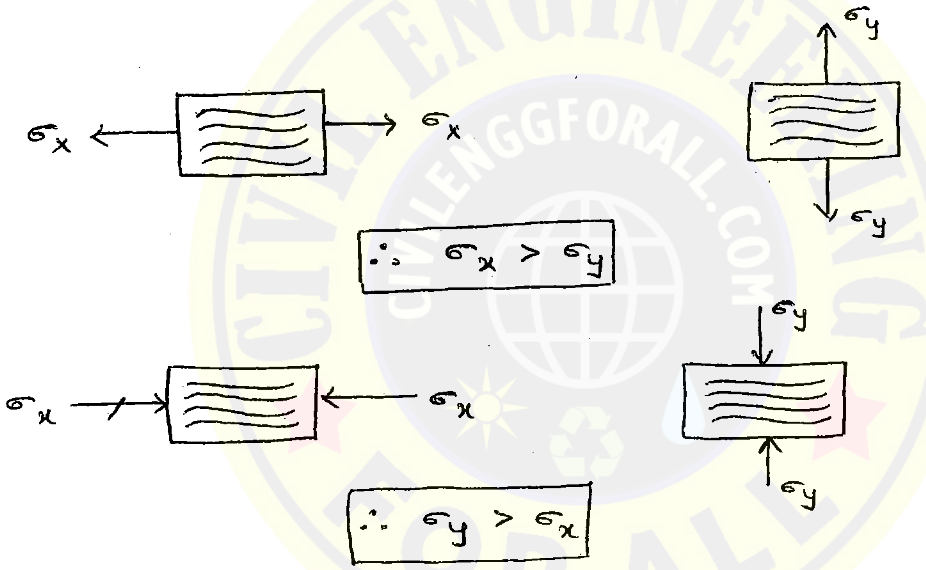
Ortho = perpendicular

tropic = direction point.



At one point in perpendicular direction but properties are different.

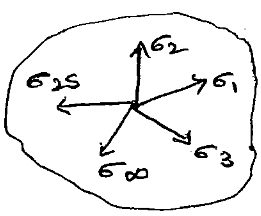
Eg:- Layered material are Orthotropic, like wood, coal, Mica, asbestos



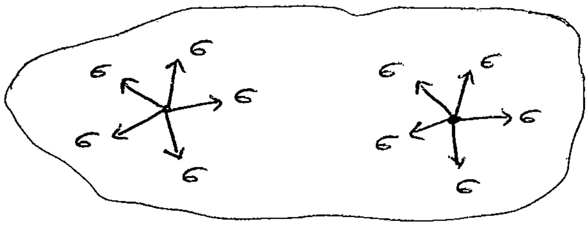
Anisotropic (or) Aleotropic (or) Non Isotropic :-

At any one point in different direction properties are different.

Ex:- vo material with voids and cracks



Homogenous + Isotropic :-



At any point in any direction property is same.

Eg:- Iron, Gold, Silver.

Assumptions:-

3. self weight is ignored.

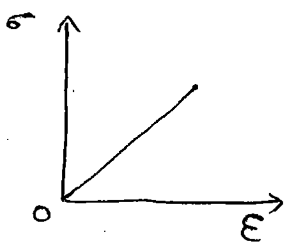
with self wt.   self wt = 0

4. Superposition principle is valid.

Algebraic sum of total effects is equal to the resultant is called super position.

Limitations:-

1. All the loads acting over a member should be within proportionality limit (Hooke's law is valid) (or) the member must be linearly elastic.

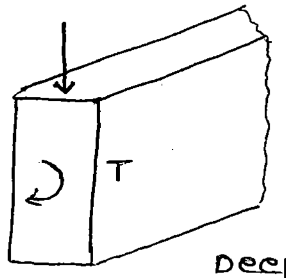


(linearly elastic)

2. The slopes and deflections due to loading must be as small as possible.

3. Super position principle is not valid for Impact load, strain energies, deep beams.

In case of deep beams due to loading torsion develops which makes distorsion in the shape. Therefore the principle is not valid.



D > 750 mm

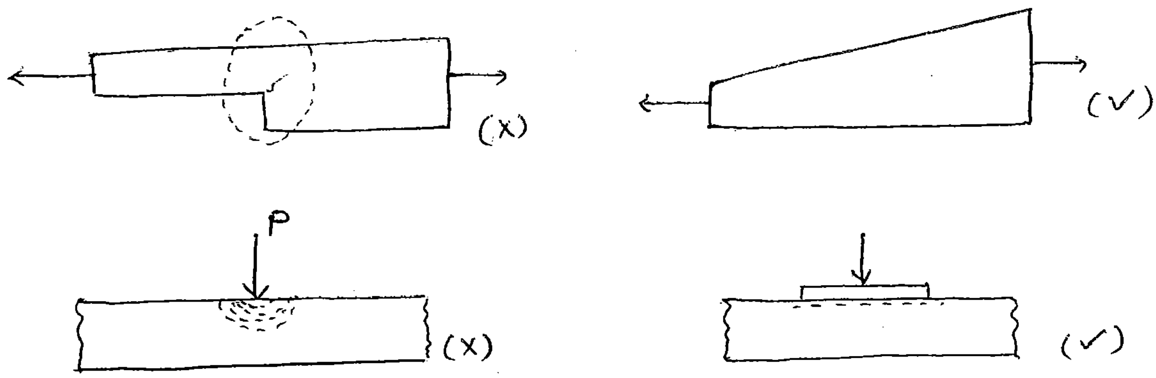
Deep beams

4. Circular shaft subjected to torsion super position is valid, non circular shaft with torsion the principle is not valid.

5. Long columns, sinking of supports, this principle is not valid.

Assumptions:-

5. St. Venants principle is valid. Sudden variation of any parameters causes stress concentration and leads to failure.



Stress :-

The resisting force per unit area is stress. The external force per unit area is pressure.

pressure is external, stress is internal

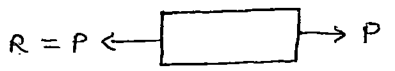
$$\sigma, p = \frac{\text{Resisting force}}{\text{unit area}}$$

$$\sigma = \frac{R}{A}$$

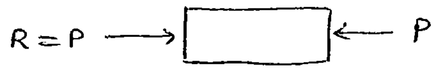
$$\sigma = \frac{P}{A}$$

Roof trusses

TIE = tension



Strut = compression



Unit :- N/m^2 (or)

SI units :-

Force = N

Distance = m, mm

time = sec

kilo, k = 10^3

M = 10^6

G = 10^9

milli = 10^{-3}

micro (μ) = 10^{-6}

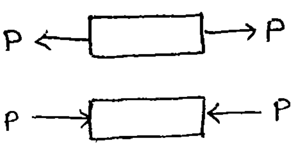
$1\ M{\cdot}Pa = 10^6\ Pa = 10^6\ N/m^2$ (or) $10^6\ N/mm^2$

$1\ G{\cdot}Pa = 10^3\ M{\cdot}pa = 10^3\ N/mm^2$

Stress

- Direct
  - Normal (P)
  - shear (τ)
- Indirect
  - Normal
  - shear

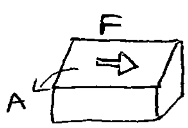
Normal (P)



$\sigma = \frac{R}{A} = \frac{P}{A}$

It is perpendicular to surface

shear (τ)

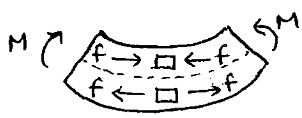


$\tau = \frac{F}{A}$

It is also called tangential force

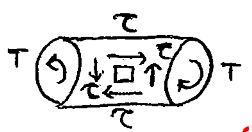
It is tangential to the surface

Normal



f = Bending stress

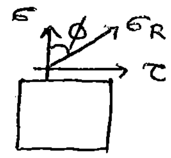
shear



shaft with torsion

τ = torsional shear stress

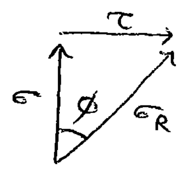
In any material stresses are broadly divided into normal and shear stresses



$$\sigma_R = \text{Resultant stress} = \sqrt{\sigma^2 + \tau^2}$$

Obliquity (ϕ) :-

Angle between normal stress and resultant stress on any plane.



$$\tan\phi = \frac{\tau}{\sigma}$$

Strains :-

1. Normal strain :- (e, ε)

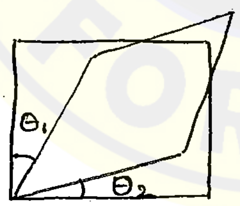
Ratio of change in dimension to original dimension

$$\boxed{\varepsilon = \frac{\delta l}{l}}$$

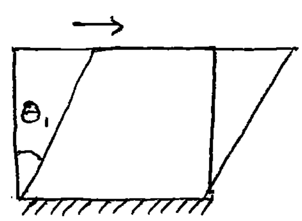
Units :- No units (or) $\frac{mm}{mm}$

2. Shear strain (ϕ, γ) :-

Angular change between any mutually perpendicular planes in radians



$$\phi = \theta_1 + \theta_2$$



$$\phi = \theta_1 + 0 = \theta_1$$

$\theta_2 = 0$

Note :-

1. Radian is a ratio, therefore shear strain is also having "no units"

3. Volumetric strain (e_v, ε_v) :-

$$\boxed{\varepsilon_v = \frac{\delta v}{v}}$$

Units :- No units

Note :-

For incompressible fluids like water, volumetric strain is zero

Strain is independent parameter, stress develops due to strain.

Material properties :-

1. Elasticity :-

A material which regains its original size and shape on removal of load.

2. Plasticity :-

A material undergoes permanent deformation at constant loading is plasticity.

Ex :- Metals loaded beyond yield point undergoes plastic deformation.

3. Ductility :-

The property by which the material can be made into thin wire.

Ex :- Soft metals.

Ductility is related to tension. It is strong in tension and weak in shear, moderate in compression.

4. Malleability :-

The property by which the material is made into thin sheets.

It is related to compression by pressing or rolling.

5. Brittle :-

It fails suddenly such a material is called as Brittle. Brittle materials are strong in compression, moderate in shear, weak in tension.

Ex:- Glass, wood, cast iron.

6. Creep :-

The permanent deformation occurs at a constant or sustained loading over a long period of time.

7. Toughness:-

Resistance of a material against sudden (or) impact loading.

8. Fatigue :-

Reduction in strength due to repeted loading is fatigue.

Stress strain curves :-

1. Low carbon steel (Fe 250)

Eg:- Mild steel

purpose of carbon :-

It increases the strength.

purpose of manganese:-

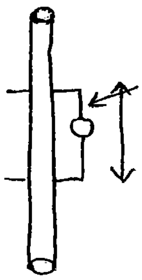
It increases the toughness.

U.T.M :-

Strain gauge (extenso meter) is used to measure the strain

A = initial / nominal c/s area

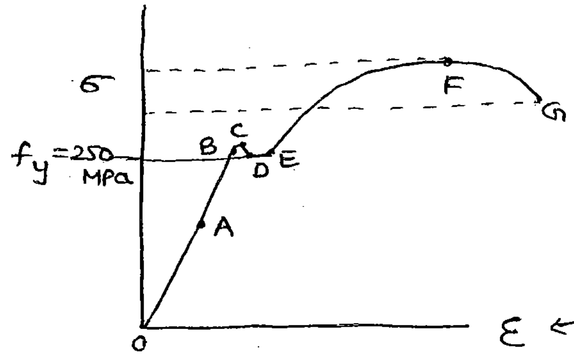
Length over which all extensions are measured



gauge length = $5.65\sqrt{A}$

→ Gauge length depends on "c/s area (or) dia of bar" (independent of length)

5



← Mild steel

ε ← independent

$$\varepsilon = \frac{\delta\,(\text{Gauge length})}{G.L}$$

$$\sigma = \frac{R}{A}$$

σ = Nominal (or) initial (or) Engineering stress (stress)

$$\sigma_0 = \frac{R}{A_0}$$

σ_0 = True stress (or) Instantaneous (or) actual stress

A_0 = initial (or) nominal cross section.

→ A = proportionality limit

Upto 'A' stress is linearly proportional to strain (Hooke law is valid) and 'OA' is a straight line.

→ B = Elastic limit

Upto 'B' material is elastic (can regain back to original shape and size).

A to B graph is slightly curved. Therefore super position principle and Hooke's law are not valid.

∴ Hooke's law is valid only upto proportionality limit.

→ C = Upper yield point

In the yield zone material starts permanent deformation.

→ D = Lower yield point

→ DE = plastic zone.

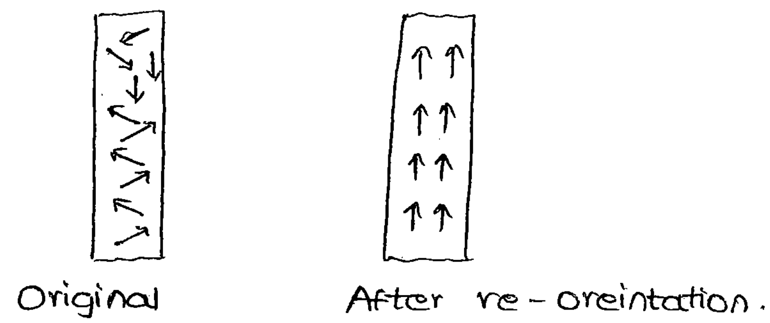
During plastic zone permanent deformation continues.

* The design stress for mild steel is corresponding to the lower yielding point.

* The position of lower yielding point is fixed and will not change with the shape of cross section.

→ F = ultimate stress

In plastic zone reorientation of molecules occur due to these steel originally alloy (non-homogeneous) becomes Homogeneous



→ G = Failure point

Zones:-/

OB = elastic zone

OA = Linearly elastic zone

AB = Non linear elastic zone

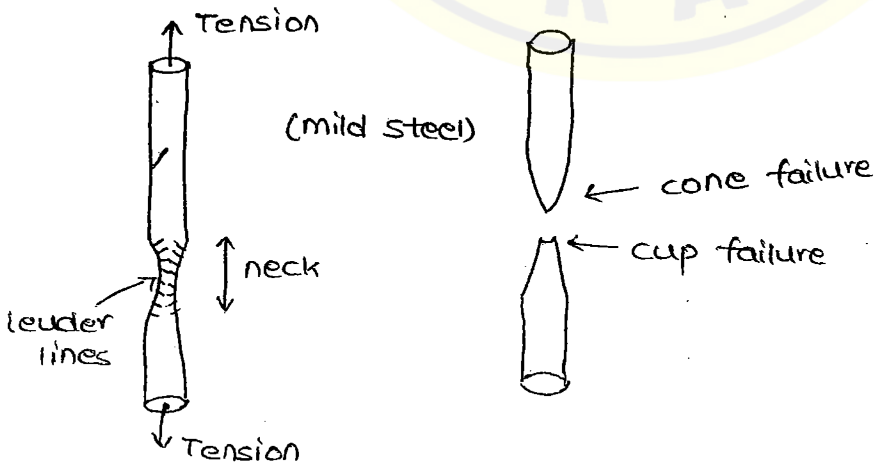
BC = Almost coincides

CD = yield zone

DE = plastic zone

EF = strain hardening zone

FG = strain softening zone (or) necking zone



The reason behind cup cone failure is shear failure. In the neck zone 45° micro cracks develop these are called Leuder lines. (Develop due to shear).

* strain at yield = 0·002 = 0·2%

* strain at failure (ε_{fail}) = 200 to 250 (yield strain)

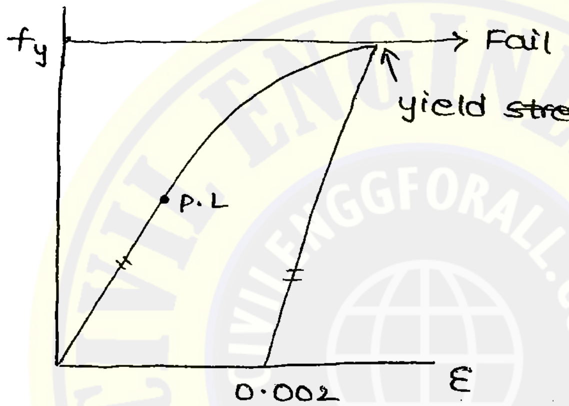
High carbon steels :-

1. High yield strength Deformed bars (HYSD) — Fe 415, Fe 500

2. Thermo Mechanically Treated (TMT) — Fe 415 → f_y = 415 M·Pa, Fe 500 → f_y = 500 M·Pa

↑ More carbon :
- Strength ↑
- Hardness ↑
- Ductility ↓
- Toughness ↓
- Brittle ↑

Offset method:-

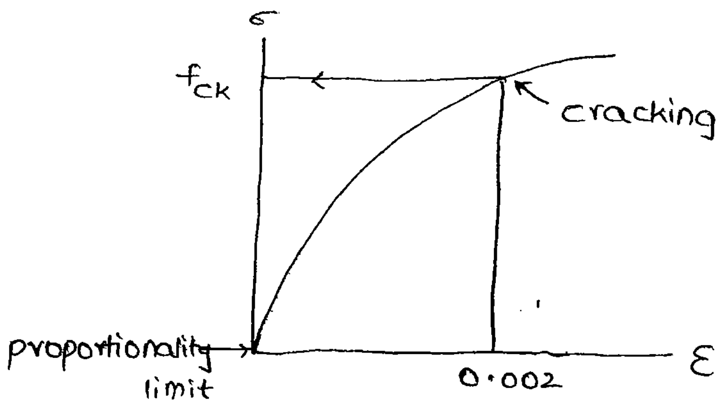


f_y = yield stress (or) proof stress

In the material where yield point is not visible on the stress strain curve, using offset method with a fixed value of strain (0·2%) can be located called Proof stress also.

Brittle :-

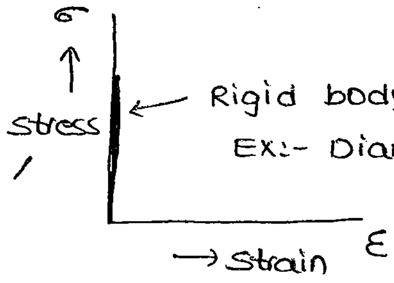
Fails suddenly. In brittle material proportionality limit coincide with origin.



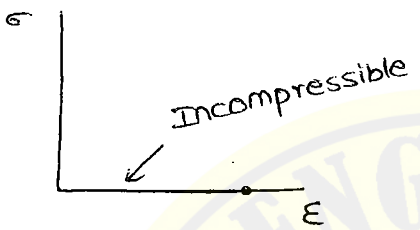
f_{ck} = cracking (or) Design stress

Idealised stress strain curves ($\sigma - \varepsilon$) :- (Assumed)

In a rigid body no dimensional changes and no volumetric changes.



Stress strain curve parallel to y-axis then it is called <u>Rigid material</u>

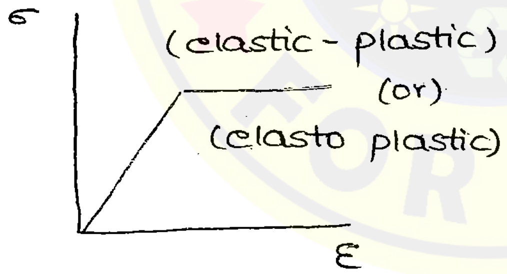


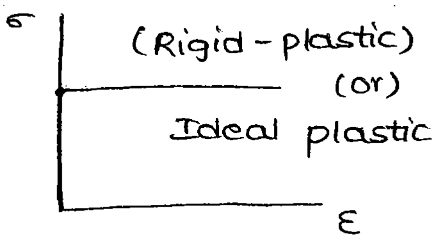
Stress strain curve is parallel to x-axis then it is called <u>Incompressible material</u>

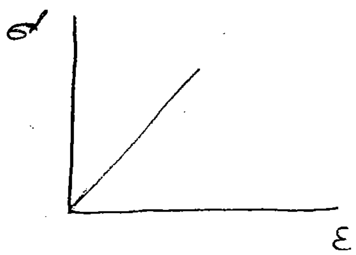
Ex:- water ($\varepsilon_v = 0$)

In a rigid body there will be no dimensional changes and no volumetric changes.

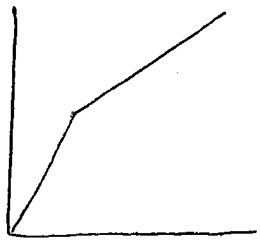
In Incompressible fluid there will be dimensional changes and no volumetric changes.



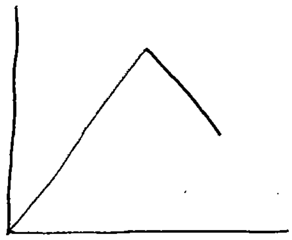




Linear elastic



Elastic - strain hardening



elastic - strain softening (necking)

** Factor of safety :-

For Ductility, $F.S = \frac{\text{yield stress}}{\text{working stress}}$ Eg :- Steel, Gold etc

For Brittle, $F.S = \frac{\text{ultimate stress}}{\text{working stress}}$ Eg:- Wood, Diamond etc

Margin of safety, $F.S \geq 1$

Scalar :-

Magnitude + No direction {pressure, work, energy}

Vector :-

Magnitude + one direction {force}

Tensor :-

Magnitude + more than one direction {stress, strain, Moment of Inertia}

Elastic constants :-

youngs modulus (E) :-

Hooke's Law within Elastic Limit

$\sigma \propto \varepsilon$ (exactly upto proportionality limit)

$\sigma = E \cdot \varepsilon$

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

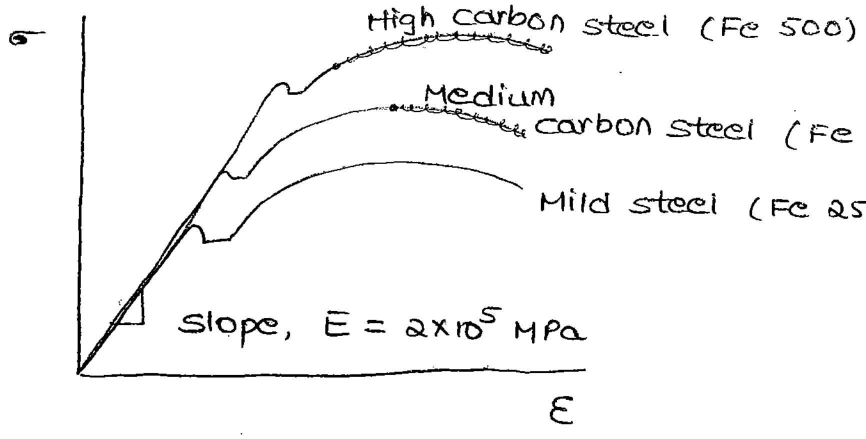
E = Non zero positive value and constant for a given material

$E_s = 200 \text{ GPa}$

$= 200 \times 10^3 \text{ MPa} \Rightarrow 2 \times 10^5 \text{ MPa}.$

Note:-

For all grades of steel young's modulus (E) is same



1. youngs modulus of Diamond ($E_{diamond}$) = 1200 G.Pa
2. youngs modulus of Rubber (E_{Rubber}) = 10 G.Pa

↑ E : ↑ elasticity

↑ slope to σ – ε curve

Modulus of Rigidity (C, N or G):-

$$\tau = G \cdot \phi$$

$$\boxed{C = N = G = \frac{\tau}{\phi}}$$

ϕ = shear strain

↑ G = ↓ ϕ

↓ distorsion in shape.

Bulk modulus (K):-

Within elastic limit

$\sigma \propto \varepsilon_v$ (exactly upto proportionality limit)

$\sigma = K \cdot \varepsilon_v$

$$\boxed{K = \frac{\sigma}{\varepsilon_v}} \qquad \boxed{\varepsilon_v = \frac{\delta v}{v}}$$

σ = volumetric stress (Normal stress all around volume)
(or)
(hydrostatic pressure)

K = Bulk modulus (or) Dialation constant.

Dialation = which causes change in volume.

$\therefore$ $\boxed{E > K > G}$ → For Isotropic material.

All are having units of stress

Poisson's ratio ~~→~~ (μ, γ, $\frac{1}{m}$) :-

$$\mu = \frac{\text{Lateral strain}}{\text{~~Longitudi~~ Linear strain}}$$

$$\boxed{\mu = \frac{\varepsilon_{lateral}}{\varepsilon_{linear}}}$$

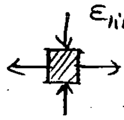
Units :- No units

Range of μ, -ve to 0.5 for General material

μ is -ve for Genetic material.

μ = 0 to 0.5 for Engineering material

μ = 0 for cork



* $\mu = \dfrac{\varepsilon_{lateral} = 0}{\varepsilon_{linear}} = 0$ (cork)

μ = 0.5 for clay, Rubber, paraffin wax

* μ = 0.25 for Isotropic material

μ = 0.3 for steel

* μ = 0.15 for concrete

Relation between elastic constants (E, K, G, μ) :-

1. $E = 2G(1+\mu)$ → ①

2. $E = 3K(1-\mu)$ → ②

3. $$\boxed{\mu = \frac{3K-2G}{6K+2G}}$$

From ① $\mu = \dfrac{E}{2G} - 1$ sub in ②

$$\boxed{E = \frac{9KG}{3K+G}}$$

	Type of material	Independent Elastic constants
1.	Homogeneous + Isotropic	2 (E & μ)
2.	Homogeneous + Orthotropic	9
3.	Homogeneous + Anisotropic	21

P.9 NO:- 7

2. Non dilatant material = In compressible fluid (or) NO change in volume

3. $E = 2G(1+\mu)$

$\mu = 0$ (Assume)

$\frac{G}{E} = 0.5$

$\mu = 0.5$ (Assume)

$\frac{G}{E} = 0.33$

$\therefore \boxed{G = 0.33 \text{ to } 0.5\, E}$

4. $\frac{E}{G} = 2(1+\mu)$

$= 2(1+0.25)$

$= 2.5$

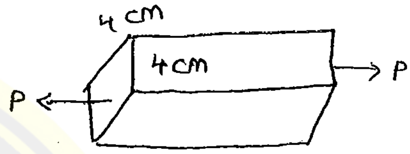
5. Given $K = G$

$\mu = \frac{3K - 2G}{6K + 2G}$

$= \frac{3-2}{6+2}$

$= \frac{1}{8} = 0.125$

6.



1 ton = 1000 kg

$P = 16000$ kg

$\sigma = \frac{P}{A}$

$= \frac{16000}{4 \times 4}$

$= 10000$ kg/cm²

$\varepsilon = \frac{\delta l}{l} = \frac{0.1}{200} = 0.0005$

$E = \frac{\sigma}{\varepsilon} = 2 \times 10^6$ kg/cm²

$\mu = \frac{1}{4}$ (given)

$E = 2G(1+\mu)$

$G = 0.8 \times 10^6$ kg/cm²

7. $\mu = \frac{\varepsilon_{lat}}{\varepsilon_{line}} = \frac{(\delta D/D)}{(\delta l/l)}$

$= \frac{\left(\frac{0.0018}{4}\right)}{\left(\frac{0.03}{20}\right)}$

$= 0.3$

8.

9. $\mu = \frac{1}{4}$

$E = 2 \times 10^5$ M·Pa

$E = 2G(1+\mu)$

$2 \times 10^5 = 2G\left(1+\frac{1}{4}\right)$

$2 \times 10^5 = 2G\left(\frac{5}{4}\right)$

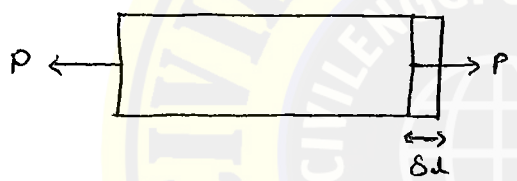
$G = 0.8 \times 10^5$ M·Pa

P.g NO:- 8

2. If $\mu = 0.5$ then $\delta v = 0$

LINEAR AND VOLUMETRIC CHANGES OF BODIES

Prismatic bar (bar with uniform c/s) subjected to axial load.



$\sigma = \frac{P}{A}$

$\epsilon = \frac{\delta l}{l}$

$E = \frac{\sigma}{\epsilon} = \frac{(P/A)}{(\delta l/l)}$

$$\boxed{\delta l = \frac{Pl}{AE}}$$

where

P = axial force

l = initial length

A = Area of cross section

E = young's modulus.

AE = Axial Rigidity

Units :- N

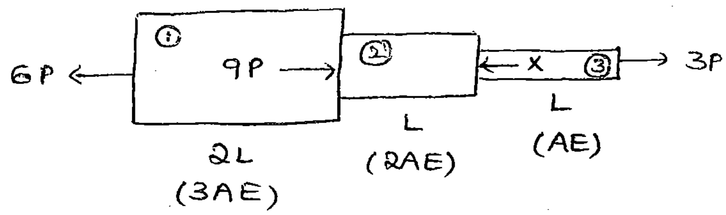
$\uparrow$ AE : $\downarrow$ δl

AE = ∞ (infinity) for rigid body

δl = 0 for perfectly rigid ...

Composite bars :-

Ex:-

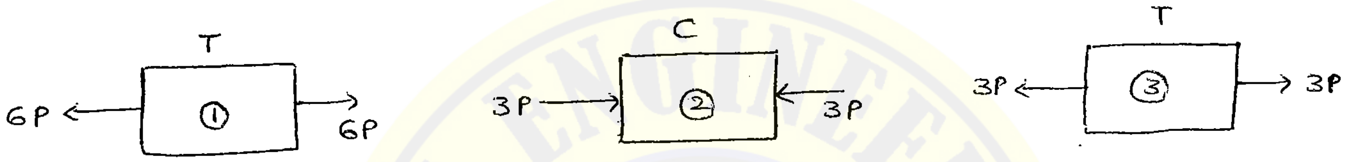


$\Sigma F_x = 0 \quad (\rightarrow \oplus)$

$+3P - x + 9P - 6P = 0$

$x = +6P$

If 'x' is -ve, change the given direction



$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

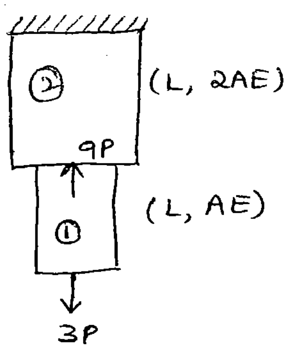
$$= \frac{6P(2L)}{3AE} + \frac{(-3P)(L)}{2AE} + \frac{(3P)(L)}{AE}$$

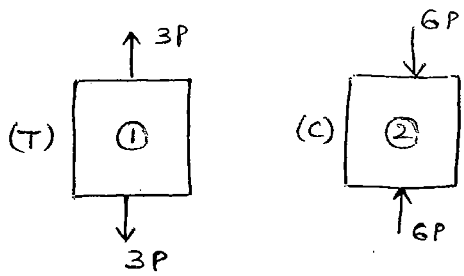
$$= \frac{24PL - 9PL + 18PL}{6AE}$$

$$= \frac{33PL}{6AE}$$

$$= +\frac{11PL}{2AE}$$ ($\uparrow$ in length) (elongation)

Ex:-





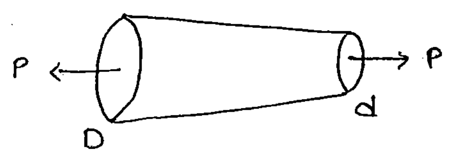
$$\delta l = \delta l_1 + \delta l_2$$

$$= \frac{3P(L)}{2AE} + \frac{(-6P)(L)}{2AE}$$

$$= 0$$

Tapering bars:-

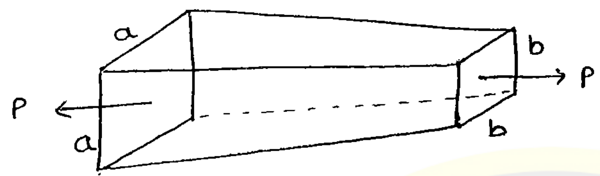
1. Circular



$$\delta l = \frac{Pl}{\frac{\pi}{4}(D \cdot d) \cdot E}$$

$$= \frac{4Pl}{\pi D \cdot d \cdot E}$$

2. Square

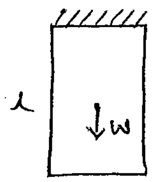


$$\delta l = \frac{Pl}{(a \cdot b)E}$$

3. Uniform thickness

$$\delta l = \frac{Pl}{t(B-b)E} \log_e\left(\frac{B}{b}\right)$$

** 4. Deformation due to self weight :-



$$\delta l_{sw} = \frac{Wl}{2AE} \rightarrow ① \quad W = \text{self weight}$$

* self weight deformation is half that of deformation due to external load.

weight density, $\gamma = \frac{W}{V}$

$W = \text{self weight} = \gamma V$

$= \gamma \cdot A \cdot l$ sub in eq ①

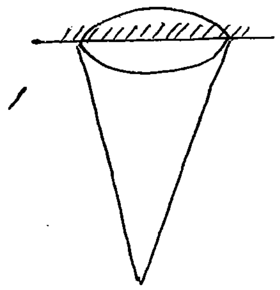
$$\delta l_{sw} = \frac{\gamma A l^2}{2AE}$$

$$\boxed{\delta l_{sw} = \frac{\gamma l^2}{2E}}$$

Note:-

self weight deformation is independent of cross sectional area is directly proportional to l^2

5. Conical bar hanging with self weight.



$$V_{cone} = \frac{1}{3} V_{cylinder}$$

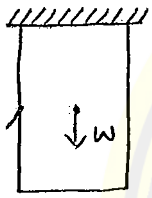
$$W_{cone} = \frac{1}{3} W_{cylinder}$$

$$\delta l_{cone} = \frac{1}{3}\left[\delta l_{SW\ cylinder}\right]$$

$$= \frac{1}{3}\left[\frac{\gamma l^2}{2E}\right]$$

$$\boxed{\delta l_{cone} = \frac{\gamma l^2}{6E}}$$

6. Self weight stress :-

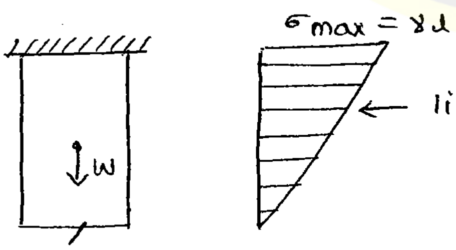


$(\sigma_{sw})_{@\ free\ end} = 0$

$(\sigma_{sw})_{@\ fixed\ end} = \frac{W}{A}$ ← σ_{max}

$\sigma_{max} = (\sigma_{sw})_{@\ fixed} = \frac{W}{A} = \frac{\gamma A l}{A} = \underline{\gamma \cdot l}$

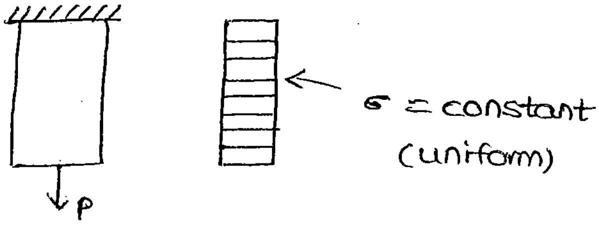
* Stress due to self weight is also independent of cross sectional area directly proportional to length.



* Stress due to self weight will be <u>linearly varied</u>.

7. Bar of uniform strength:-

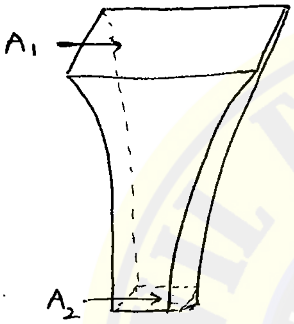
If a stress developed along a length of bar is the same then it is called Bar of <u>uniform</u> strength.



$\sigma = \frac{P}{A}$

* A weightless prismatic bar subjected to external loading is a bar of uniform strength.

In practise the members are subjected to external loads with self weight. In such a case the cross section must be tappered to have uniform strength.



$$\frac{A_1}{A_2} = e^{\left(\frac{\gamma l}{\sigma}\right)}$$

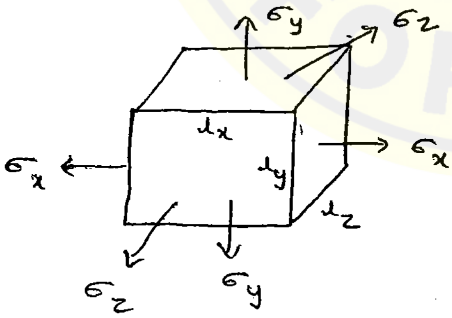
γ = weight density

σ = uniform stress

l = length

$$\boxed{\log_e\left(\frac{A_1}{A_2}\right) = \frac{\gamma l}{\sigma}}$$

Volumetric strains:-



Tensile = +ve

compression = −ve

strains:-

$$\frac{\delta l_x}{l_x} = \varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

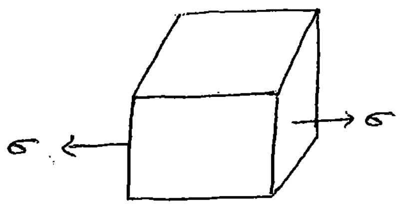
$$\frac{\delta l_y}{y} = \varepsilon_y = \frac{\sigma_y}{E} - \mu\cdot\frac{\sigma_z}{E} - \mu\cdot\frac{\sigma_x}{E}$$

$$\frac{\delta l_z}{z} = \varepsilon_z = \frac{\sigma_z}{E} - \mu\cdot\frac{\sigma_x}{E} - \mu\cdot\frac{\sigma_y}{E}$$

Volumetric strain:-

$$\frac{\delta v}{v} = \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Eg:-

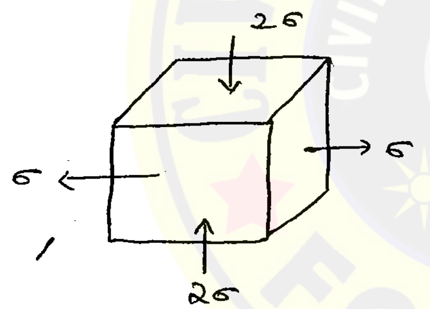


$$\sigma_x = \sigma$$
$$\sigma_y = 0$$
$$\sigma_z = 0$$

$$\varepsilon_x = \frac{\sigma}{E} - \mu(0) - \mu(0)$$
$$= \frac{\sigma}{E}$$
$$\varepsilon_y = 0 - 0 - \mu \cdot \frac{\sigma}{E}$$
$$\varepsilon_z = 0 - \mu \cdot \frac{\sigma}{E} - 0$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$
$$= \frac{\sigma}{E} - \mu \cdot \frac{\sigma}{E} - \mu \cdot \frac{\sigma}{E}$$
$$\varepsilon_v = \frac{\sigma}{E}\left[1 - 2\mu\right]$$

Eg:-



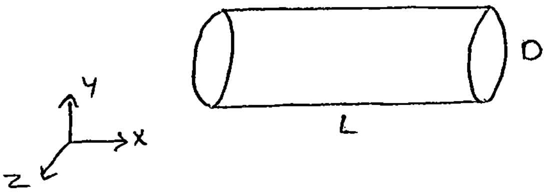
$$\sigma_x = \sigma$$
$$\sigma_y = -2\sigma$$
$$\sigma_z = 0$$

Tension = +ve
compression = −ve

$$\varepsilon_x = \frac{\sigma}{E} - \frac{\mu(-2\sigma)}{E} - 0$$
$$= \frac{\sigma}{E} + \frac{2\mu\sigma}{E}$$
$$\varepsilon_y = \frac{-2\sigma}{E} - \mu(0) - \frac{\mu(\sigma)}{E}$$
$$= \frac{-2\sigma}{E} - \frac{\mu\sigma}{E}$$
$$\varepsilon_z = 0 - \frac{\mu(\sigma)}{E} - \frac{\mu(-2\sigma)}{E}$$
$$= -\frac{\mu\sigma}{E} + \frac{2\mu\sigma}{E}$$
$$= \frac{\sigma\mu}{E}$$

$$\varepsilon_v = \frac{\sigma}{E} + \frac{2\mu\sigma}{E} - \frac{2\sigma}{E} - \cancel{\frac{\mu\sigma}{E}} + \cancel{\frac{\sigma\mu}{E}}$$
$$= \frac{2\mu\sigma}{E} - \frac{\sigma}{E} \Rightarrow \frac{\sigma}{E}\left[2\mu - 1\right]$$

Volumetric strain of a cylinder:-



$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{\delta l}{l} + \frac{\delta D}{D} + \frac{\delta D}{D}$$

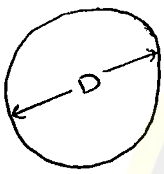
$$\varepsilon_v = \varepsilon_l + \varepsilon_h + \varepsilon_h$$

$$\varepsilon_v = \varepsilon + 2\varepsilon_h$$

ε_l = longitudinal (or) axial strain

ε_h = hoop (or) circumferential strain

Volumetric strain for a sphere:-

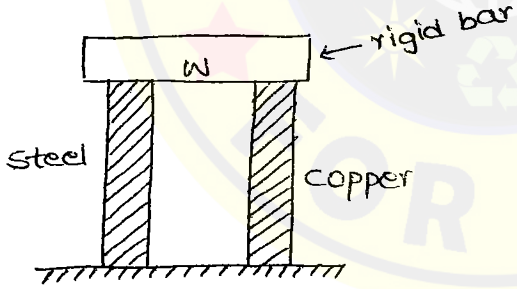


$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{\delta D}{D} + \frac{\delta D}{D} + \frac{\delta D}{D}$$

$$\boxed{\varepsilon_v = 3\varepsilon_h}$$

Composite members:-



Equilibrium equation

$$P_s + P_c = W$$

Compatability condition

$$\delta l_s = \delta l_c$$

$$\left(\frac{Pl}{AE}\right)_s = \left(\frac{Pl}{AE}\right)_c$$

Note:-

To analyse a composite member equilibrium equations and compatability conditions are required.

P.g NO:- 10

1. Given $A = 2\,cm^2 = 200\ mm^2$ $\quad E = 2\times10^5\ N/mm^2$ (or) M.Pa

$P = 40{,}000\,N$ $\quad l = 1000\ mm$

$$\delta l = \frac{Pl}{AE} = \frac{40000\times1000}{200\times2\times10^5}$$

$$\delta l = 1\ mm$$

2. Given $l = 1000$ mm $\quad E_s = 2\times10^5$ M·Pa

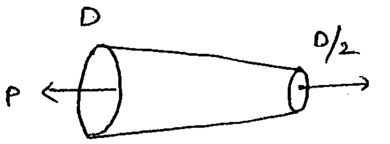
$\delta l = 1$ mm

$$\delta l = \frac{Pl}{AE}$$

$$1 \text{ mm} = \left(\frac{P}{A}\right)\left(\frac{1000}{2\times10^5}\right)$$

$$200 \text{ M·Pa} = \frac{P}{A} = \sigma$$

3.

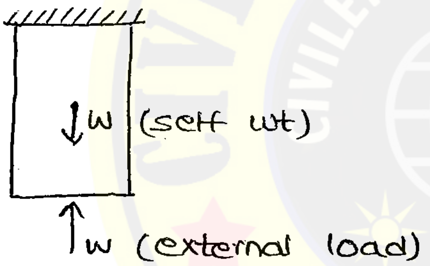


$$\delta l = \frac{Pl}{AE}$$

$$= \frac{Pl}{\frac{\pi}{4}\left(D\cdot\frac{D}{2}\right)\cdot E}$$

$$\delta l = \frac{8Pl}{\pi E D^2}$$

4.



$$\delta l = +\left(\delta l_{sw}\right) - \left(\delta l_{external}\right)$$

$$= \frac{Wl}{2AE} - \frac{Wl}{AE}$$

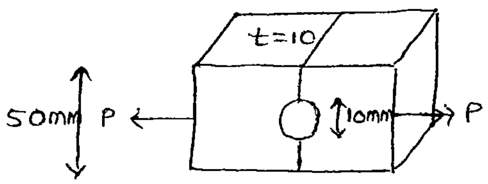
$$= \frac{-Wl}{AE} \quad \text{(contraction)}$$

6. $$\sigma_{sw} = \frac{W}{A} = \frac{\gamma A l}{A}$$

$$\sigma_{sw} = \gamma l$$

If 'l' is doubled then stress is doubled.

7. Given $P = 1000$ N



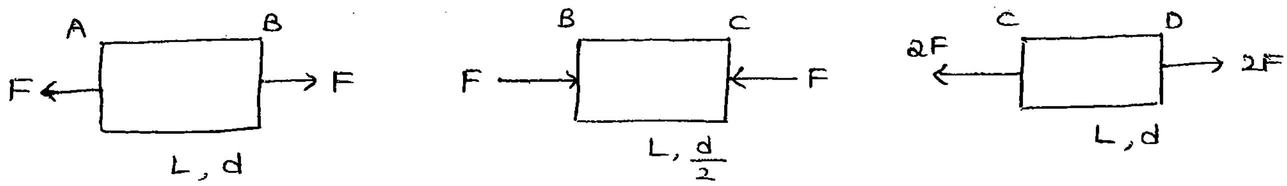
$$\sigma_{max} = \frac{P}{A_{min}}$$

$$= \frac{\cancel{1000}}{\cancel{\frac{\pi}{4}[50^2 - 10^2]}} \qquad \frac{\cancel{1000}}{\cancel{(50\times10)}}$$

$$= \frac{1000}{(50-10)\,t} = 2.5 \text{ Mpa}$$

$(b-D)\cdot t$

8.



$$\sigma_{AB} = \frac{P}{A} = \frac{F}{\frac{\pi d^2}{4}}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{F}{\frac{\pi (d/2)^2}{4}}$$

$$\sigma_{CD} = \frac{P}{A} = \frac{2F}{\frac{\pi d^2}{4}}$$

$$AB : BC : CD = \frac{4F}{\pi d^2} : \frac{16F}{\pi d^2} : \frac{8F}{\pi d^2}$$

$$= 1 : 4 : 2$$

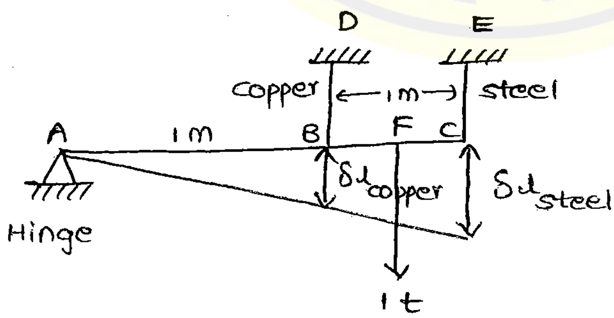
9.

$$\frac{\delta l_{max}}{\delta l_{min}} = \frac{A_{max}}{A_{min}}$$

$$= \frac{\frac{\pi}{4}(3)^2}{\frac{\pi}{4}(1)^2}$$

$$= 9 : 1$$

11.



$$\tan\theta = \frac{\delta l_s}{2} = \frac{\delta l_c}{1}$$

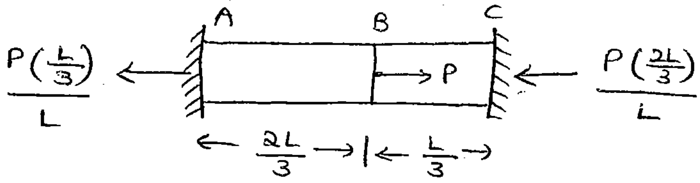
$$\delta l_s = 2\,\delta l_c$$

$$\left(\frac{Pl}{AE}\right)_s = 2\left(\frac{Pl}{AE}\right)_c$$

$$\frac{P_s\, l_s}{2\times 2\times 10^6} = 2\left[\frac{P_c \cdot l_c}{4\times 2\times 10^6}\right] \Rightarrow \frac{(P_s\, l_s)}{(P_c \cdot l_c)} = 2 \Rightarrow \frac{P_c}{P_s} = 0\cdot$$

12. $\dfrac{AB}{BC} = \dfrac{\frac{PL}{3L}}{\frac{2PL}{3L}}$

$= 1:2$



13. Given

$E_A = 4 \qquad E_B = 5$

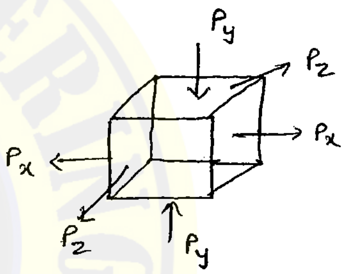
$\delta l = \dfrac{Pl}{AE}$

$\delta l \propto \dfrac{1}{E}$

$\dfrac{\delta l_A}{\delta l_B} = \dfrac{E_B}{E_A} = \dfrac{5}{4}$

$5:4$

14. $\sigma_x = +P_x \qquad \sigma_y = -P_y \qquad \sigma_z = +P_z$

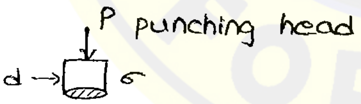


$\varepsilon_x = \dfrac{\sigma_x}{E} - \mu\dfrac{\sigma_y}{E} - \mu\cdot\dfrac{\sigma_z}{E}$

$= \dfrac{P_x}{E} - \mu\dfrac{(-P_y)}{E} - \mu\dfrac{(+P_z)}{E}$

$+$: tensile, $-$: compression

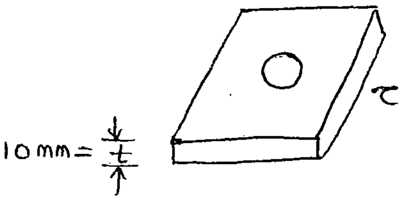
** 15.



punching force = σ resisting shear force

σ (cross sectional area of punching head) = τ (shearing area)

$\sigma\left(\frac{\pi}{4}d^2\right) = \tau\,(\pi\cdot d\cdot t)$



Given condition:

$\sigma = 4\tau$

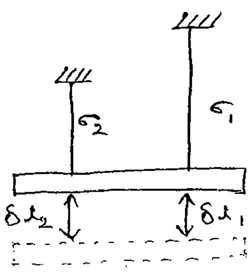
$\therefore d = t = 10\text{ mm}$

$d = t = 10\text{mm}$

16. Given condition

$\delta l_{①} = \delta l_{②}$

$\dfrac{P_1 l_1}{A_1 E_1} = \dfrac{P_2 l_2}{A_2 E_2}$



For same material

$$E_1 = E_2$$

$$\frac{\sigma_1 (6)}{E} = \frac{\sigma_2 (3)}{E}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{1}{2}$$

1.4. THERMAL STRESSES (2M)

1. It is a indirect stress
2. 'α' is a strain developed in a member for 1 unit change in temperature

α = coefficient of linear or thermal expansion

It is a material property. constant for a material.

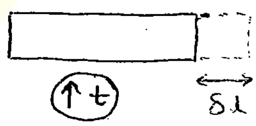
$$\alpha_{steel} = \alpha_{concrete} = 12\times10^{-6}/^\circ C$$

$$\alpha_{brass} = 21\times10^{-6}/^\circ C$$

** $\uparrow \alpha$: $\uparrow$ sensitive for temperature changes : $\uparrow$ change in dimensions

1. Prismatic bar (uniform c/s) free to expand or contract :-

Strain developed, $\varepsilon_t = \alpha t$



$$\varepsilon_t = \frac{\delta l}{l} = \alpha t$$

Free expansion due to temperature, $\boxed{\delta l = l \alpha t}$

For free to expand or contract stress developed is zero

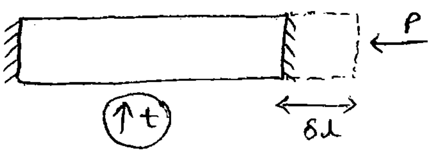
** Note:-

Free expansion or contraction, "No" stresses will be developed

$$\boxed{\sigma_t = 0}$$

2. Prismatic bar fixed at ends :-

Free expansion = expansion prevented



$$L \times t = \frac{Pl}{AE}$$

$$\boxed{\sigma_t = \alpha t E}$$

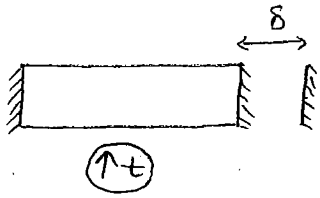
** Nature of stress :-

$$\sigma_t = \alpha \cdot t \cdot E \quad \begin{cases} \uparrow t : \text{compression} \\ \downarrow t : \text{tension} \end{cases}$$

3. Prismatic bar with yielding supports :- (by δ)

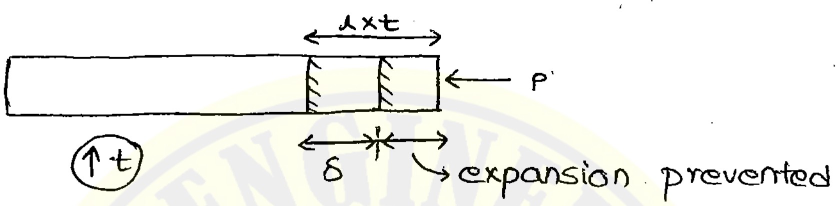
Free expansion due to temperature

$$\delta l = l \alpha t$$



a. If $l\alpha t \ngtr \delta \longrightarrow$ <u>No stresses</u>

b. If $l\alpha t > \delta \longrightarrow$ <u>stress develops</u>



δ = expansion allowed

$l \times t$ = total expansion required

Expansion prevented causes stress

$$(l\alpha t - \delta) = \frac{Pl}{AE}$$

$$\boxed{\sigma_t = \frac{(l\alpha t - \delta)E}{l}} \quad \begin{cases} \uparrow t : \text{compression} \\ \downarrow t : \text{tension} \end{cases}$$

EX:- A steel bar of 10m length is having $E = 2\times10^5$ Mpa $\alpha = 12\times10^{-6}/^\circ c$. The bar is at a room temperature of $20^\circ c$, it is heated to $80^\circ c$. Determine the temperature stress if the bar is

i. Free to expand

ii. Expansion prevented

iii. supports yield by 5mm

iv. supports yield by 10mm

A. Given $l = 10m$, $E = 2\times10^5$ Mpa, $\alpha = 12\times10^{-6}/^\circ c$

$t = 80 - 20$

$t = 60^\circ$

i. For free to expand, $\sigma_t = 0$

ii. For expansion prevented

$$\sigma_t = \alpha t E$$

$$= 12\times10^{-6} \times 60 \times 2\times10^5$$

$$= 144 \text{ Mpa (compression)}$$

iii. yielding, $\delta = 5\,mm$

Free expansion, $\delta l = l \alpha t$

$$= (10,000)(12 \times 10^{-6})(60)$$

$$\delta l = 7.2\ mm$$

Here $l\alpha t > \delta$ – stress develops

$7.2 > 5$, so stress develops

$$\sigma_t = \frac{(l\alpha t - \delta) E}{l}$$

$$= \frac{(7.2 - 5)\ 2\times10^5}{10,000}$$

$$= 44\ Mpa.\ \text{(compression)}$$

iv. yielding, $\delta = 10\ mm$

Free expansion, $\delta l = l\alpha t$

$$= 10,000 \times 12\times10^{-6} \times 60$$

$$= 7.2\ mm$$

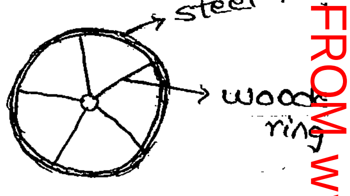
$l\alpha t < \delta$ so no stress develops

$7.2 < 10$

$\therefore \sigma_t = 0$

Hoop stress:- $(d < D)$

It is also called circumference stress.



d = initial dia of steel ring

D = Dia of rigid wooden wheel

D = final dia of steel ring

Hoop strain, $\varepsilon_h = \dfrac{\pi D - \pi d}{\pi d}$

$$= \frac{D-d}{d}$$

Hoop stress, $\sigma_h = \left(\dfrac{D-d}{d}\right) \cdot E$

$$\sigma_h = \varepsilon_h \cdot E$$

** Nature of stress:-

Tension in steel ring

wooden wheel in compression

Minimum increase in temperature of steel ring

$$\varepsilon_t = \varepsilon_h$$

$$\alpha t = \frac{D-d}{d}$$

$$t = \left(\frac{D-d}{d}\right) \times \frac{1}{\alpha}$$

Ex:- A min steel ring dia of 999 mm is to be fitted over a wooden wheel of 1000 mm dia. Determine temperature stress developed due to fixing. Also determine min. increase in temperature for proper fixing.

A. Given $d = 999$ mm, $D = 1000$ mm

$E = 2 \times 10^5$ Mpa, $\alpha = 12 \times 10^{-6}/^\circ c$ } steel

$$\sigma_h = \frac{D-d}{d} \times E$$

$$= \frac{1000-999}{999} \times 2 \times 10^5$$

$$\sigma_h = 200{\cdot}2 \text{ Mpa}$$

{ Tension in steel ring
compression in wooden wheel }

Minimum increase in temperature

$$\varepsilon_t = \varepsilon_h$$

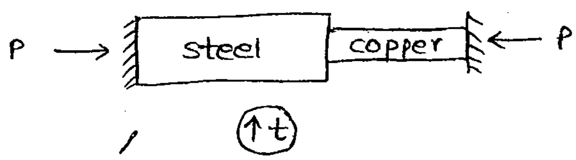
$$t = \left(\frac{D-d}{d}\right) \times \frac{1}{\alpha}$$

$$= 200{\cdot}2 \times 12 \times 10^{-6}$$

$$= 83{\cdot}4^\circ c$$

Composite bars:-

1.



$$\alpha_{copper} > \alpha_{steel}$$

$$P_{steel} = P_{copper} = P$$

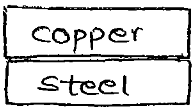
Stress, $\sigma_s = \frac{P}{A_{steel}}$

$\sigma_{copper} = \frac{P}{A_{copper}}$

$\uparrow t$: Both in compression

$\downarrow t$: Both in tension

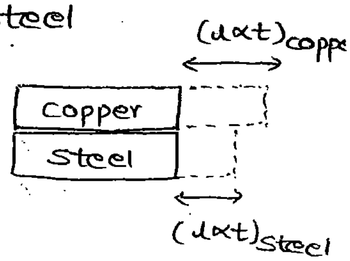
2.



→ No bond in between ~~us~~ copper and steel

If temperature increases, $\uparrow t$

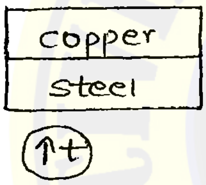
$\alpha_{copper} > \alpha_{steel}$



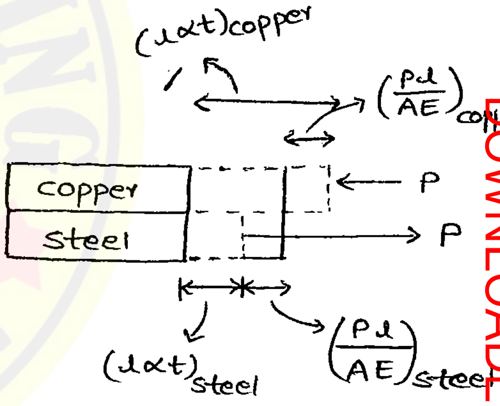
For free expand,

Stress : σ_{steel} , σ_{copper} will be <u>zero</u>

3.



(Bond)



$\alpha_{copper} > \alpha_{steel}$

Compatability condition

$$\delta l_{steel} = \delta l_{copper}$$

$$(l\alpha t)_{steel} + \left(\frac{Pl}{AE}\right)_{steel} = (l\alpha t)_{copper} - \left(\frac{Pl}{AE}\right)_{copper}$$

but $P_{steel} = P_{copper} = P =$ constant

Nature of stresses:-

Material	$\uparrow t$	$\downarrow t$
$\uparrow\alpha$, copper	compre	T
$\downarrow\alpha$, steel	T	C

Note:-
In a good composite material 'α' value must be nearly same so that no stresses due to temperature.

EX:- R.C.C.

P.g NO:- 15

6. $\uparrow t = 20^\circ c$

$\sigma_t = (\alpha t) E$

$= 1.5 \times 10^{-6} \times 20 \times 2 \times 10^6$

$= 60$ Mpa (compression)

7. $\sigma_t = \alpha t E$

$2000 \text{ kg/cm}^2 = (12.5 \times 10^{-6}) \times t \times (2 \times 10^6$

$t = 80^\circ c$

P.g NO:- 19

10. $P = P_c + P_s \longrightarrow ①$

$P = \sigma_c A_c + \sigma_{Asc} \cdot A_{sc}$

$400 \times 10^3 = (6) A_c + (120) A_{sc} \Rightarrow A_c = \frac{400 \times 10^3}{6} - \frac{120 A_{sc}}{6}$

compatability condition

$\delta l_c = \delta l_{sc}$

$$\frac{P_c l}{A_c E_c} = \frac{P_s l}{A_s E_s}$$

$$\frac{P_c}{P_s} = \frac{A_c}{A_s}\left(\frac{E_c}{E_s}\right) \qquad \therefore m = \frac{E_s}{E_c}$$

$$\frac{P_c}{P_s} = \frac{A_c}{A_s}\left(\frac{14}{200}\right)$$ sub in eq ①

$$P = P_s + P_s\left[\frac{A_c}{A_s}\left(\frac{14}{200}\right)\right]$$

$$P = P_s\left[1 + \frac{A_c}{A_s}\left(\frac{14}{200}\right)\right] \rightarrow ②$$

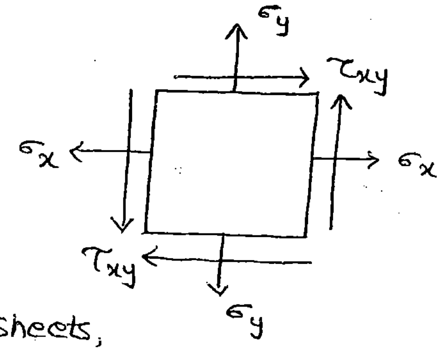
sub 'A_c' value in ②

UNIT - 2

COMPLEX STRESS AND STRAIN

Plane stress (or) 2D system:-

In any member or element normal stresses are balanced by force equilibrium, shear stresses are balanced by moment equilibrium.



Eg:- thin members, plates, beams, sheets, shafts, etc.

Note:-

In 2D system number of independent stress compone are <u>three</u>.

Stress : Tensor

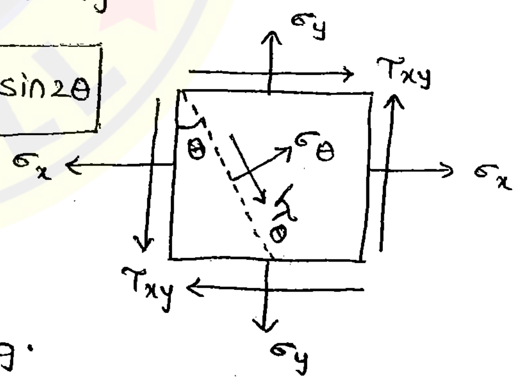
stress : one magnitude + more than one diarection

$$2D\ \text{tensor} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}_{2\times 2}$$

2M *** Stresses induced by Direct stresses on a plane inclined at θ with the verticle shown in fig :-

$$\boxed{\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta}$$

$$\boxed{\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta}$$



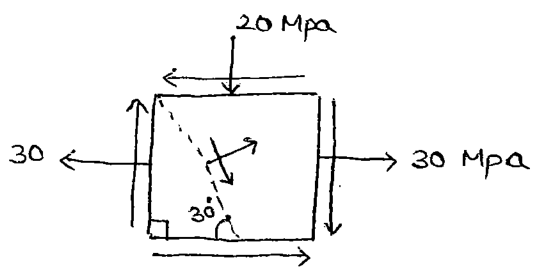
(Standard element)

Sign conventions, as shown in fig.

Resultant stress on inclined plane

$$\boxed{\sigma_R = \sqrt{\sigma_\theta^2 + \tau_\theta^2}}$$

Eg:-



$\sigma_x = +30$ Mpa

$\sigma_y = -20$ Mpa

$\tau_{xy} = -10$ Mpa

$\theta = 90 - 30° = 60°$

$$\sigma_\theta = \frac{(30)+(-20)}{2} + \frac{30-(-20)}{2}\cos 2(60) + (-10)\sin(2\times 60)$$

$$= -16.16\ \text{Mpa (compression)}$$

$$\tau_\theta = \frac{30-(-20)}{2}\sin(2\times 60) - (-10)\cos(2\times 60^\circ)$$

$$= +16.65\ \text{Mpa (Tension)}$$

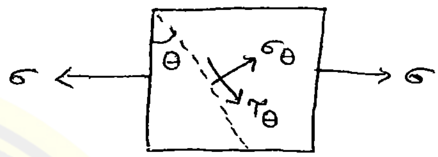
$$\sigma_R = \sqrt{\sigma_\theta^2 + \tau_\theta^2}$$

$$= 20.19\ \text{Mpa.}$$

Ex:- Given $\sigma_x = \sigma$

$\sigma_y = 0$

$\tau_{xy} = 0$



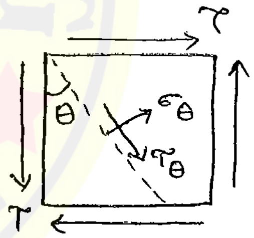
$$\sigma_\theta = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\theta = \sigma\left[\frac{1+\cos 2\theta}{2}\right] = \sigma(\cos^2\theta)$$

$$\tau_\theta = \frac{\sigma}{2}\sin 2\theta = \frac{\sigma}{2}[2\sin\theta\cos\theta] = \sigma(\sin\theta\cos\theta)$$

Ex:- Given

$\sigma_x = \sigma_y = 0$

$\tau_{xy} = \tau$



(pure shear condition)

$$\sigma_\theta = \tau\sin 2\theta$$

$$\tau_\theta = -\tau\cos 2\theta$$

Principal stresses:- (2D system)

The maximum or minimum magnitude of normal stress.

Used in the design

Major principal stress $\longrightarrow$ $\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Minor principal stress $\longrightarrow$ $\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

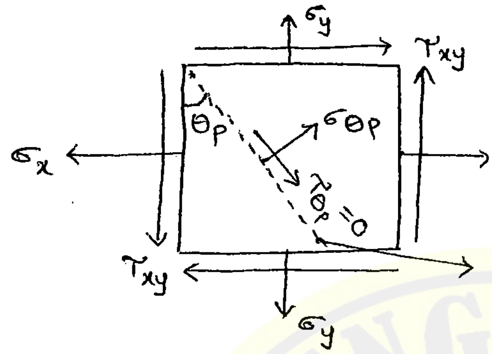
principal plane :-

The plane on which principal stresses are acting. On principal plane shear stress will be zero.

In 2-D system there will be two mutually perpendicular (90°) planes. On both the planes shear stress is zero.

∴ principal plane are always mutually perpendicular

Ex:-



$$\therefore \tau_{\theta_p}=0=\frac{\sigma_x-\sigma_y}{2}\sin 2\theta_p-\tau_{xy}\cos(2\theta_p)$$

$$\tan 2\theta_p=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

θ_p = angle of major principal plane

θ_p+90° = angle of minor principal plane

Maximum shear stress:-

$$\boxed{\tau_{max}=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

(or)

$$\boxed{\tau_{max}=\frac{\sigma_1-\sigma_2}{2}}$$

Maximum shear stress plane:-

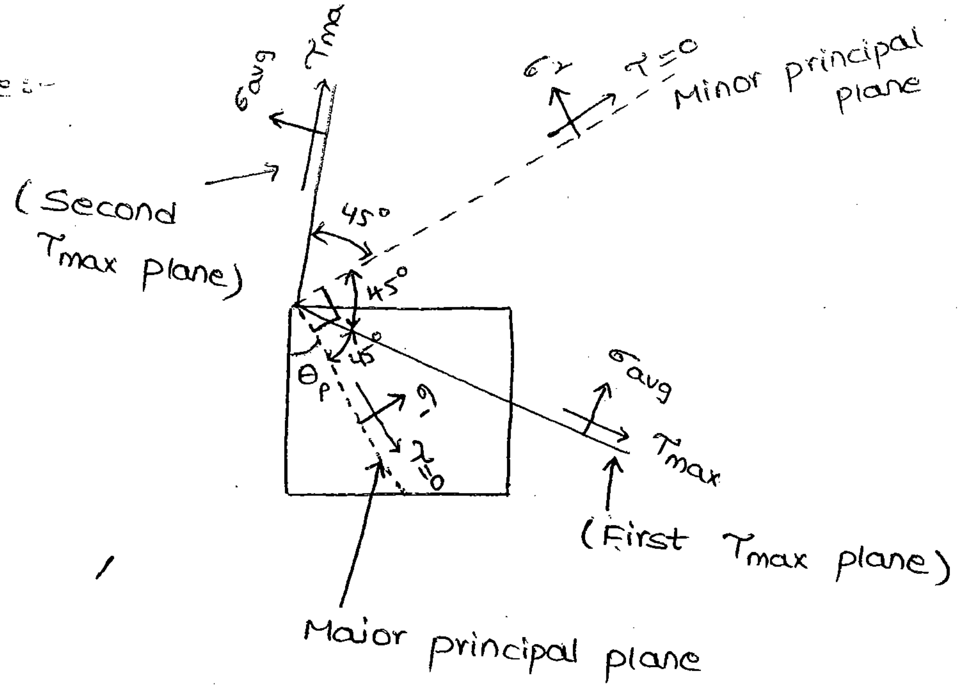
The plane on which maximum shear stress will be acting.

In 2-D system there are two τ_{max} planes separated by 90°. Angle between any ~~two~~ principal plane and nearest τ_{max} plane is 45°.

On τ_{max} plane normal stress will also be acting

$$\boxed{\sigma'=\sigma_{avg}=\frac{\sigma_1+\sigma_2}{2}\text{ (or) }\frac{\sigma_x+\sigma_y}{2}}$$

Note :-



$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

*** Mohr's circle :-

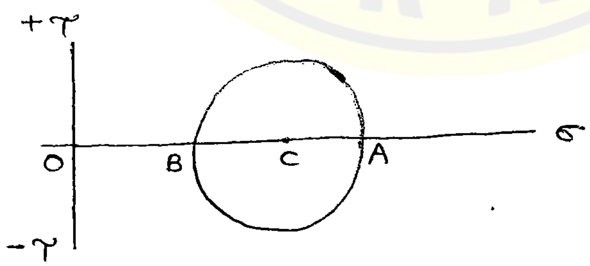
1. It is a graphical method.
2. It is developed for 2D system

parameters required to draw a circle :-

1. Distance from origin to centre of Mohr circle

$$OC = \sigma' = \sigma_{avg} = \frac{\sigma_1 + \sigma_2}{2} \text{ (or) } \boxed{\frac{\sigma_x + \sigma_y}{2}}$$

2. Radius of Mohr circle $= \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$ (or) $\boxed{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$



$OA = \sigma_1 = OC + CA$ $\therefore OA = OC + CA$

$\sigma_1 = \sigma_{avg} + \tau_{max}$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\therefore$ Normal stress on τ_{max} plane is "OC".

$OB = \sigma_2$

$\sigma_2 = OC - BC$

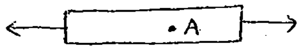
$= \sigma_{avg} - \tau_{max}$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

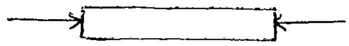
$\therefore$ Radius of Mohr's circle is τ_{max}

Case: 1

Uniaxial stress system (1-D):-



TIE : Tension

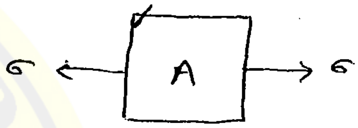


STRUT : compression

$\sigma_x = \sigma$

$\sigma_y = 0$

$\tau_{xy} = 0$



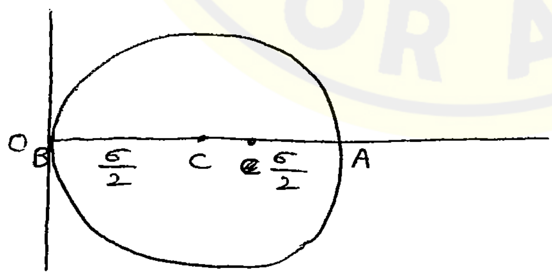
Element in a tie

$OC = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

$= \frac{\sigma}{2}$

Radius $= \tau_{max} = \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + 0^2}$

$\tau_{max} = \frac{\sigma}{2}$



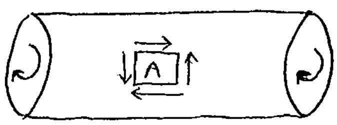
$\sigma_1 = \frac{\sigma}{2} + \frac{\sigma}{2} = \sigma = OA$

$\sigma_2 = 0$

On any plane shear stress is zero i.e., principal plane. The corresponding normal stress is principal stress.

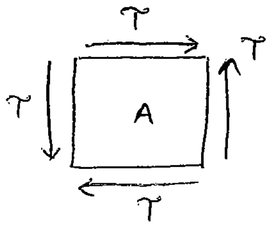
Case: 2

pure shear



Element on a shaft subjected to torsion.

If the applied shear is in clockwise direction the "complementary shear stress" develops in anticlockwise direction and vice versa.



$\sigma_x = 0$

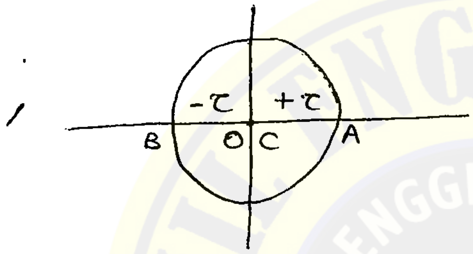
$\sigma_y = 0$

$\tau_{xy} = \tau$

$$OC = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Radius, $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\tau_{max} = \tau$



$\sigma_1 = OA = +\tau$

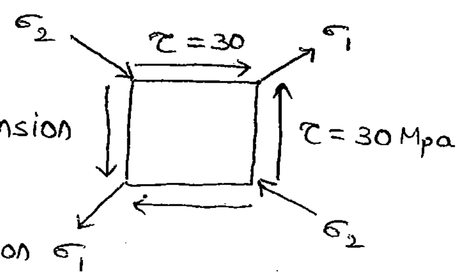
$\sigma_2 = OB = -\tau$

**

1. For pure shear normal stress on τ_{max} plane, $\sigma_{avg} = 0$
2. For pure shear centre of Mohr circle coincides with "Origin"

Ex:- Identify the pure shear condition [c]

Ex:- Find stresses.



$\sigma_1 = +30$ (Tensile) (or) Diagonal tension

$\sigma_2 = -30$ (compressive) (or) Diagonal compression

Note:-

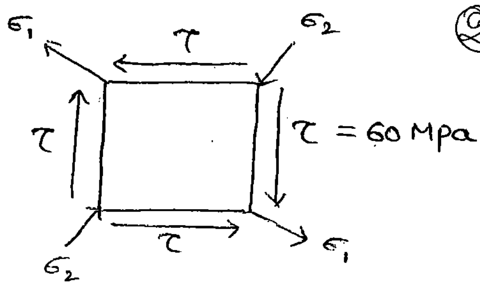
1. Diagonal tension and Diagonal compression develops in case of pure shear.

Ex:- $\sigma_1 = +\tau$

$= +60$ Mpa (Tension)

$\sigma_2 = -\tau$

$= -60$ Mpa (compression)



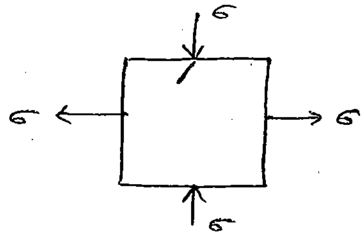
Case : 3

pure shear

$\sigma_x = +\sigma$

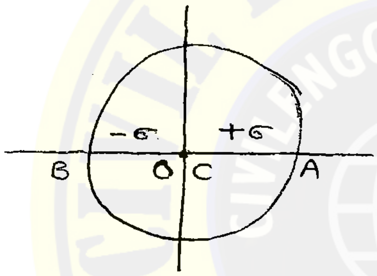
$\sigma_y = -\sigma$

$\tau_{xy} = 0$



$OC = \sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2} = 0$

Radius, $\tau_{max} = \sigma$



$OA = \sigma_1 = +\sigma$ (tension)

$OB = \sigma_2 = -\sigma$ (compression)

case:2 case:3

(Both are pure shear elements)

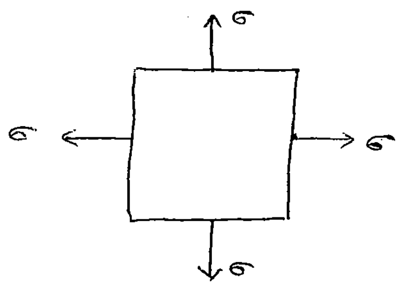
** case : 4

Isotropic condition

$\sigma_x = \sigma$

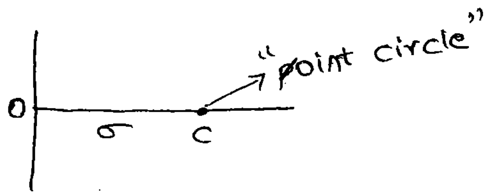
$\sigma_y = \sigma$

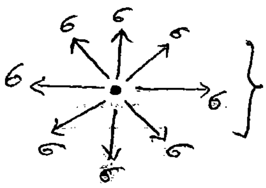
$\tau_{maxy} = 0$



$OC = \sigma_{avg} = \sigma$

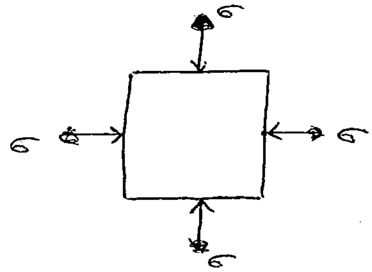
radius $= \tau_{max} =$ zero (0).





} @ a point in any direction property same (Isotropic condition)

In this case all the planes are principal planes without shear stress. The corresponding normal stress is the principal stress.



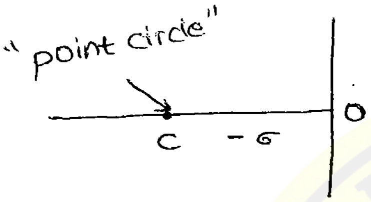
$\sigma_x = -\sigma$

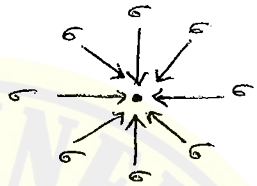
$\sigma_y = -\sigma$

$\tau_{xy} = 0$

$OC = \sigma_{avg} = -\sigma$

Radius, $\tau_{max} = 0$





Note :-

On a submerged body maximum shear stress is zero, Mohr circle drawn will be just a point.

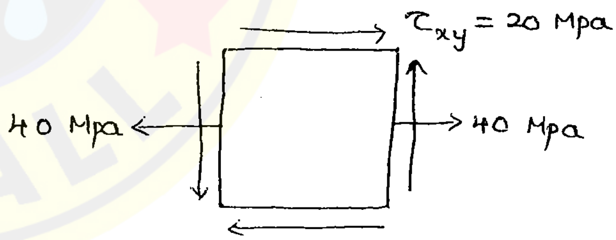
Case : 5

General case of beams

$\sigma_x = 40$ Mpa

$\sigma_y = 0$

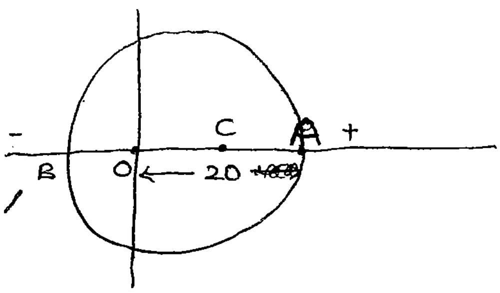
$\tau_{xy} = 20$ Mpa



$OC = \sigma_{avg} = 20$ Mpa

Radius, $\tau_{max} = \sqrt{20^2 + 20^2}$

$= 28.28$ Mpa.



$OA = \sigma_1 = OC + CA$

$= \sigma_{avg} + \tau_{max}$

$= 20 + 28.28$

$= 48.28$ Mpa (Tension)

$OB = \sigma_2 = OC - BC$

$= \sigma_{avg} - \tau_{max}$

$= 20 - 28.28$

$= -8.28$ (compression)

Note:-

In beams principal stresses will be always opposite in nature.

Strain system:-

stresses	strains
σ_x	ε_x
σ_y	ε_y
τ_{xy}	$\phi_{xy}/2$

Strains on inclined plane:-

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta) + \left(\frac{\phi_{xy}}{2}\right)\sin(2\theta)$$

$$\cancel{\left(\frac{\phi_{xy}}{2}\right)} \left(\frac{\phi_\theta}{2}\right) = \frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta - \left(\frac{\phi_{xy}}{2}\right)\cos(2\theta)$$

principal strains:-

Major principal strain, $\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_y}{2} + \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$

Minor principal strain, $\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} - \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$

principal planes:-

The plane on which principal stresses and principal planes will be acting.

On this plane shear stress as well shear strain is zero

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2\left(\frac{\phi_{xy}}{2}\right)}{\varepsilon_x - \varepsilon_y}$$

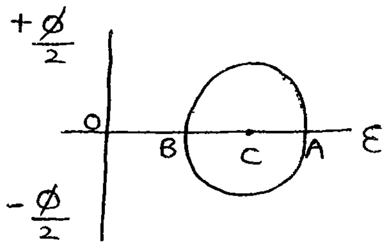
$$= \frac{\phi_{xy}}{\varepsilon_x - \varepsilon_y}$$

** Maximum shear strain :-

$$\frac{\phi_{max}}{2} = \frac{\varepsilon_1 - \varepsilon_2}{2}$$

$$\therefore \boxed{\phi_{max} = \varepsilon_1 - \varepsilon_2}$$

Mohr circle of strains :-



$OA = \varepsilon_1$

$OB = \varepsilon_2$

** Radius of Mohr circle of strains $= \frac{\phi_{max}}{2}$

Strain gauges :-

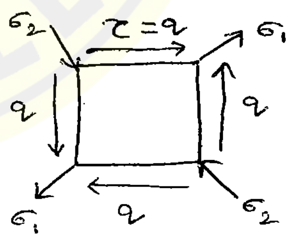
Strain measuring device is called strain gauge.

Types :-

1. Mechanical
2. Electrical
3. Digital.

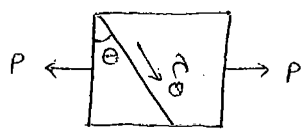
In a 2-D system or plane stress system min. three strain gauges are required to analyse the member.

P.g NO:- 33



1. $\sigma_1 = +q$ (tension)

   $\sigma_2 = -q$ (compression)



2. $\sigma_x = P$

   $\sigma_y = 0$

   $\tau_{xy} = 0$

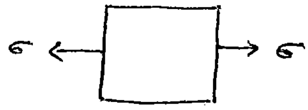
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tau_\theta = \frac{P}{2}\sin 2\theta$$

3. $\sigma = \dfrac{P}{A}$

$= \dfrac{10,000}{5}$

$\sigma = 2000 \text{ kg/cm}^2$



$$\text{Radius} = \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{2000}{2}\right)^2 + 0}$$

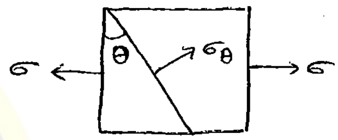
$$= \sqrt{1000^2}$$

$$= 1000 \text{ kg/cm}^2$$

4. $\sigma_1 = 10 \text{ N/mm}^2$ (or) Mpa

$\sigma_2 = -10 \text{ N/mm}^2$

5. Given



$\sigma_x = \sigma$

$\sigma_y = 0$

$\tau_{xy} = 0$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\theta$$

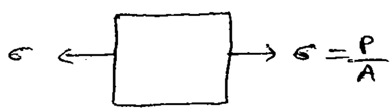
$$= \frac{\sigma}{2}\left[1 + \cos 2\theta\right]$$

$$\sigma_\theta = \sigma\cos^2\theta$$

7. Given $P = 10,000$ kg, $\tau_{max} = 500 \text{ kg/cm}^2$

$$\sigma = \frac{P}{A}$$

$$\text{Radius} = \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$= \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + 0^2}$$

$$500 = \frac{\sigma}{2}$$

$$\sigma = 1000 \text{ kg/cm}^2$$

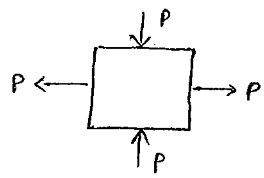
$$1000 = \frac{10,000}{A}$$

$$A = 10 \text{ cm}^2$$

Side of a square $= \sqrt{A} = \sqrt{10}$ cm

9. On τ_{max} plane normal stress, $\sigma_{avg} = \dfrac{\sigma_1+\sigma_2}{2}$

11. Radius $= \tau_{max} = \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

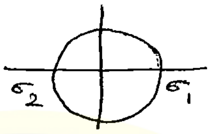


$$= \sqrt{\left(\frac{P-(-P)}{2}\right)^2+0}$$

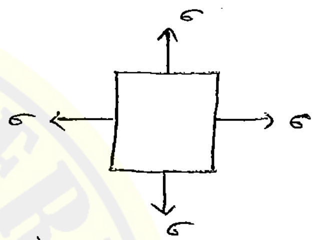
$$\tau_{max} = P$$

12. $\sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

13. $\tau_{max} = \sigma_1$



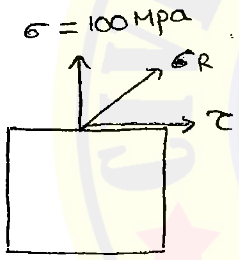
14. Resultant stress $= \sqrt{\sigma^2+\tau^2}$

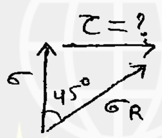


$$= \sqrt{\sigma^2+0}$$

$$= \sigma$$

15.



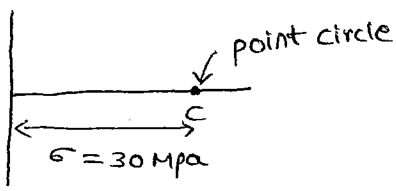


$$\tan 45^\circ = \frac{\tau}{\sigma}$$

$\tau = 100$ Mpa (or) N/mm^2

16. $\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$

$\sigma_x = 30$ Mpa $\quad \sigma_y = 30$ Mpa $\quad \tau_{xy} = 0$



17. $\tau_{max} = \dfrac{\sigma_1-\sigma_2}{2}$

$(50, 50) = \dfrac{50-50}{2} = 0$

$(150, 50) = \dfrac{150-50}{2} = 50$

$(0, 50) = \dfrac{+50}{2} = 25$

$(-50, -200) = \dfrac{-50+200}{2} = 75$

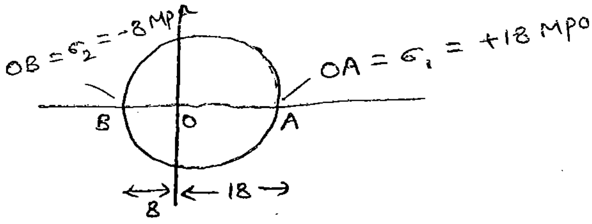
18. $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$= \frac{18-(-8)}{2}$

$\tau_{max} = 13.0$

$\sigma_1 = 18$ Mpa

$\sigma_2 = -8$ Mpa



$\tau_{avg} = \frac{\sigma_1+\sigma_2}{2} = \frac{18-8}{2} = 5.0$

19. $\varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \longrightarrow ①$

$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \longrightarrow ② \times \mu$

$\mu\cdot\varepsilon_2 = \mu\frac{\sigma_2}{E} - \mu^2\frac{\sigma_1}{E} \longrightarrow ③$

solve ① & ③

$(\varepsilon_1 + \mu\cdot\varepsilon_2) = \frac{\sigma_1}{E} - \mu^2\cdot\frac{\sigma_1}{E}$

$\sigma_1 = \frac{E(\varepsilon_1+\mu\varepsilon_2)}{1-\mu^2}$

$= \frac{1.92\times10^5(0.0006+0.2(0.0002))}{1-0.2^2}$

$= 128$ Mpa (or) N/mm^2

20. Given $\sigma_x = 110$ $\sigma_y = 30$ $\tau_{xy} = 30 = \tau_{yx}$

Radius, $\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{110-30}{2}\right)^2 + 30^2}$

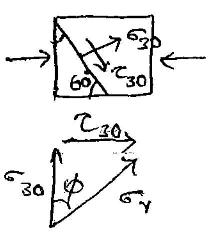
$= \sqrt{40^2+30^2}$

$= 50$

P.9 NO:-41

Feb

1.



$\sigma = \frac{P}{A} = \frac{25,000}{4} = 6250$ kg/cm^2

$\sigma_x = 6250$ kg/cm^2

$\sigma_y = 0$, $\tau_{xy} = 0$

$\sigma_R = \sqrt{\sigma_{30}^2 + \tau_{30}^2} = \cancel{\sqrt{6250^2+0^2} = 6250}$

Obliquity $\tan\phi = \frac{\tau_{30}}{} =$

UNIT - 3

SHEAR FORCE AND BENDING MOMENT (8 to 12 M)

To analyse a beam the number of equlibrium equations are:

1. $\Sigma F_x = 0$
2. $\Sigma F_y = 0$
3. $\Sigma M_z = 0$

Types of supports :-

1. Roller support

(One reaction)

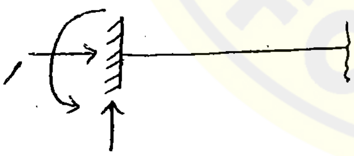
Ex:- Old bridges

2. Hinge support

(Two reactions)

3. Fixed support

It is also called Built-in support

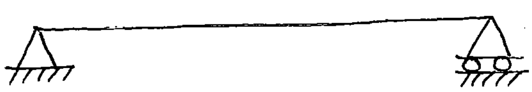


(Three reactions)

Eg:- welded joints

Types of Beams

1. Simply supported beam :-



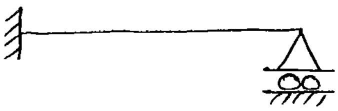
Ex:- Bridge girders

We reduce secondary stresses so we placed one hinged and one roller. If both sides is hinged it is a indeterminate structure.

2. Cantilever beam :-

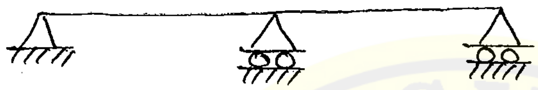
One end is fixed and other end is free

3. propped cantilever beam :-



One end is fixed and other end is roller

4. Continuous beam :-

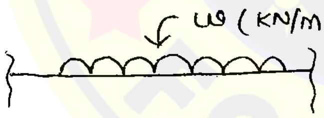


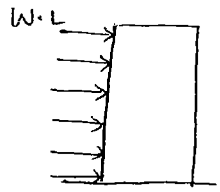
Type of Loading :-

1. point or concentrated Load :-

Eg :- wheel load, hooks

2. Uniformly distributed load (U.D.L) :-

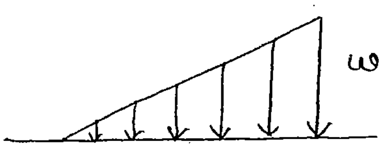


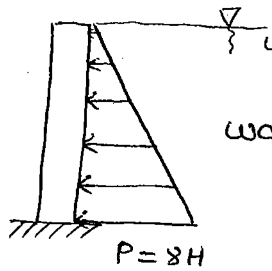


Eg :- L.L, D.L, wind load, snow load

3. Uniformly varied load (U.V.L) :-

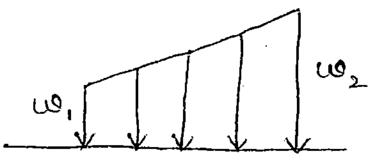
a. Triangular :-

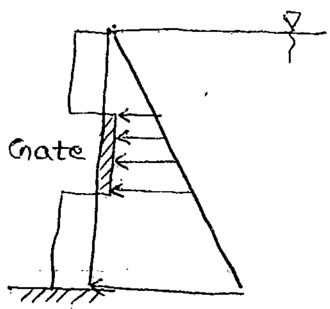




water pressure on side walls

b. Trapezoidal :-



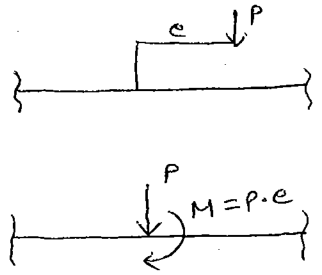


- Couple :-

Effect of a couple is rotation

$M = P \cdot d$

Bracket connection:-



Shear force diagram:-

A diagram showing variation of verticle force or shear force along the length of the beam.

Bending Moment diagram:-

A diagram showing the variation of bending moment along the length of a beam.

Shear force at a point (or) section of beam:-

Algebraic sum of verticle forces either to the left (or) to the right of a beam.

Bending moment at a point (or) section of beam:-

Algebraic sum of moments either to the left (or) to the right of a section.

Sign convention:-

1. Sagging is +ve
2. Hogging is -ve

Relation between intensity of load (ω, KN/m):-

1. $F = \dfrac{dM}{dx}$ $\quad \dfrac{KN\text{-}m}{m}$

SF (F, KN)

BM (M, KN-m)

2. $\omega = \dfrac{dF}{dx}$ $\quad \dfrac{KN}{m}$

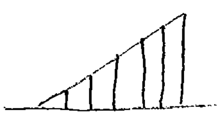
3. $\omega = \dfrac{d^2M}{dx^2}$ $\quad \dfrac{KN\text{-}m}{m^2}$

Conclusions :-

1. Rate of change of Bending moment is SF, Rate of change of SF is Intensity of load (or) Rate of loading (w)
2. Slope of BMD is SF, slope of SFD is Intensity of load
3. $F = \frac{dM}{dx} \Rightarrow dM = F \cdot dx$

   The change of B·M between any two points is equal to "area of SFD between the same two points" (F·dx)
4. $w = \frac{dF}{dx} \Rightarrow dF = w \cdot dx$

   The change in SF between the two points is equal to "area of loading diagram between the same two points" (w·dx)

Loading	SFD	BMD
 No variation of load	Uniform (or) constant (or) Horizontal straight line ($x^0 = 1$)	Linear or inclined straight line (x^1)
 Udl	Linear (or) inclined straight line (x^1)	2° parabola (or) square parabola (x^2)
 U·V·L	2° para (or) parabola (or) square parabola (x^2)	3° parabola (or) cubic parabola (x^3)

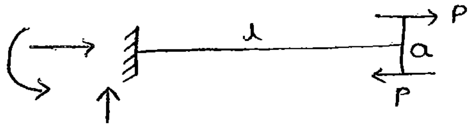
Bending moment to be maximum (by magnitude) (max +ve or max −ve)

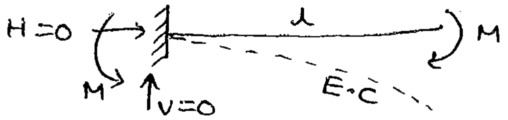
$$\frac{dM}{dx} = 0$$

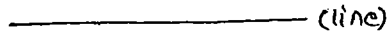
$$F = 0$$

** At the point of Max. BM (+ (or) −ve), Shear force must be zero. At the point of Max. SF, BM need not be zero.

Cantilever beam with couple:-



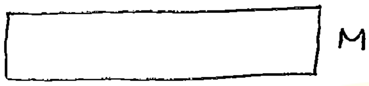




SFD

pure bending zone

Cantilever beam subjected to couple, "SF will be zero", B.M = non zero constant and maximum.



BMD

Note:-

In practise "pure bending" is not possible. (self wt. causes SF).

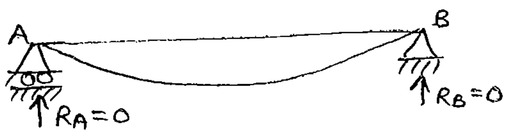
EC = Elastic curve or Deflected shape

For cantilever : tension top. Main steel on top

** In "pure bending" Elastic curve is "circular arc" (R=const).

In other cases Elastic curve is parabola.

Simply supported beam:-



Σf_y (or) $\Sigma V = 0$

$R_A + R_B = 0$

$\Sigma M_A = 0$ (c.w is +ve)

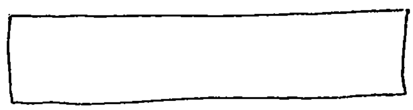
$+M - M - R_B(l) = 0$

$R_B = 0$

$R_A = 0$

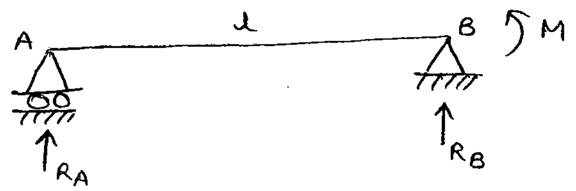
SFD

pure bending



BMD

EX:-



$\Sigma V = \Sigma f_y = 0$

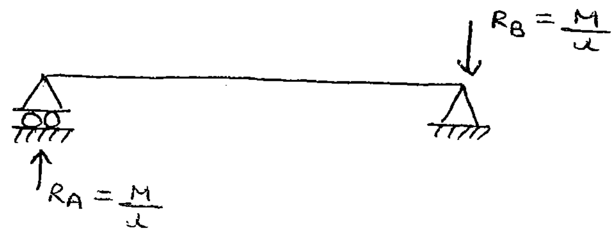
$R_A + R_B = 0$

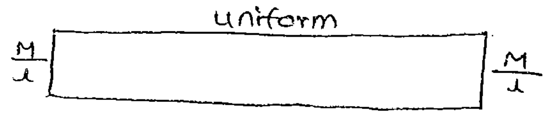
$\Sigma M_A = 0$

$-M - R_B(l) = 0$

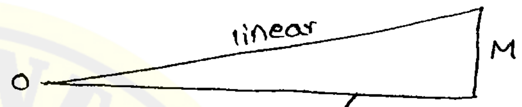
$R_B = \frac{-M}{l}$

$R_A = \frac{M}{l}$





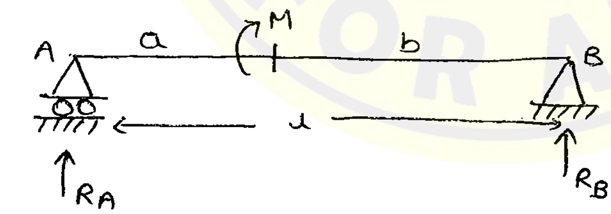
SFD



BMD

1. At the point of concentrated moment (or) couple in a beam there will be a verticle in in BMD whose length is "Magnitude of moment"
2. At the point of point load (or) reaction there will be a verticle line in SFD whose length is equal to "Magnitude of Force or Reaction".

EX:-



$\Sigma F_y = 0$

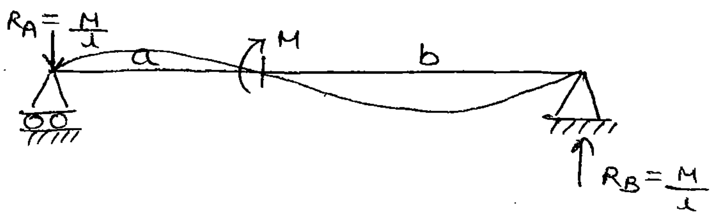
$R_A + R_B = 0$

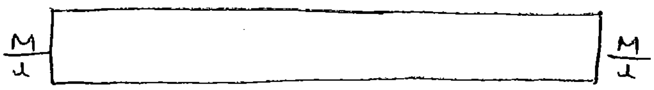
$\Sigma M_A = 0$

$+M - R_B(l) = 0$

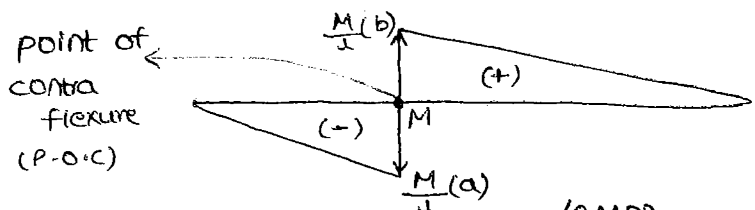
$R_B = \frac{M}{l}$

$R_A = -\frac{M}{l}$



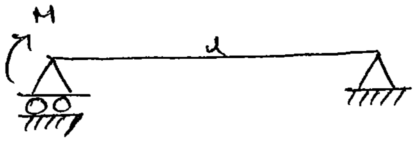


SFD

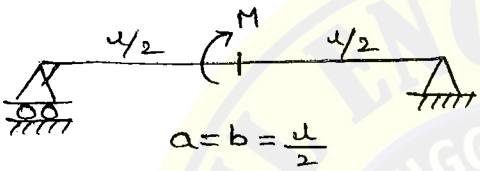


(BMD)

Ex:-



Ex:-



point of contraflexure (P.O.C) :-

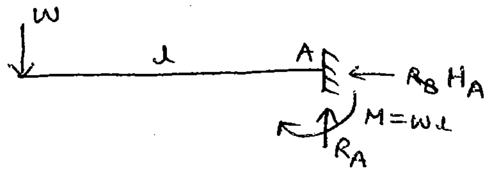
The point at which B.M changes its sign (or) curvature of the beam reverses its direction

Note:-

The point where SF changes sign is not given a specific name.

Cantilever with point load :-

Ex:-



$\Sigma V = 0$

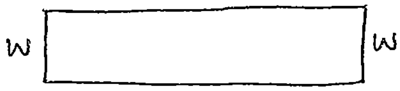
$R_A - W = 0$

$R_A = W$

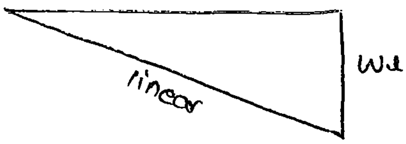
$\Sigma M_A = 0$

$-Wl + M = 0$

$M = Wl$

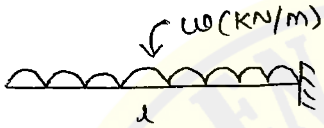


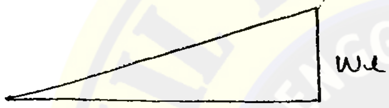
(SFD)



(BMD)

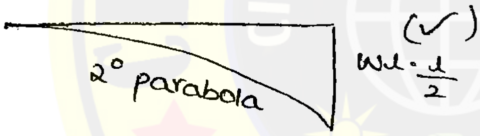
Cantilever with Udl :-





(SFD)

Slope = 0



(BMD)

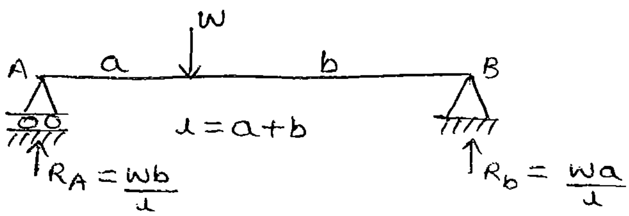
$F = \left(\frac{dM}{dx}\right)$ slope of BMD

$SF = 0$

$\left(\frac{dM}{dx}\right)_{\text{slope of BMD}} = 0$

Slope ≠ 0

Simply supported beam with point load :-

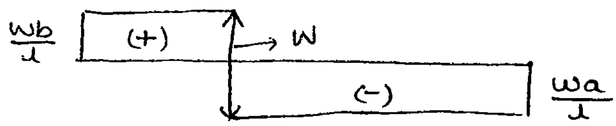


Assume $(b > a)$

Max SF = Max reaction

$R_A = \frac{Wb}{l}$

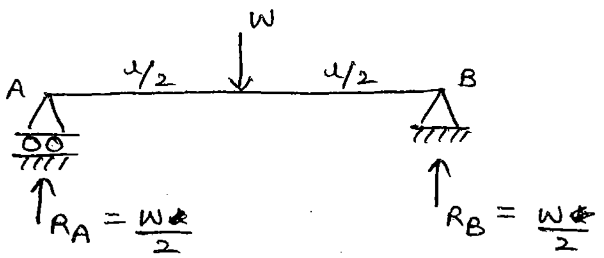
Max BM $= \frac{Wab}{l}$

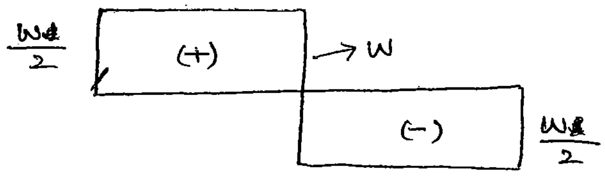


(SFD)

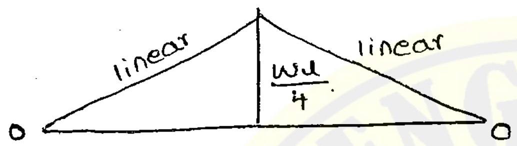
(BMD)

Ex:-



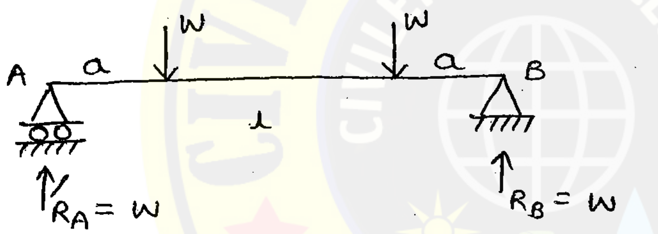


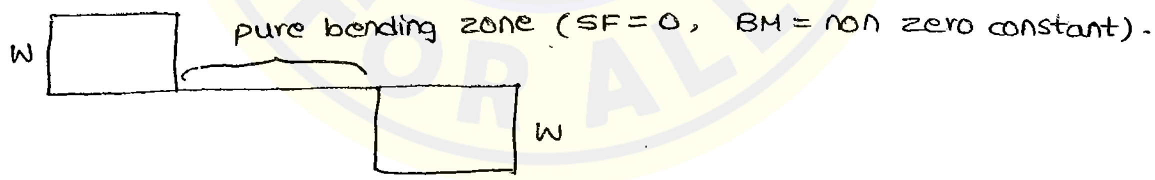
(SFD)



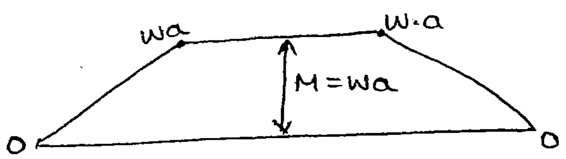
(BMD)

Ex:-





(SFD)



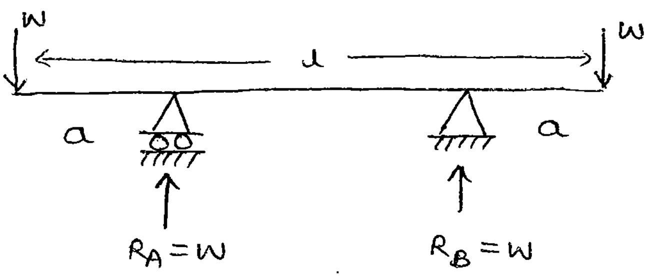
(BMD)

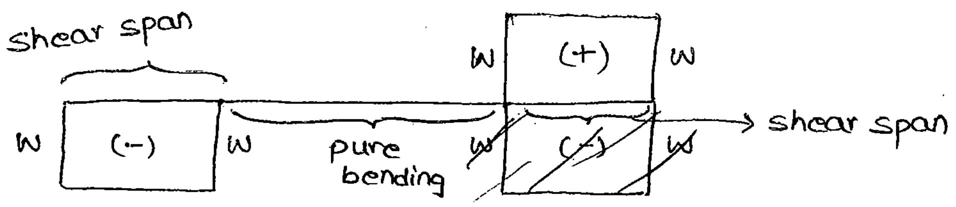
Shear span (a):-

The zone in which SF is non zero constant (In the shear span zone B·M may not be zero).

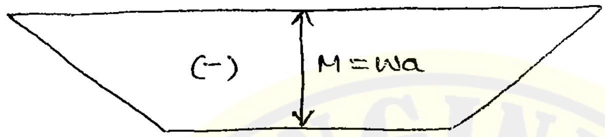
In lab two point load testing is very common. For testings related to BM: pure ~~BM~~ bending zone is used, for shear testings shear span is used.

EX:-



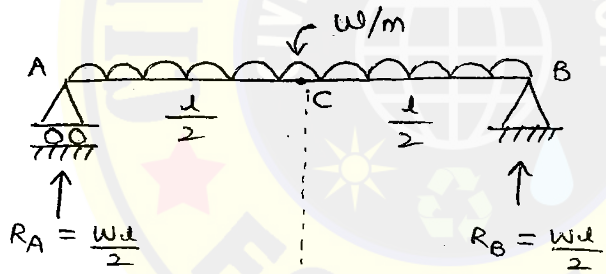


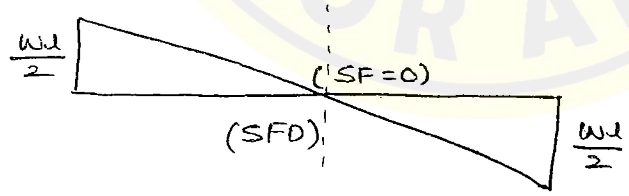
(SFD)



(BMD)

~~Ex~~:- Simply supported beam with Udl :-



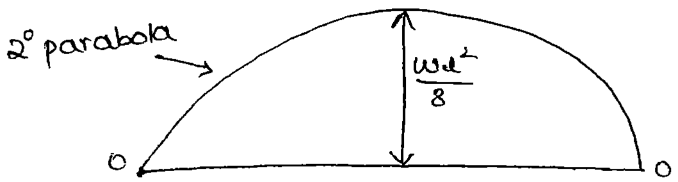


(SFD)

Max BM @ C @ SF = 0

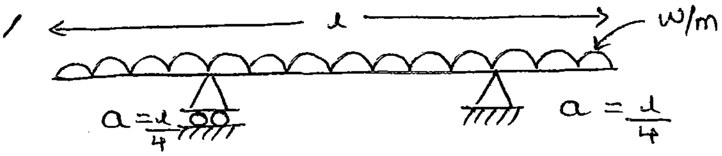
$$M_c = R_A\left(\frac{l}{2}\right) - w\left(\frac{l}{2}\right)\left(\frac{l}{4}\right)$$

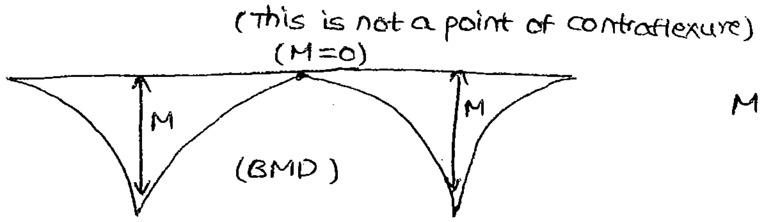
$$M_c = \frac{wl^2}{8}$$



BMD

EX:- (Over hanging beams)





Max BM @ supports

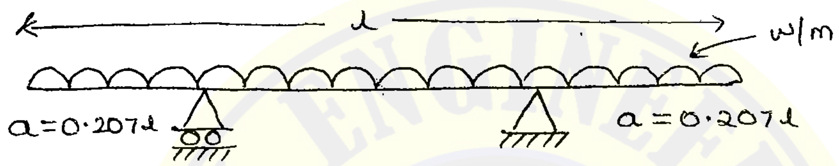
$= w \times a \times \frac{a}{2}$

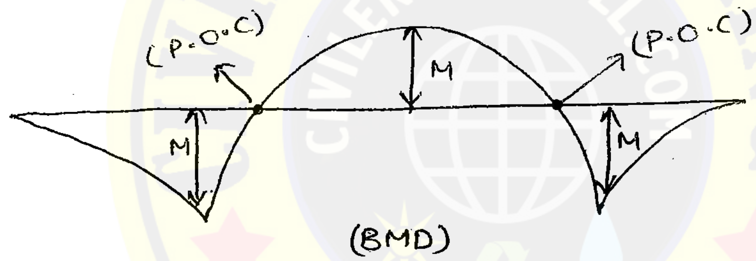
$= w\left(\frac{l}{4}\right)\left(\frac{l}{8}\right)$

$M = \frac{wl^2}{32}$

Compared to simply supported with udl beams B.M is reduced by $\frac{32}{8} = 4$ times

EX:-





$w(a)\left(\frac{a}{2}\right) = \frac{wa^2}{2}$

$= \frac{w(0.207l)^2}{2}$

$M = \frac{wl^2}{46.67}$

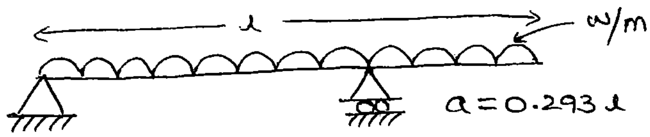
compared to S.S. with udl beams B.M is reduced by $\frac{46.67}{8} = 5.8$ times

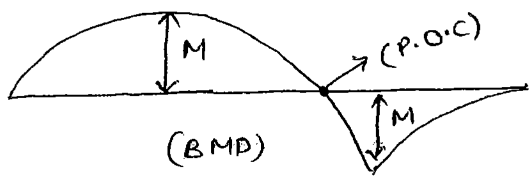
If $a = 0.207l$ the minimum B.M develops in the beam are sagging moment = Hogging moment

Note:-

1. Beam with two over hangs the maximum no. of contra-flexures are possible = 2

EX:-





$M = w(a)\left(\frac{a}{2}\right)$

$= \frac{wa^2}{2} = \frac{w(0.293l)}{2}$

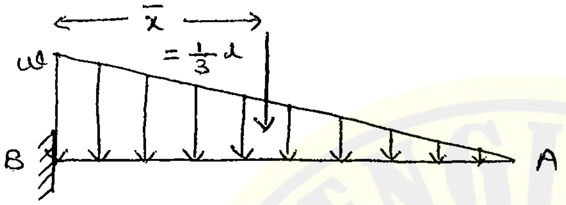
$M = \frac{wl^2}{23.3}$

S.No	Beam	No. of p.o.c possible
1.	Cantilever beam	zero (0)
2.	Simply supported beam	zero (0)
3.	Single Overhang beam	One
4.	Double Overhang beam	Two
5.	Propped cantilever beam	One
6.	Fixed beams	Two

Cantilever beam with U.V.L :-

** Ex:-

(case: 1)



$SF_A = 0$

SF_B = area of triangl

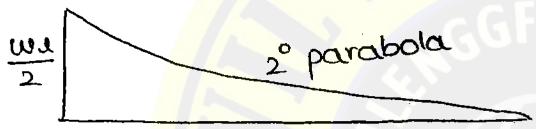
$= \frac{1}{2} w \cdot l$

$M_A = 0$

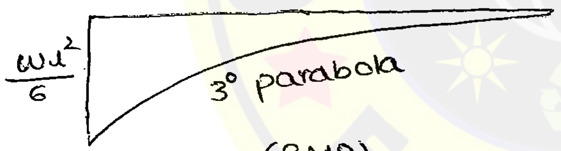
M_B = area of triangle ($\bar{x}$)

$= \frac{1}{2} \times w \times l \left(\frac{1}{3} l\right)$

$= \frac{wl^2}{6}$



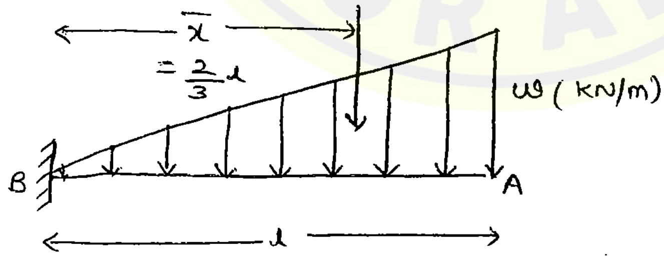
(SFD)



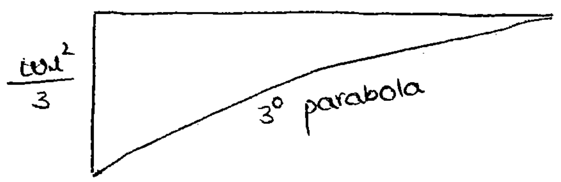
(BMD)

** Ex:-

(case: 2)



(SFD)



$M_A = 0$

M_B = area of Δ^{le} ($\bar{x}$)

$= \frac{1}{2} \times w \times l \left(\frac{2}{3} l\right)$

$= \frac{wl^2}{3}$

$F = \left(\frac{dM}{dx}\right)$ slope of BMD

SF = 0 then BM = 0

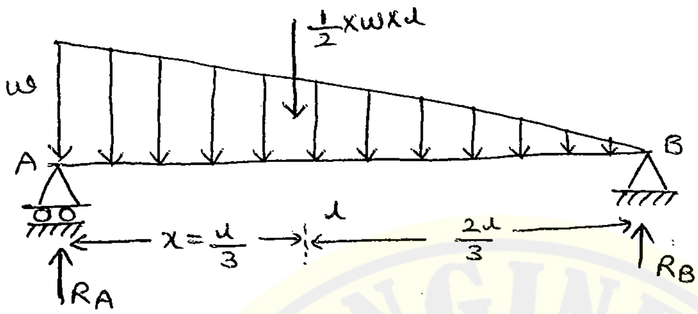
**Note :-

$$\frac{(\text{Max SF})_1}{(\text{Max SF})_2} = 1$$

$$\frac{(\text{Max BM})_1}{(\text{Max BM})_2} = \frac{1}{2}$$

Simply supported beam with U.V.L :-

Ex:-



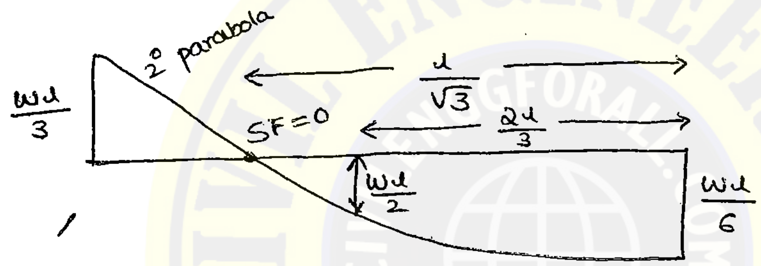
$\Sigma F_y = 0$

$R_A + R_B = \frac{wl}{2}$

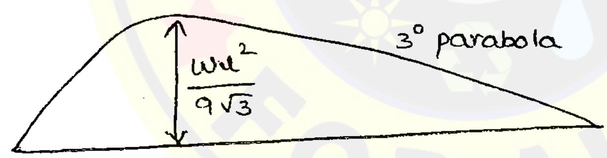
$-R_B l + \frac{wl}{2}\left(\frac{l}{3}\right) = 0$

$R_B = \frac{wl}{6}$

$R_A = \frac{wl}{3}$



(SFD)



(BMD)

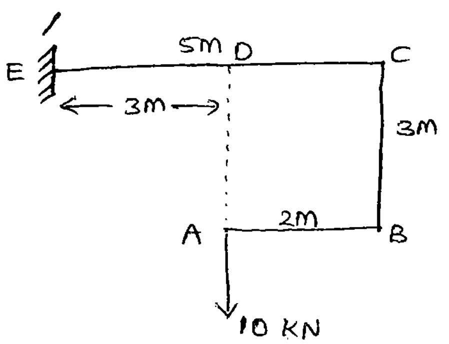
** Max. SF in beam = Max. Reaction $= \frac{wl}{3}$

Max Bending moment $= \frac{wl^2}{9\sqrt{3}}$

Max Bending moment from tail occurs @ $\frac{l}{\sqrt{3}}$

Frames:-

Ex:-



$M_A = 0$

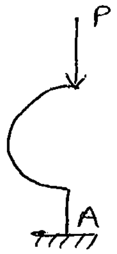
$M_B = 10 \times 2 = 20$ kN

$M_C = 10 \times 2 = 20$ kN

$M_D = 0$

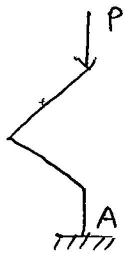
$M_E = 10 \times 3 = 30$ kN

Ex:-



$M_A = 0$

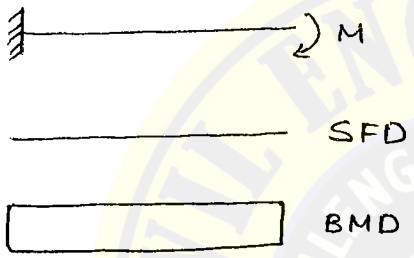
Line of action of force passes through a point then moment is zero



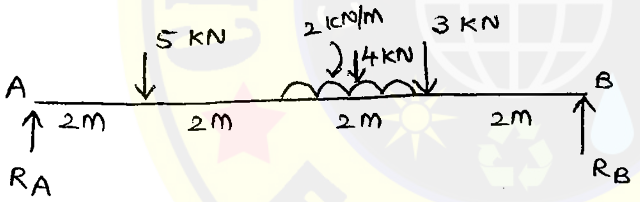
$M_A = 0$

P.g No:- 47

4.



8.



$\Sigma F_y = 0$

$R_A + R_B = 12 \text{ kN}$

$\Sigma M_A = 0$

$-8R_B + 18 + 20 + 10 = 0$

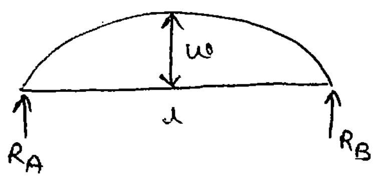
$R_B = 6 \text{ kN}$

$R_A = 6 \text{ kN}$

$\frac{R_A}{R_B} = \frac{6}{6}$

$\frac{R_A}{R_B} = 1$

10.



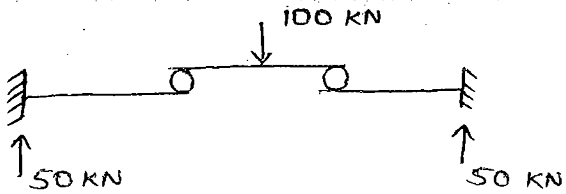
Symmetrical load

$R_A = R_B = \frac{\text{Total load}}{2} = \frac{\text{area in parabola}}{2}$

$= \frac{\left(\frac{2}{3} w l\right)}{2}$

$= \frac{wl}{3}$

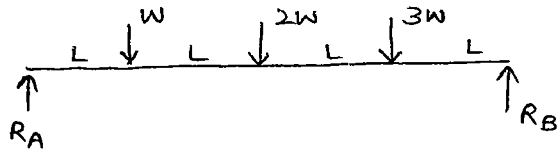
18.



Due to symmetry

Max SF = max reaction = 50 KN

19. Max. SF = Max. reaction = R_B

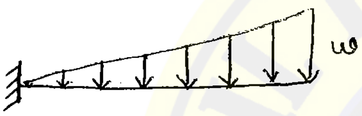


$\Sigma M_A = 0$

$$-R_B(4l) + w(l) + 2w(2l) + 3w(3l) = 0$$

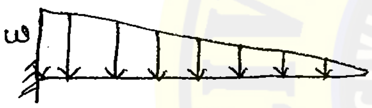
$$R_B = \frac{14}{4} w$$

21. 1.



$$\frac{(\text{Max BM})_1}{(\text{Max BM})_2} = \frac{wl^2/3}{wl^2/6} = \frac{2}{1}$$

2.



23. $a = \frac{l}{4} = 0.25 l$

$= 25\% \, l$

UNIT - 4

CENTRE OF GRAVITY AND MOMENT OF INERTIA (2-4M)

Centroid (or) centre of area :-

The point through which entire area is to be concentrated.
It is applicable to thin members

Centre of mass :-(or) centre of gravity :-

The point through which entire mass or weight will be acting. It is applicable to solids.

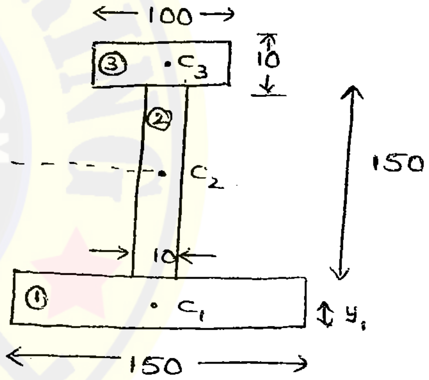
Compound areas :-

$$\bar{y} = \frac{\Sigma A_i y_i}{A_i} \qquad \bar{x} = \frac{\Sigma A_i x_i}{A_i}$$

Ex:- Locate centroid from base

A.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$



$$= \frac{(150\times10)\left(\frac{10}{2}\right) + (10\times150)\left(\frac{150}{2}+10\right) + (100\times10)\left(\frac{10}{2}+150+10\right)}{(1500)+(1500)+(1000)}$$

$$= \frac{(1500\times5)+(1500\times85)+(1000\times165)}{4000}$$

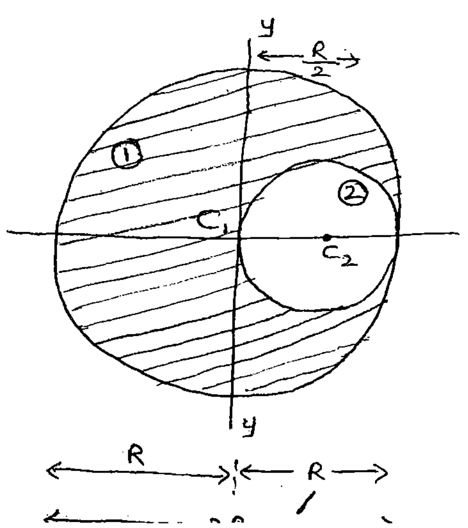
$= 75$ mm from base

Ex:- Locate centroid from y-axis:-

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{\pi R^2(0) + \pi\left(\frac{R}{2}\right)^2\left(\frac{R}{2}\right)}{\pi R^2 - \pi\left(\frac{R}{2}\right)^2}$$

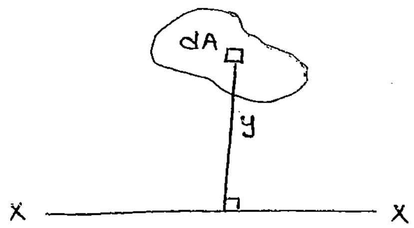
$= -\frac{R}{6}$ (on -ve side of x-axis)



Moment of Inertia :-

Second moment of a given area about a reference axis is moment of Inertia.

$$I_x = \int dA \cdot y^2$$



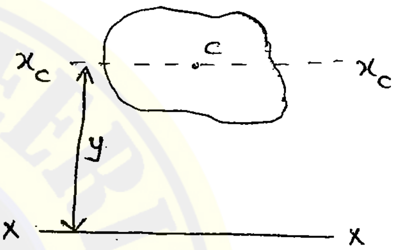
Unit : mm^4 (or) cm^4 (or) m^4

$I \uparrow$: $\uparrow$ stability : $\uparrow$ strength

Moment of Inertia is used in members subjected to Bending moment (or) Twisting moment.

Parallel axis theorem:-

$$I_x = I_{xc} + Ay^2$$



This theorem is used to shift centroidal moment of inertia to any other parallel axis.

The least possible moment of inertia is the centroidal moment of Inertia only.

Perpendicular axis theorem:-

$$I_x = \int dA \cdot y^2$$

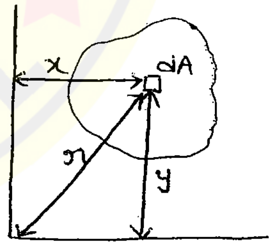
$$I_y = \int dA \cdot x^2$$

$$I_z = \int dA \cdot r^2 \qquad r^2 = x^2 + y^2$$

$$= \int dA\,(x^2 + y^2)$$

$$= \int dA\, x^2 + \int dA\, y^2$$

$$I_z = I_x + I_y$$



Here $I_z = J = I_p$ = polar moment of Inertia

Moment of inertia about an axis perpendicular to the plane of area is polar moment of inertia

In bending problems I_x and I_y are used, In torsion problems I_z will be used.

Product of Inertia :-

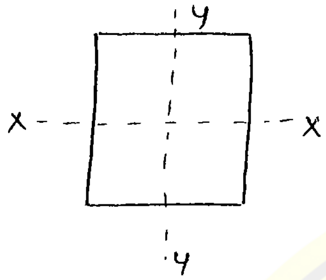
Moment of inertia about any two mutually perpendicular axis in the plane of area is product of inertia.

$$I_{xy} = \int dA \cdot x \cdot y$$

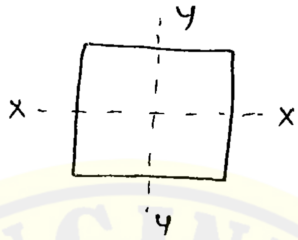
Unit :- m^4, cm^4, mm^4

I_x, I_y, I_z are always non zero positive values

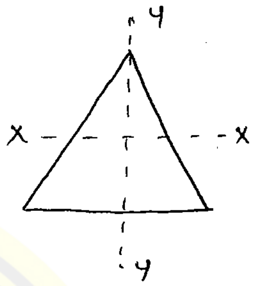
Then I_{xy} can be positive, negative (or) zero also



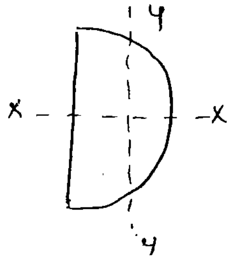
$I_{xy} = 0$



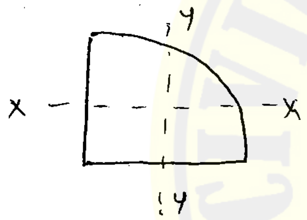
$I_{xy} = 0$



$I_{xy} = 0$



$I_{xy} = 0$



$I_{xy} \neq 0$

Any one axis is symmetrical then $I_{xy} = 0$. Not symmetrical any one axis, $I_{xy} \neq 0$

Principal Moment of Inertia :-

Maximum or Minimum moment of inertia

Stresses	Inertia
σ_x	I_x
σ_y	I_y
τ_{xy}	I_{xy}

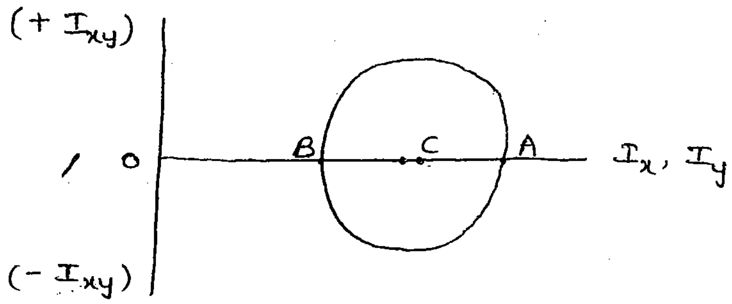
Major, $I_{max} = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$

Minor, $I_{min} = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$

principal axis :-

The axis about which principal moment of inertia will be acting, about these axis product of inertia (I_{xy}) is zero. Therefore principal axis are symmetrical axis.

Mohr circle of Inertia :-



$OA = \sigma_1 = I_1 = I_{max}$

$OB = I_2 = I_{min}$

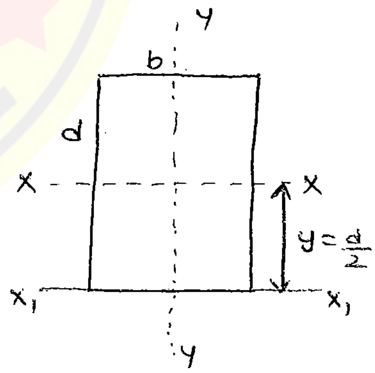
$OC = I_{avg} = \frac{(I_1+I_2)}{2}$

** Radius of Mohr circle is "maximum product of inertia."

I_{xy} is used in Unsymmetrical bending

Moment of Inertia of simple figures:-

1. $I_{xx}' = \frac{bd^3}{12}$ $\quad I_{yy} = \frac{db^3}{12}$



$I_{xy} = 0$

$I_z = I_{xx} + I_{yy}$

$= \frac{bd}{12}[d^2+b^2]$

$I_{x_1} = \frac{bd^3}{12} + bd\left(\frac{d}{2}\right)^2$

$= \frac{bd^3}{3}$

Radius of gyration r (or) $k = \sqrt{\frac{I}{A}}$

$$\boxed{\begin{aligned} r_x &= \sqrt{\frac{I_x}{A}} \\ r_y &= \sqrt{\frac{I_y}{A}} \end{aligned}}$$

Radius of gyration used in columns

$$r_x = \sqrt{\frac{I_x}{A}}$$

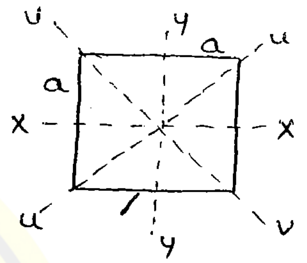
$$= \sqrt{\frac{(bd^3/12)}{bd}}$$

$$= \frac{d}{\sqrt{12}}$$

$$r_x = \frac{d}{2\sqrt{3}}$$

$$r_y = \frac{b}{2\sqrt{3}}$$

2. Square section:

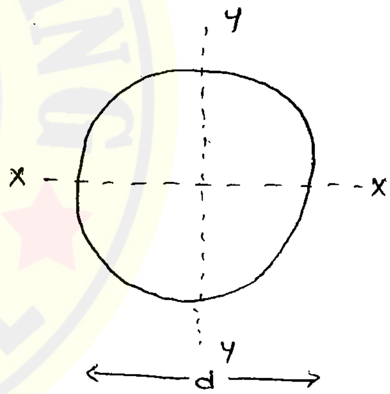


$$I_x = \frac{a \cdot a^3}{12} = \frac{a^4}{12}$$

$$I_y = \frac{a^4}{12}$$

$$\boxed{I_x = I_y = I_u = I_v = \frac{a^4}{12}}$$

3. Circle section:-



$$I_x = I_y = \frac{\pi d^4}{64}$$

$$\boxed{J = I_z = I_x + I_y = \frac{\pi d^4}{32}}$$

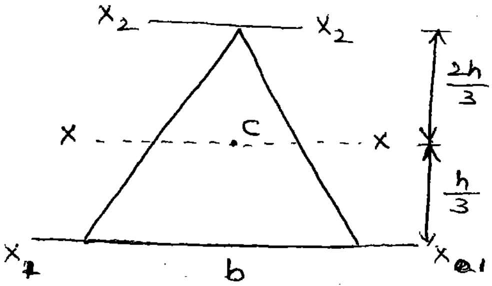
$I_{xy} = 0$ (symmetrical)

$$r \text{ (or) } k = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{\pi d^4/64}{\pi d^2/4}}$$

$$= \frac{\cancel{\pi} d}{4}$$

4. Triangular section:-



$$I_x = \frac{bh^3}{36}$$

$$I_{x_1} = \frac{bh^3}{36} + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{12}$$

$$I_{x_2} = \frac{bh^3}{36} + \left(\frac{1}{2}bh\right)\left(\frac{2h}{3}\right)^2$$

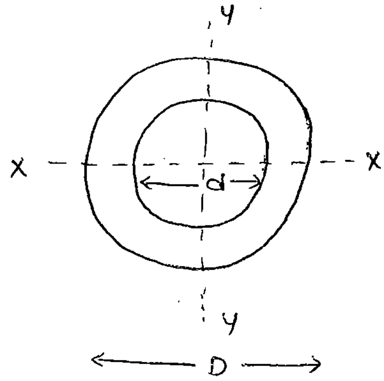
$$= \frac{bh^3}{}$$

5. Hollow circle :-

$$I_x = I_y = \frac{\pi}{64}(D^4 - d^4)$$

$$I_z = J = I_x + I_y$$

$$** \quad \boxed{J = \frac{\pi}{32}(D^4 - d^4)}$$



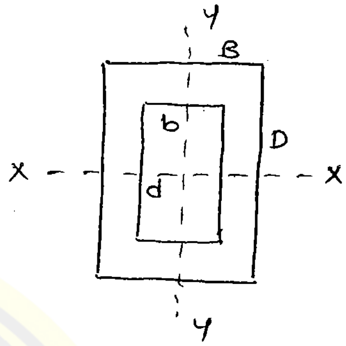
6. Hollow Rectangle :-

$$I_x = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}\left[BD^3 - bd^3\right]$$

$$I_y = \frac{DB^3}{12} - \frac{db^3}{12}$$

$$= \frac{1}{12}\left[DB^3 - db^3\right]$$



Slenderness Ratio:-

$$\lambda = \frac{l}{r_{min}}$$

l = effective length of column

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

It is used in columns.

UNIT - 5

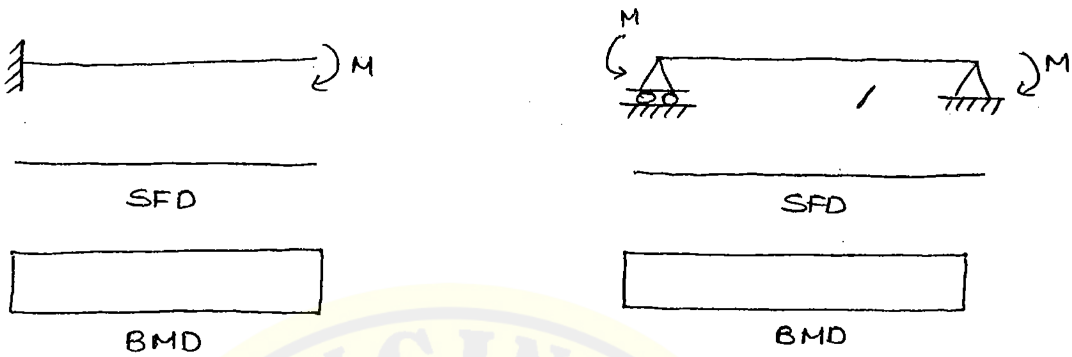
THEORY OF SIMPLE BENDING (6 to 8 M)

Bending moment :-

Non zero constant and maximum

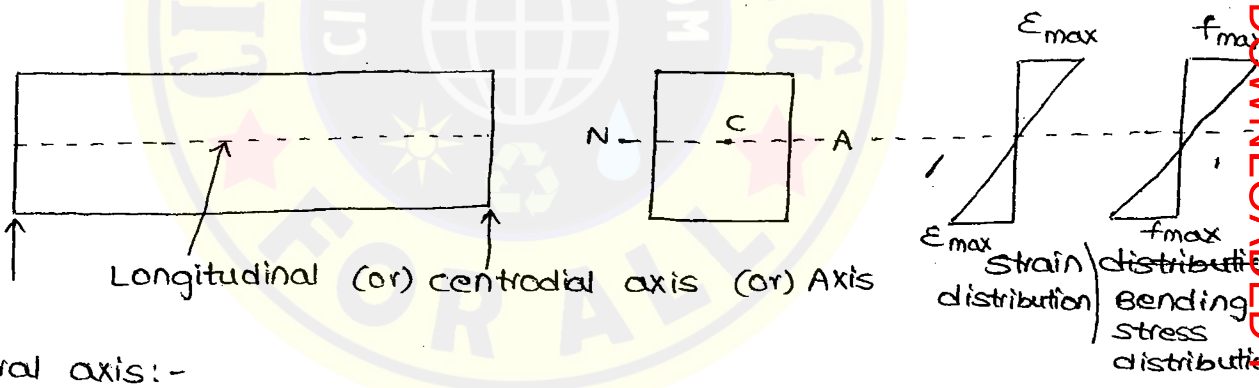
SF = 0

Ex:-



Pure (or) simple bending is not possible in practise but shear and bending are separated using superposition principle.

Longitudinal axis :-



Neutral axis:-

Neutral axis is in the cross section passing through the centroid where bending stress is zero.

Angle between centroidal and neutral axis will be 90°

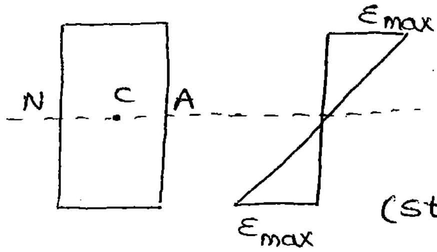
Neutral plane :-

The plane containing centroidal and neutral axis.

Assumptions :-

** 1. plane cross section remains plane after bending. As per Bernoulli's assumption there should not be any distorsion in the cross section. (Euler bernoulli's assumption)

2. As per Bernoulli "strain is linear" with zero at the centroidal axis and maximum at the extreme fibre



(strain distribution)

3. Homogeneous + Isotropic and Hooke's Law is valid.

4. To eliminate shear beam is assumed to have thin layers and no shear between the layers.

5. youngs modulus in tension zone = youngs modulus in compression zone.

$$E_{Tension} = E_{compression}$$

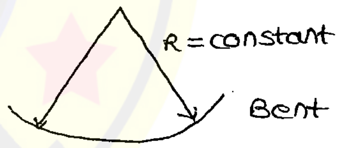
Material properties are constant in tension and in compression.

6. Radius of curvature is large as compared to ~~pure bending~~. width or depth.

$$R >>> b \text{ (or) } d$$

** Bent or deflected shape in pure bending is "Arc of a circle"

$\boxed{\text{Radius} = \text{constant}}$



** In the beams slopes and deflections are very small (super position principle is valid) (≈ 0)

Bending (or) Flexural equation:-

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$$

E = young's modulus

R = Radius of curvature

M = Bending moment

I = Moment of inertia about Neutral Axis. (Here N.A is bending axis)

f = flexural (or) Bending stress

y = distance from N.A.

$$\frac{M}{I} = \frac{f}{y}$$

$$\boxed{f = \frac{M}{I} \cdot y}$$

$$\boxed{f \propto y}$$

*** Note:-**

As per Bending equation <u>stress</u> is linear in the cross section.

As per Bernoulli equation <u>strain</u> is linear in the cross section.

**** Section modulus (z):-**

First moment of area about a reference axis ∅ is section modulus (z).

$$\boxed{z = \frac{I}{y_{max}}}, \quad m^3$$

Units :- m^3, cm^3, mm^3

$$\boxed{\uparrow z \;:\; \uparrow \text{strength in bending}}$$

Flexural Rigidity (EI):-

Product of youngs modulus and Moment of Inertia is Flexural Rigidity.

Units :- $N-m^2$, $N-mm^2$

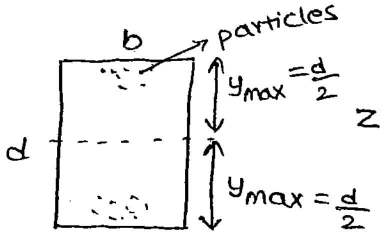
$$\boxed{\uparrow EI \;:\; \uparrow \text{stiff} \;:\; \uparrow \text{rigid}} \;:\; \boxed{\downarrow \text{slopes and} \downarrow \text{deflections}}$$

Axial rigidity (AE):-

Units :- N

$$\boxed{\uparrow AE \;:\; \downarrow \delta l} \qquad \delta l = \frac{Pl}{AE}$$

Ex:-

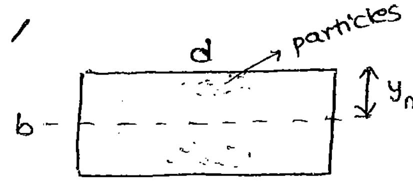


$$z = \frac{I}{y_{max}} = \frac{(bd^3/12)}{(d/2)}$$

$$\boxed{z = \frac{bd^2}{6}}$$

case (i):

EX:-



$y_{max} = \frac{b}{2}$ (case : ii)

$$z = \frac{I}{y_{max}}$$

$$= \frac{\frac{db^3}{12}}{\left(\frac{b}{2}\right)}$$

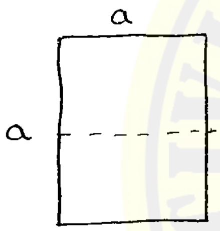
$$\boxed{z = \frac{db^2}{6}}$$

Note :-

If particles are away from the Neutral axis then beam is very strong

case (i) > case (2)

Ex :-

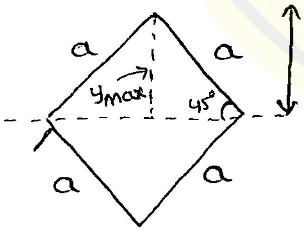


$y_{max} = \frac{a}{2}$

$$z = \frac{I}{y_{max}}$$

$$= \frac{\left(\frac{a \cdot a^3}{12}\right)}{\left(\frac{a}{2}\right)} = \frac{a^3}{6}$$

Square with side horizontal



$y_{max} = \frac{a}{\sqrt{2}}$

$$\sin 45^\circ = \frac{y_{max}}{a}$$

$$y_{max} = \frac{a}{\sqrt{2}}$$

Square with diagonal horizontal

$$z = \frac{I}{y_{max}}$$

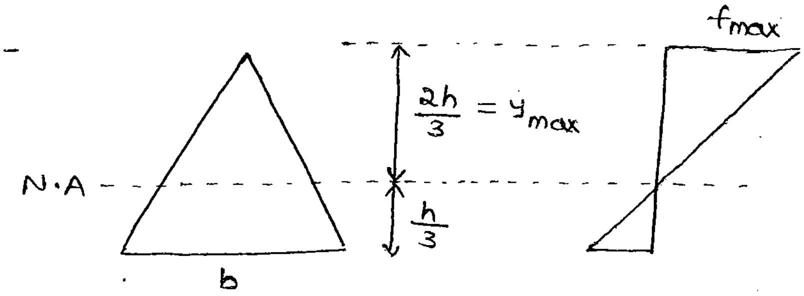
$$= \frac{\frac{a \cdot a^3}{12}}{(a/\sqrt{2})} = \frac{a^3}{6\sqrt{2}}$$

$$\frac{(\text{strength})_{square}}{(\text{strength})_{diagonal}} = \frac{z_{square}}{z_{diagonal}}$$

$$= \sqrt{2} = 1.414$$

$\therefore (\text{strength})_{square} = (41.4\%) \uparrow (\text{strength})_{diagonal}$

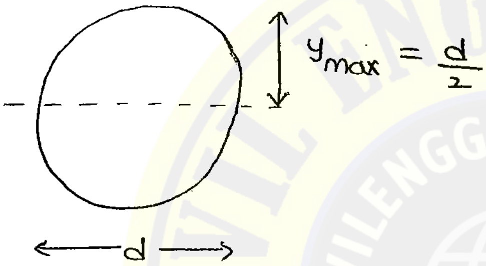
EX:-



$$z = \frac{I}{y_{max}}$$

$$= \frac{\frac{bh^3}{36}}{\left(\frac{2h}{3}\right)}$$

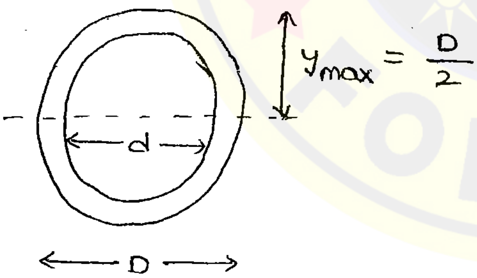
$$z = \frac{bh^2}{24}$$



$$z = \frac{I}{y_{max}}$$

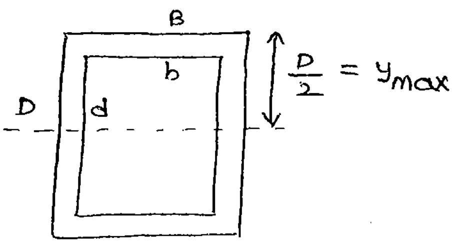
$$= \frac{\frac{\pi d^4}{64}}{(d/2)}$$

$$z = \frac{\pi d^3}{32}$$



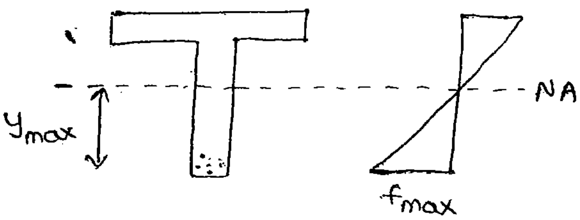
$$z = \frac{I}{y_{max}}$$

$$= \frac{\frac{\pi (D-d)^4}{64}}{(D/2)} = \frac{\pi}{32D}(D^4 - d^4)$$

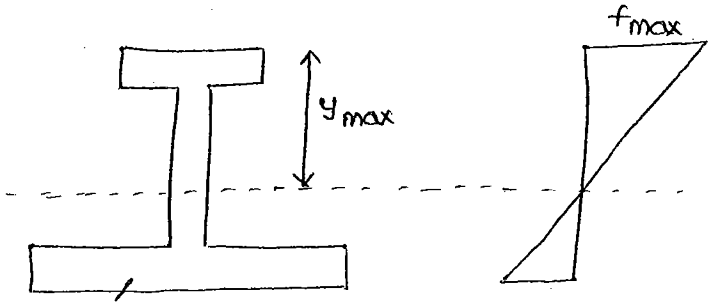


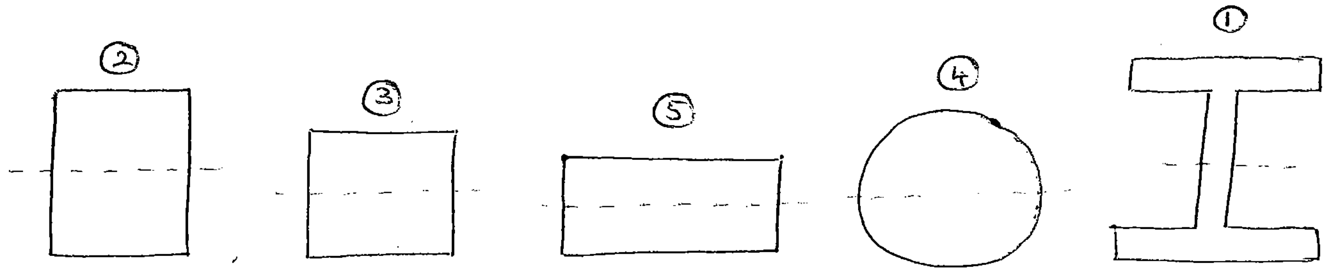
$$z = \frac{I}{y_{max}}$$

$$= \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}} = \frac{1}{6D}(BD^3 - bd^3)$$



NA divides area equally. In this beam NA goes upper side compare to bottom side. particles are far away from N.A then the Section have a y_{max}.





All are having same c/s area
(Ranking as per bending strength)

Composite beams :- (Flitched beams)

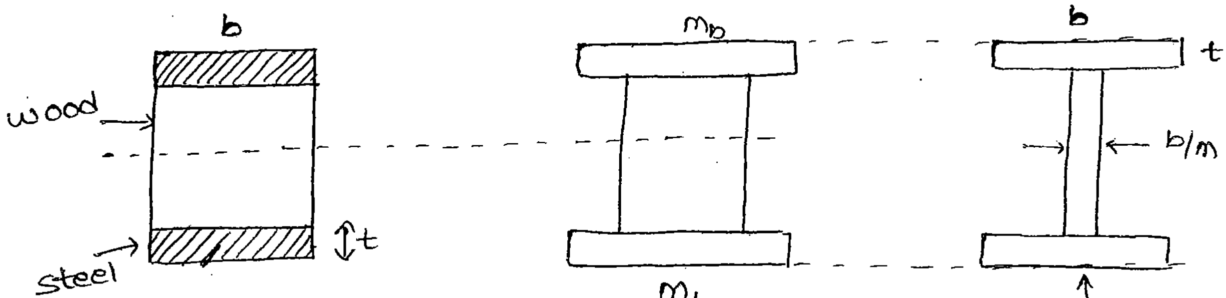
Bending equation is not useful for composite beams in such a case the entire beam can be converted into single material using modular ratio.

Eg:- R.C.C. (Made of different ~~di~~ material bonded together)

Modular ratio (m) :-

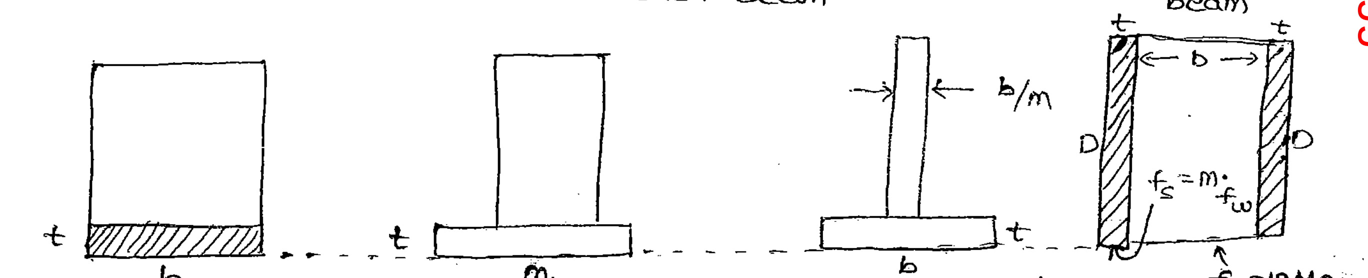
$$m = \frac{E_{strong}}{E_{weak}}$$

$$m = \frac{E_s}{E_w}$$



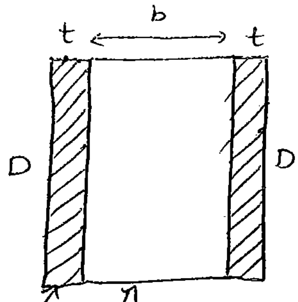
equivalent ~~Ex:-~~ wooden beam

Equivalent steel beam



Equi. in wood

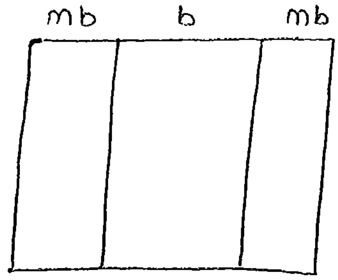
Equivalent in steel



$f_w = 10$ Mpa

$f_s = m \cdot f_w$

(not a good combination)



Equivalent in wood

Beam of uniform strength:-

Along the length of a beam if the bending stress developed is the same then it is beam of uniform strength.

From bending equation

$$\frac{M}{I} = \frac{f}{y}$$

$$M = f \cdot z$$

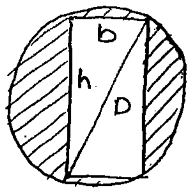
$$f = \text{constant}$$

$$M \propto z$$

For rectangular section

$$M \propto \frac{bd^2}{6} \quad \left.\begin{matrix} M \propto b \\ M \propto d^2 \end{matrix}\right.$$

Dimensions of a rectangle cut from a circular log:-



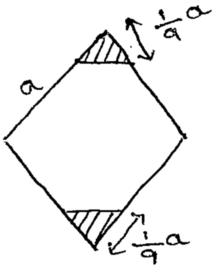
$$** \boxed{\frac{b}{h} = \frac{1}{\sqrt{2}}}$$

$$\frac{b}{D} = \frac{1}{\sqrt{3}} \qquad \frac{h}{D} = \frac{\sqrt{2}}{\sqrt{3}}$$

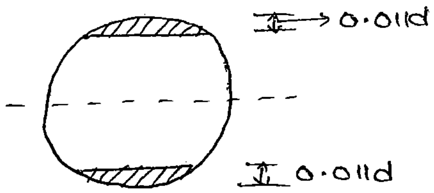
Note:-

For a beam rectangular is better. For a column circular is better.

EX:-)



strong ↑ by 5.35%



strong by 0.7%

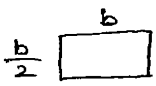
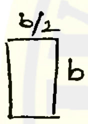
P.g No:-76

5.

A	B
b	~~d~~ b
2d	~~b~~ d

$$\frac{Z_A}{Z_B} = \frac{\frac{b(2d)^2}{6}}{\frac{bd^2}{6}} = 4$$

11.

A	B
	

$$\frac{f_A}{f_B} = \frac{Z_B}{Z_A}$$

$$= \frac{\left[\frac{(b/2)(b)^2}{6}\right]}{\left[\frac{(b)(b/2)^2}{6}\right]}$$

$$= 2$$

$$\therefore \frac{M}{I} = \frac{f}{y}$$

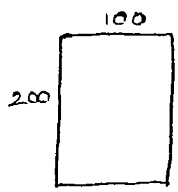
$$f = \frac{M}{Z}$$

$$f \propto \frac{1}{Z}$$

12. same area

Stronger
↓ stress

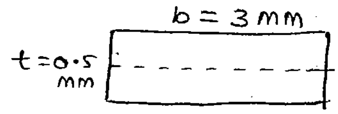
13. $M = 4 \times 10^7$ N-mm



$$\frac{M}{I} = \frac{f}{y}$$

$$f = \frac{M}{I} \cdot y = \frac{M}{Z} = \frac{4 \times 10^7}{\left(\frac{100 \times 200^2}{6}\right)} = 60 \text{ Mpa}$$

14. Steel



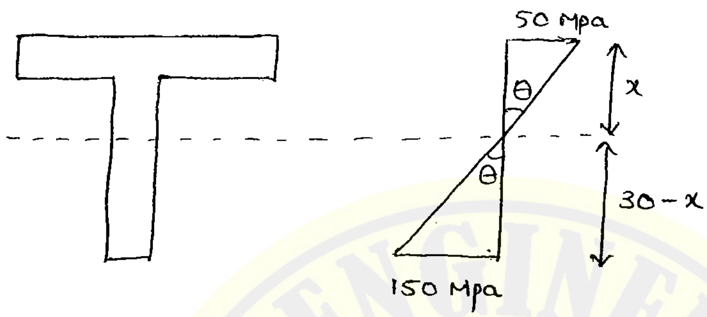
$$R = \frac{500}{2} = 250 \text{ mm}$$

$$\frac{E}{R} = \frac{f}{y_{max}}$$

$$\frac{2\times10^5}{250} = \frac{f}{\left(\frac{0.5}{2}\right)}$$

$$f = 200 \text{ Mpa}$$

15.



$$\tan\theta = \frac{50}{x} = \frac{150}{30-x}$$

$$50(30-x) = 150x$$

$$x = 7.5 \text{ cm}$$

16. $\sigma_w = 7 \text{ N/mm}^2$

$$\sigma_s = m(\sigma_w)$$

$$= 20\times7$$

$$= 140 \text{ N/mm}^2$$

17.

$$\frac{M}{I} = \frac{f}{y}$$

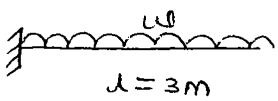
$$M = Wl$$

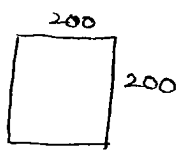
$$M = f\cdot z$$

$$Wl = f\cdot\frac{bd^2}{6}$$

$$b = \frac{6Wl}{fd^2}$$

18.





$$M = \frac{wl^2}{2}$$

$$z = \frac{200\times200^3}{6}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$M = f\cdot z$$

$$= 5\left(\frac{200\times200^3}{6}\right)$$

$$M = 6.66\times10^6 \text{ N-mm}$$

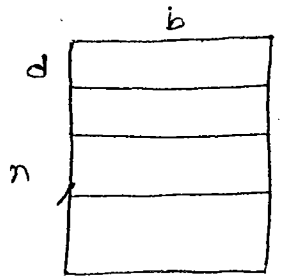
$$= 6.66 \text{ kN-m}$$

$$M = \frac{wl^2}{2}$$

$$6.66 = \frac{w(3m)^2}{2}$$

$$W =$$

19.



loosely placed (or) sliced beams

$$\boxed{\frac{(\text{Strength})_{\text{sliced}}}{(\text{strength})_{\text{solid}}} = \frac{1}{n}}$$

$$(\text{Strength})_{\substack{\cancel{\text{steel}} \\ \text{Sliced}}} = n(\text{strength of each slice})$$

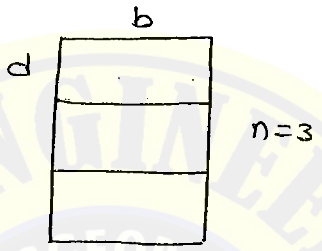
$$= n(Z \text{ of each slice})$$

$$= n\left(\frac{bd^2}{6}\right)$$

$$(\text{Strength})_{\text{solid}} = (Z)_{\text{total}}$$

$$= \frac{b(nd)^2}{6}$$

20.



$$\frac{E}{R} = \frac{M}{I}$$

$$\rho = \frac{1}{R} = \frac{M}{EI}$$

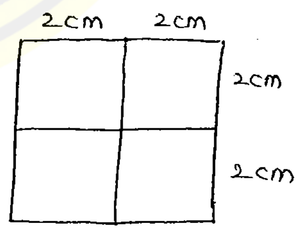
$$\rho \propto \frac{1}{I}$$

$$\frac{\rho_{\text{sliced}}}{\rho_{\text{solid}}} = \frac{I_{\text{solid}}}{I_{\text{sliced}}}$$

$$= \frac{b(3d)^3/12}{3(bd^3/12)}$$

$$= 9$$

21.



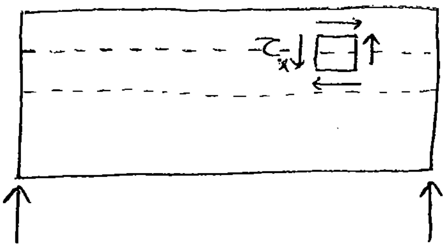
$$\frac{(\text{Strength})_{\text{solid}}}{(\text{Strength})_{\text{sliced}}} = \frac{(Z)_{\text{solid}}}{(Z)_{\text{sliced}}}$$

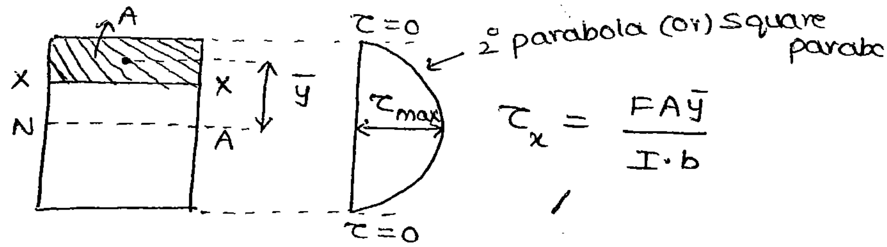
$$= \frac{\frac{4\times 4^2}{6}}{4\left(\frac{2\times 2^2}{6}\right)}$$

$$= 2$$

UNIT - 6

SHEAR STRESS DISTRIBUTION IN BEAMS (4-5 M)





$$\tau_x = \frac{F A \bar{y}}{I \cdot b}$$

F = Shear Force

A = Area above or below x - x

$\bar{y}$ = distance from centroid of area (A) from N.A

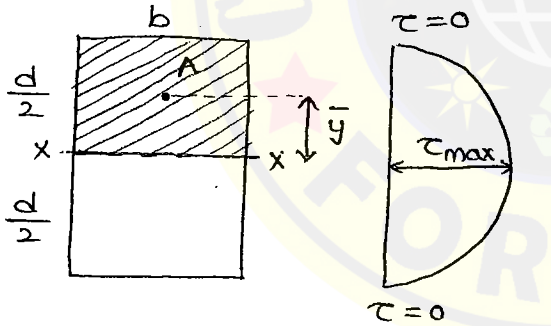
I = Moment of Inertia of total cross section.

b = width of c/s at section x - x

Average or Nominal shear stress:-

$$\tau_a = \frac{F}{c/s \text{ area}}$$

*1. Rectangular (or) square beam:-



$$\tau_{max} = \frac{F A \bar{y}}{I b}$$

$$= \frac{F\left(b \cdot \frac{d}{2}\right)\left(\frac{d}{4}\right)}{\frac{bd^3}{12}(b)}$$

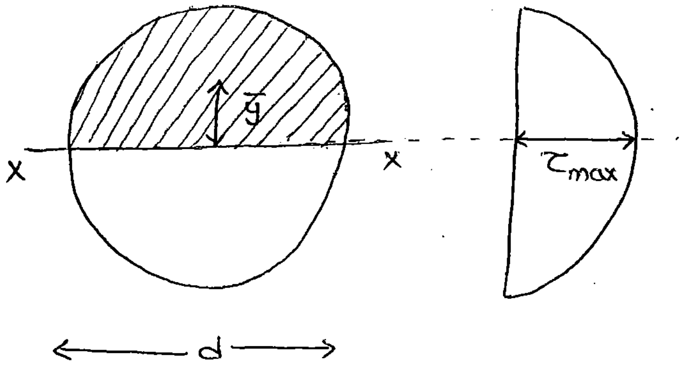
$$\boxed{\tau_{max} = \frac{3F}{2bd}}$$

$$\tau_{avg} = \frac{F}{bd}$$

$$\boxed{\frac{\tau_{max}}{\tau_{avg}} = \frac{3}{2} = 1.5}$$

$$\boxed{\tau_{max} = 50\% \text{ of } \tau_{avg}}$$

* 2. Solid circular cross section:-



$$\bar{y} = \frac{4R}{3\pi} = \frac{4\left(\frac{d}{2}\right)}{3\pi} = \frac{2d}{3\pi}$$

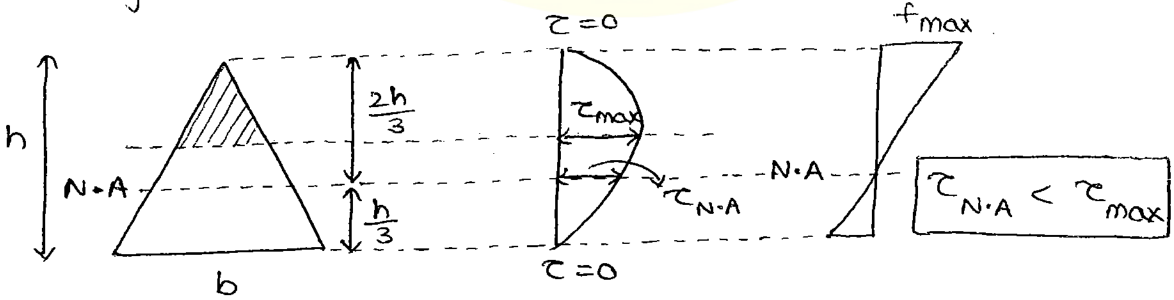
$$\tau_{max} = \frac{FA\bar{y}}{Ib} = \frac{F\left(\frac{1}{2}\cdot\frac{\pi}{4}d^2\right)\left(\frac{2d}{3\pi}\right)}{\left(\frac{\pi}{64}d^4\right)(d)} = \frac{16F}{3\pi d^2}$$

$$\tau_{avg} = \frac{F}{\left(\frac{\pi}{4}d^2\right)}$$

$$\boxed{\frac{\tau_{max}}{\tau_{avg}} = \frac{4}{3} = 1.33}$$

$$\boxed{\tau_{max} = 33\% \text{ of } \tau_{avg}}$$

* 3. Triangular section:-



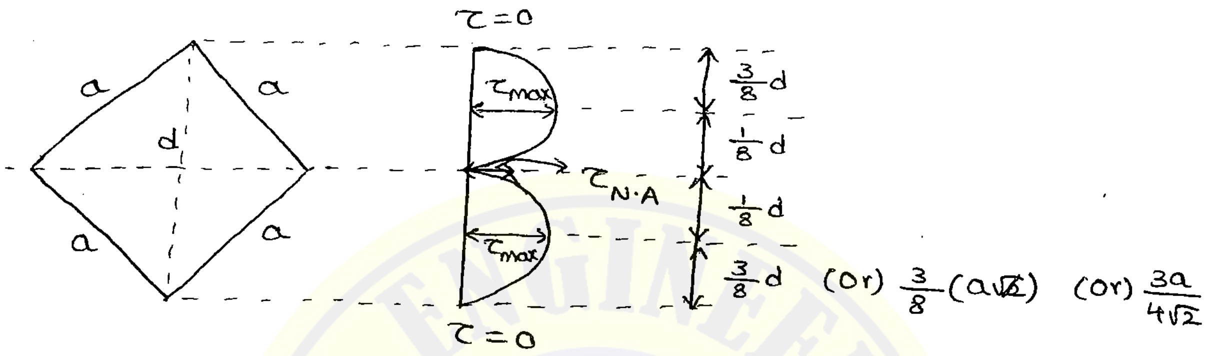
At the point of maximum bending stress shear stress is zero, at the point of maximum shear stress bending stress need not be zero.

$$\boxed{\frac{\tau_{max}}{\tau_{avg}} = 1.5}$$ same as rectangle or square

$$\boxed{\frac{\tau_{N.A}}{\tau_{avg}} = \frac{4}{3}}$$ same as solid circular section

$$\boxed{\frac{\tau_{max}}{\tau_{N.A}} = \frac{9}{8}}$$

4. Diamond section:-



diagonal, $d = a\sqrt{2}$

$$\boxed{\frac{\tau_{max}}{\tau_{avg}} = \frac{9}{8}}$$

$$\frac{\tau_{N.A}}{\tau_{avg}} = 1$$

$$\frac{\tau_{max}}{\tau_{N.A}} = \frac{9}{8}$$

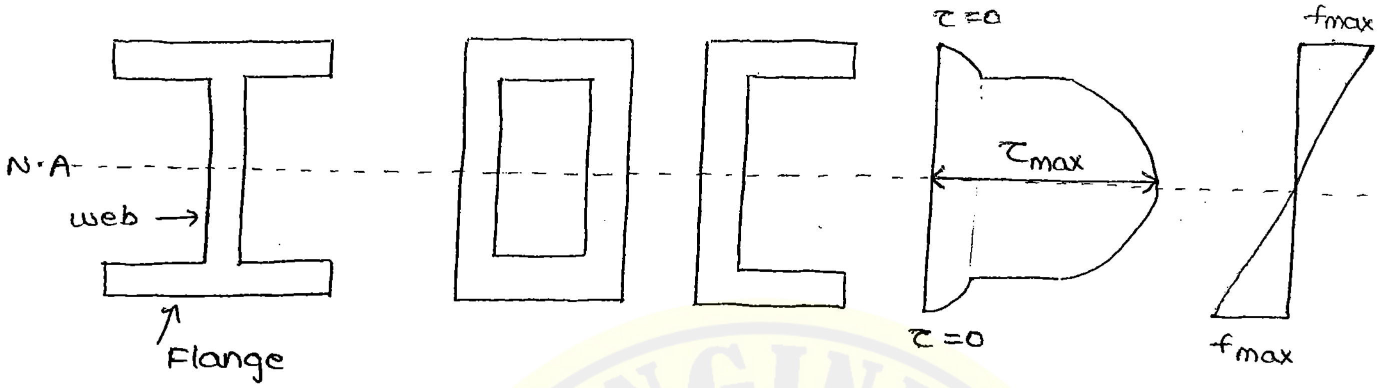
Section	τ_{max}/τ_{avg}	$\tau_{N.A}/\tau_{avg}$
Rectangle / square	$\frac{3}{2} = 1.5$	$3/2 = 1.5$
Circular	$\frac{4}{3} = 1.33$	$4/3 = 1.33$
Triangle	$\frac{3}{2} = 1.5$	$4/3 = 1.33$
Diamond	$\frac{9}{8} = 1.125$	1.0

Flanged beams:-

$$\tau = \frac{FA\bar{y}}{Ib}$$

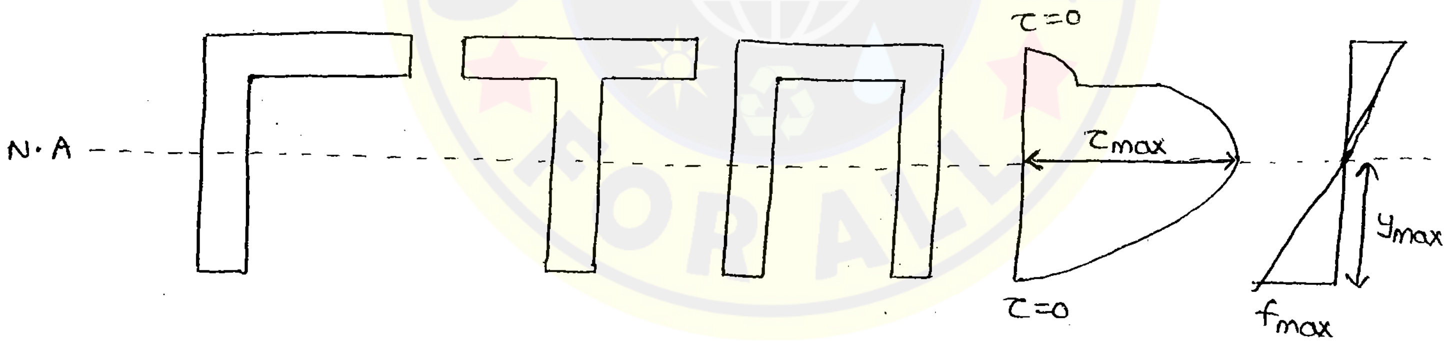
$\downarrow b : \uparrow \tau$

$\uparrow b : \downarrow \tau$

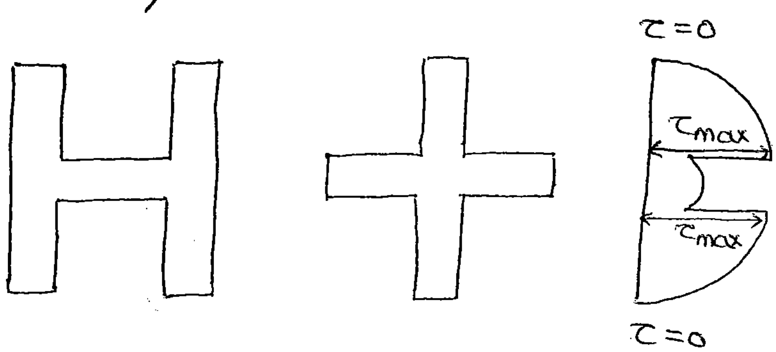


In flanged beams web will be resisting shear force, Flange is to resist Bending Moment.

Angle (or) T - section :-



H - section (or) plus section:-



Note:-

τ_{max} is not maximum at Neutral axis is Triangle, diamond, H-section, plus section.

P.g NO:- 85

2. $\dfrac{\tau_{avg}}{\tau_{max}} = \dfrac{2}{3}$

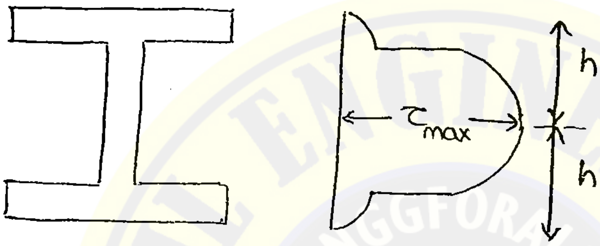
4. $\tau_{max} = \dfrac{3}{2}(\tau_{avg})$

$= \dfrac{3}{2}\left[\dfrac{F}{bd}\right]$

$3 = \dfrac{3}{2}\left[\dfrac{50\times10^3}{100\times d}\right]$

$d = 250$ mm

11.



$\tau_{max} = 1.0\,h$

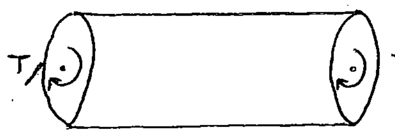
UNIT - 7

TORSION

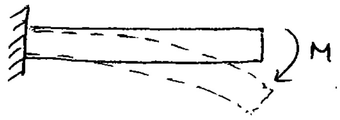
Pure Torsion :- (T)

Non zero constant and maximum. (SF = 0 AF = 0 BM = 0)

Eg:-



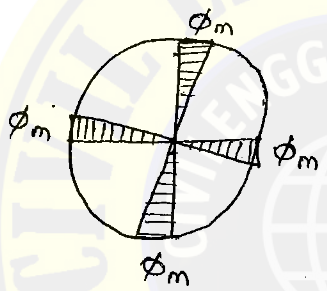
Torsion about axis



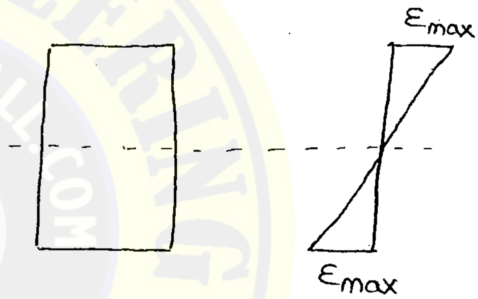
BM along the axis

Assumptions :-

1. plane cross section remain plane after torsion
2. As per Bernoulli there should not be any distorsion in the shape of cross section.



c/s of shaft



Due to bending

3. As per Bernoulli shear strain is linear in the cross section with zero at the centre of shaft and maximum at extreme fibres (or) At outer surface of the shaft
4. Bernoulli's assumption is valid for circular shafts only (both solid and hallow circular shafts)
5. Torsion is uniform (or) constant along the length of the shaft.
6. Radii remains straight after torsion. If the radii lines are straight before and after torsion the shear strain distribution will be linear and Bernoulli's assumption is valid.

Torsion formula:-

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$$

T = Torque

$J = I_p = I_z = I_x + I_y$ (polar M·I)

G = Rigidity modulus

θ = angle of twist in c/s in radians

l = length of shaft

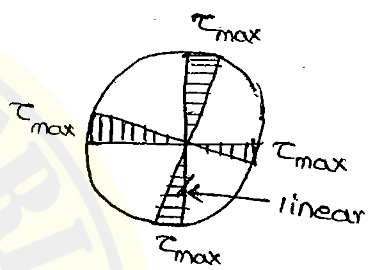
τ = Torsional shear stress

r = radial distance from centre of shaft

$$\frac{T}{J} = \frac{\tau}{r}$$

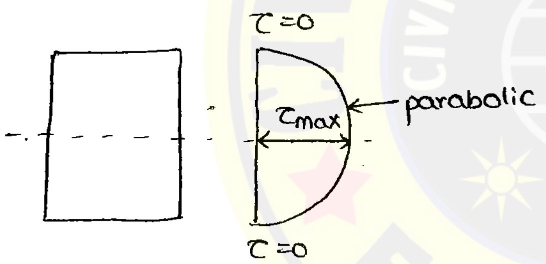
$$\tau = \frac{T}{J}\cdot r$$

$$\tau \propto r$$



c/s of shaft

Torsional shear stress distributio



Flexural shear stress distribution.

Limitation:-

1. The torsion equation is applicable for circular shafts only (solid and hallow shafts)

$$\boxed{\frac{T}{J} = \frac{G}{(l/\theta)} = \frac{\tau}{r}}$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{f}{y}}$$

Bending	Torsion
1. Section modulus, $Z = \frac{I}{y_{max}}$ $\uparrow Z$: More strength in bending Units :- m^3, cm^3, mm^3	1. polar (section) modulus, $Z_p = \frac{J}{r_{max}}$ $\uparrow Z_p$: $\uparrow$ strength in torsion Units :- m^3, cm^3, mm^3
2. Flexural Rigidity = EI Units :- $\frac{N}{mm^2} \cdot mm^4$ (or) $N-mm^2$ $\uparrow EI$: $\uparrow$ rigid : $\uparrow$ stiffness : $\downarrow$ slopes & $\downarrow$ deflection	2. Torsional rigidity = GJ Units :- $N-mm^2$ $\uparrow GJ$: $\uparrow$ rigid : $\uparrow$ stiffness : $\downarrow \theta$

Axial force :-

Axial rigidity = AE

Units : N

$\uparrow AE$: $\downarrow \delta l$

$$\delta l = \frac{Pl}{AE}$$

For springs :-

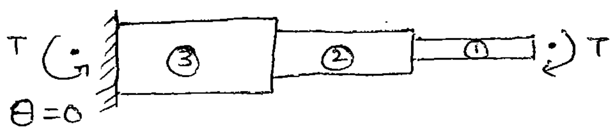
1. Axial stiffness, $K = \frac{W}{\delta}$

(Load per unit deflection)

2. Torsional stiffness, $K_T = \frac{T}{\theta}$

(Torque per unit angle of Twist)

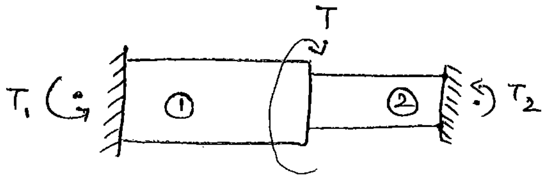
Shafts in series :-



$$T_1 = T_2 = T_3 = T$$

$$\theta_{max} \text{ (@ free end)} = \theta_1 + \theta_2 + \theta_3$$

Shafts in parallel:-

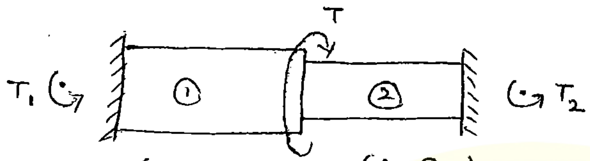


$$T_1 + T_2 = T$$

@ the junction (where torsion is applied)

$$\theta_1 = \theta_2$$

EX:-



$$T_1 + T_2 = T \longrightarrow ①$$

$$\theta_1 = \theta_2$$

From torsion equation

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\boxed{\theta = \frac{Tl}{JG}}$$

$$\frac{T_1 l_0}{2GJ} = \frac{T_2 l_0}{GJ}$$

$T_1 = 2T_2$ sub in ①

$$2T_2 + T_2 = T$$

$$T = 3T_2$$

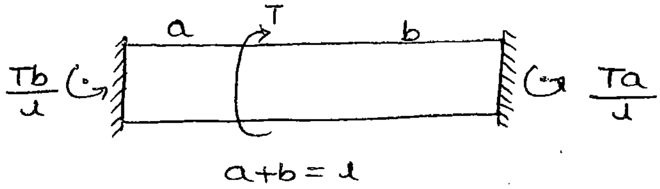
$$T_2 = \frac{T}{3}$$

$$T_1 = \frac{2T}{3}$$

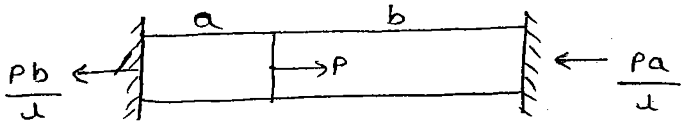
θ at the junction $= \theta_1 = \theta_2 = \dfrac{\left(\frac{2T}{3}\right)l}{2GJ} = \dfrac{\left(\frac{T}{3}\right)l}{GJ}$

$$\theta_1 = \theta_2 = \frac{TL}{3GJ}$$

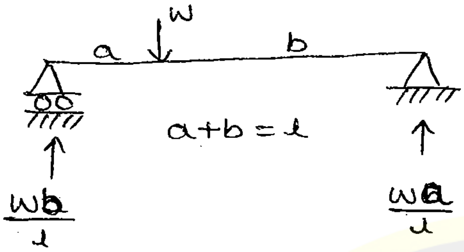
EX:-



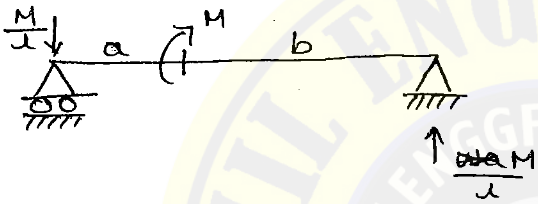
EX:-



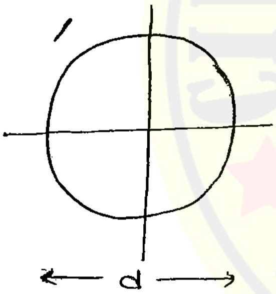
EX:-



EX:-

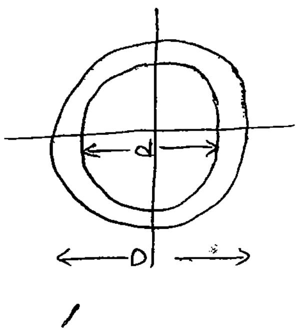


EX:-



$$z = \frac{I}{y_{max}} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$$

$$z_p = \frac{J}{y_{max}} = \frac{\frac{\pi d^4}{32}}{(d/2)} = \frac{\pi d^3}{16}$$



$$z = \frac{\frac{\pi}{64}(D^4-d^4)}{(D/2)}$$

$$z = \frac{\pi}{32D}(D^4-d^4)$$

$$z_p = \frac{\frac{\pi}{32}(D^4-d^4)}{(D/2)} = \frac{\pi}{16D}[D^4-d^4]$$

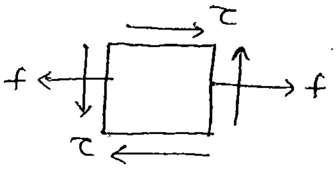
For transmitting section area is same hallow shaft is better than solid circular shaft and it is efficient.

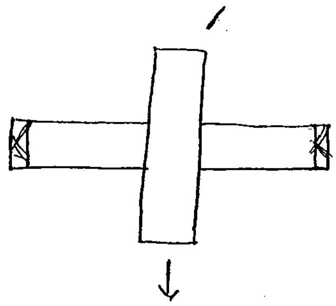
Combined stresses :-

In practise shafts are subjected to Bending Mome in addition to torsion in such a case the design of shaft is based on principal stresses

Diameter of solid shaft 'd'

Element on shaft





$$\sigma_x = f = \frac{M}{Z} = \frac{M}{\left(\frac{\pi d^3}{32}\right)} = \frac{32M}{\pi d^3}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \tau = \frac{T}{Z_p}$$

$$= \frac{T}{\frac{\pi d^3}{16}}$$

$$= \frac{16T}{\pi d^3}$$

$$\frac{f}{y} = \frac{M}{I} \qquad \frac{\tau}{r} = \frac{T}{J}$$

$$f = \frac{M}{I} \cdot y \qquad \tau = \frac{T}{J} \cdot r$$

$$= \frac{M}{Z} \qquad = \frac{T}{Z_p}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\sigma_{1,2} = \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]}$$

Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$\tau_{max} = \frac{\frac{16}{\pi d^3}\left[M + \sqrt{M^2+T^2}\right] - \frac{16}{\pi d^3}\left[M - \sqrt{M^2+T^2}\right]}{2}$$

$$\boxed{\tau_{max} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]}$$

Equivalent Bending Moment:-

$$M_e = \frac{M + \sqrt{M^2+T^2}}{2}$$

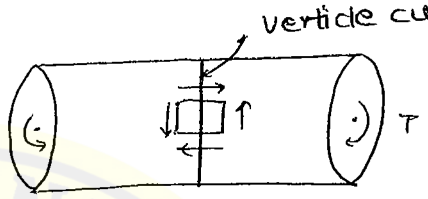
Equivalent Torsion:-

$$T_e = \sqrt{M^2+T^2}$$

Failure criteria:-

Ductile Shafts. It is weak in shear

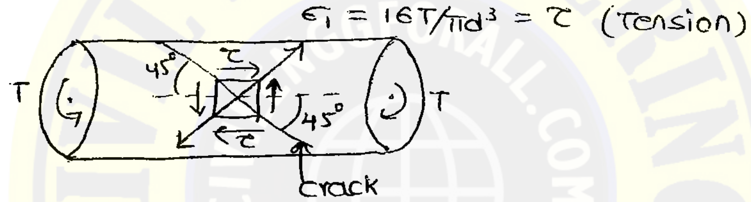
Eg:- Mild steel. 90° crack



Brittle:-

Eg:- cast Iron, concrete

Brittle material are weak in tension



In Brittle material 45° crack develops in the axis of shaft

P.g No:-94

8. $\frac{\tau}{f} = \frac{\left(\frac{16T}{\pi d^3}\right)}{\left(\frac{32M}{\pi d^3}\right)}$

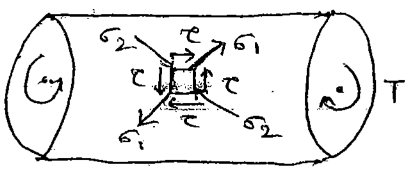
$= \frac{T}{2M}$

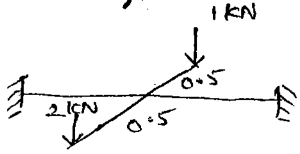
9. Chalk: Brittle

Clock wise Torsion: clockwise 45° crack

Anti c.w. Torsion: Anti c.w. 45° crack.

** 10. If a member is subjected to Torsion the maximum stresses (both shear and Normal) develops on the surface, the stress develop in the cross section is zero.





11. Net torsion $= 2(0.5) - 1(0.5)$

$= 0.5$ KN-m

12. $\dfrac{T}{J} = \dfrac{\tau}{r}$

$\tau = \dfrac{T}{J} \cdot r$

$= \dfrac{T}{Z_p}$

$\tau = \dfrac{16T}{\pi d^3}$

$5 = \dfrac{16T}{\pi (40)^3}$

$T = 62.8 \times 10^3$ N-mm

$= 62.8$ N-m

13. $\tau = \dfrac{16T}{\pi d^3}$ (only for solid shaft)

$= \dfrac{16(800 \times 100 \text{ kg-cm})}{\pi (10 \text{ cm})^3}$

$\tau = 407.4$ kg/cm^2

14. $\theta = 0.6$ degrees

$= 0.6 \times \dfrac{\pi}{180}$ radians $= 0.01$ radians

$\dfrac{T}{J} = \dfrac{G\theta}{l}$

$\dfrac{11{,}000}{\frac{\pi}{32}(5)^4} = \dfrac{G(0.01)}{50}$

$G = 0.856 \times 10^6$ kg/cm^2

15. Power Transmission

$P = \dfrac{2\pi NT}{60}$

T = N-m (or) Joule (J)
N = rpm
P = avg. power (N-m/s) (or) $\frac{J}{S}$ (or) watt

KN-m/s → KW

1 H.P = 746 W = 0.746 KW

If T ~~kg~~ is given in kg-m

$P = \dfrac{2\pi NT}{4500}$ where

T = kg-m
N = rpm
P = H.P

$80 = \dfrac{2\pi(60)T}{4500}$

$T_{avg} = \dfrac{3000}{\pi}$ kg-m

$$\text{Maximum Torque} = 30\%\,(T_{avg})$$

$$= 1.3\left(\frac{3000}{\pi}\right)$$

$$= \frac{3900}{\pi}$$

16. Given $N = 150$ rpm

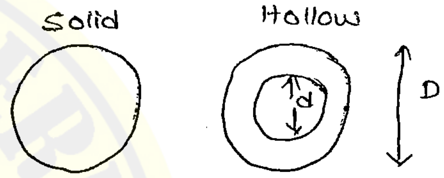
$T = 150$ kg-m

$$P = \frac{2\pi N T}{4500}$$

$$= \frac{2\pi \times 150 \times 150}{4500}$$

$$P = 10\pi$$

** 17.



$d = \frac{D}{2}$

$$\frac{(\text{Strength})_{solid}}{(\text{Strength})_{hollow}} = \frac{(Z_p)_{solid}}{(Z_p)_{hollow}}$$

$$= \frac{\frac{\pi}{16}D^3}{\frac{\pi}{16D}(D^4 - d^4)}$$

$$= \frac{\pi D^3}{16} \times \frac{16D}{\pi\left(D^4 - \left(\frac{D}{2}\right)^4\right)}$$

$$= \frac{\pi D^4}{\pi\left(\frac{15D^4}{16}\right)}$$

$$= \frac{16}{15}$$

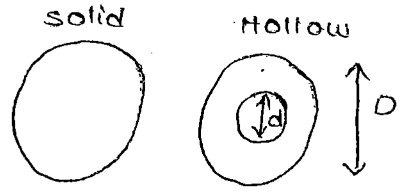
18. $P_A = P_B$

$$\frac{2\pi N_A T_A}{4500} = \frac{2\pi N_B T_B}{4500}$$

$$N_A T_A = N_B T_B$$

$$100(10) = 150(T_B)$$

$$T_B = \frac{1000}{150} = 6.67 \text{ KN-m.}$$



19.

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\tau = \frac{T}{J} \cdot r$$

$$\tau = \frac{T}{Z_p}$$

Given $T_s = T_h$

$$\tau \propto \frac{1}{Z_p}$$

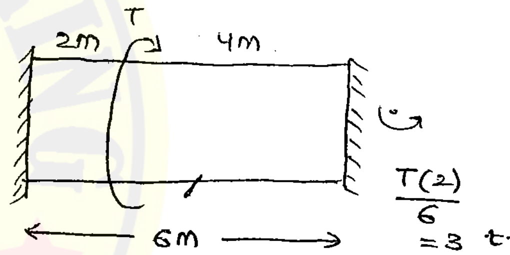
$$\frac{\tau_s}{\tau_h} = \frac{(Z_p)_h}{(Z_p)_s}$$

$$= \frac{\frac{\pi}{16D}\left[D^4 - d^4\right]}{\frac{\pi}{16} D^3}$$

$$\boxed{\frac{\tau_s}{\tau_h} = \frac{D^4 - d^4}{D^4}}$$

20. $T = 9$ t-m

$$6 \text{ t-m} = \frac{T(4)}{6}$$



21.

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{8 \times 10^6 \text{ N-mm}}{\frac{\pi}{32}(10)^4} = \frac{(8 \times 10^4)(\theta)}{(1000 \text{ mm})}$$

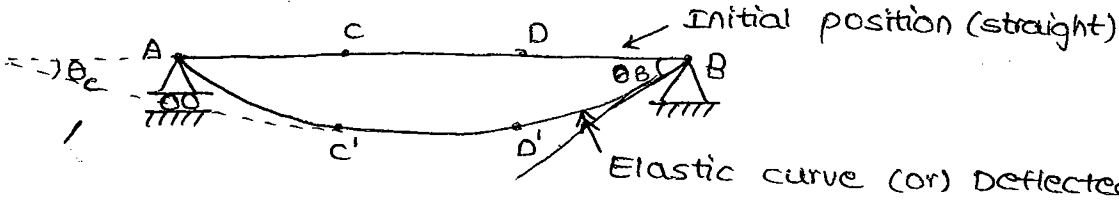
$$\theta = \frac{32}{1000\pi} \text{ radians.}$$

UNIT - 9

SLOPE AND DEFLECTIONS (6 to 8 M)

Beam:-

Horizontal member subjected to transverse (or) vertical load.



Loading	Shape of Elastic curve
No load	Straight
pure bending	Arc of a circle (R = const)
BM + SF both are acting	parabola

Deflection:-

The displacement of a point from initial position to the final position on elastic curve

Note:-

At any rigid support deflection is zero

$y_c = CC'$, $y_D = DD'$, $y_A = 0$, $y_B = 0$

The angle made by a tangent drawn to the elastic curve with the initial position is the slope. (should in radian) θ.

Methods of determining slope and deflections:-

1. Double Integration (Macauley's) method:-

Curvature equation, $EI \dfrac{d^2y}{dx^2} = M \rightarrow (1)$

$$\frac{d^2y}{dx^2} = \text{curvature}(\rho) = \frac{1}{R} = \frac{M}{EI}$$

R = Radius of curvature

Integrate the eq ①

$$EI\left(\frac{dy}{dx}\right) = \int M + c_1 \longrightarrow \text{slope equation}$$

Integrate on both sides

$$EI(y) = \iint M + c_1 x + c_2 \longrightarrow \text{Deflection equations}$$

c_1 and c_2 are integration constants which can be Calculated by boundary conditions.

Two boundary conditions are required for c_1 and c_2

** Relations:-

$$F = \frac{dM}{dx}$$

$$w = \frac{dF}{dx}$$

y = Deflection

$$\frac{dy}{dx} = \text{slope}$$

$$\frac{d^2y}{dx^2} = \text{curvature}(\rho) = \frac{1}{R} = \frac{M}{EI} = M \quad (EI \text{ is unity})$$

$$\frac{d^3y}{dx^3} = \frac{dM}{dx}\cdot\frac{1}{EI} = \frac{F}{EI} = F \quad (EI \text{ is unity})$$

$$\frac{d^4y}{dx^4} = \frac{dF}{dx}\cdot\frac{1}{EI} = \frac{w}{EI} = w \quad (\text{if } EI \text{ is unity})$$

Note:-

At the point of maximum magnitude of deflection (max. positive value (or) maximum negative value) slope must be zero. But at the point of maximum slope, deflection need not be zero.

B.M to be maximum

$$\boxed{\frac{dM}{dx} = 0 \quad F = 0}$$

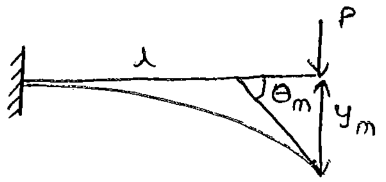
y to be maximum

$$\boxed{\frac{dy}{dx} = 0 \quad \text{slope} = 0}$$

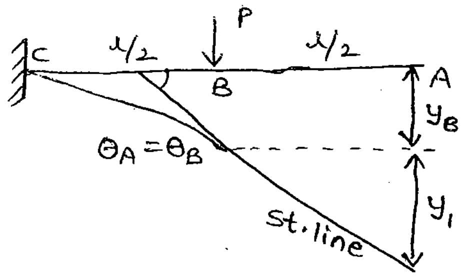
The above condition is not valid if the beam is subjected to concentrated Moments, and in cantilevers.

Cantilever :-

At free end

1. 

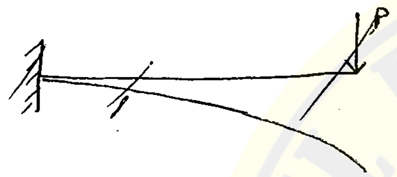
$$\theta_m = \frac{Pl^2}{2EI} \qquad y_m = \frac{Pl^3}{3EI}$$

2. 

$$\theta_B = \frac{P\left(\frac{l}{2}\right)^2}{2EI} = \frac{Pl^2}{8EI}$$

$$y_B = \frac{P(l/2)^3}{3EI} = \frac{Pl^3}{24EI}$$

$$\theta_A = \theta_B = \frac{Pl^2}{8EI}$$

~~3.~~ 

From Δle

$$\tan\theta_B = \tan\theta_A = \frac{y_1}{\left(\frac{l}{2}\right)}$$

$$\theta_B = \theta_A = \frac{2y_1}{l}$$

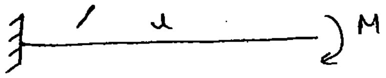
$$y_1 = \frac{\theta_A l}{2}$$

$\because$ For small angles $\tan\theta \approx \theta$

$$y_{max} = y_A = y_B + y_1$$

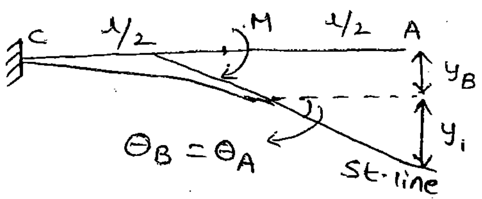
$$= \frac{Pl^3}{24EI} + \frac{Pl^2}{8EI}\left(\frac{l}{2}\right)$$

$$y_{max} = \frac{5Pl^3}{48EI}$$

3. 

$$\theta_{max} = \frac{Ml}{EI}$$

$$y_{max} = \frac{Ml^2}{2EI}$$

4. 

$$\theta_B = \frac{M(l/2)}{EI} = \frac{Ml}{2EI}$$

$$y_B = \frac{M\left(\frac{l}{2}\right)^2}{2EI} = \frac{Ml^2}{8EI}$$

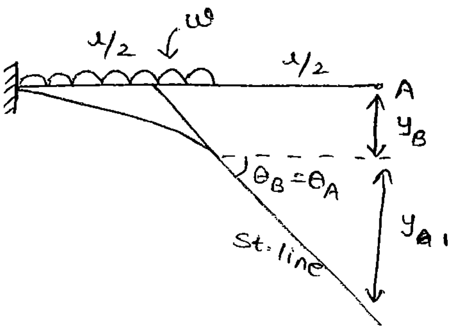
$$\theta_B = \theta_A = \frac{Ml}{2EI}$$

$$y_A = y_B + y_1$$

$$= y_B + \theta_B\left(\frac{l}{2}\right)$$

$$= 3Ml^2/8EI$$

5.



$$\theta_B = \frac{w(l/2)^3}{6EI} = \frac{wl^3}{48EI}$$

$$y_B = \frac{w(l/2)^4}{8EI} = \frac{wl^4}{128EI}$$

$$\theta_A = \theta_B = \frac{wl^3}{48EI}$$

$$\tan\theta_B = \theta_B = \frac{y_1}{\left(\frac{l}{2}\right)}$$

$$y_1 = \theta_B\left(\frac{l}{2}\right)$$

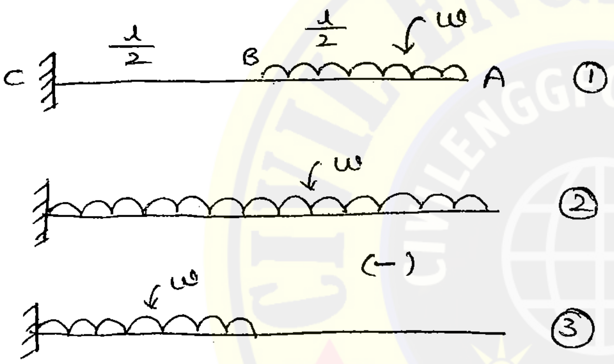
$$y_A = y_B + y_1$$

$$= \frac{wl^4}{128EI} + \theta_B\left(\frac{l}{2}\right)$$

$$= \frac{wl^4}{128EI} + \frac{wl^3}{48EI}\left(\frac{l}{2}\right)$$

$$= \frac{7wl^4}{384EI}$$

6.



① = ② − ③

$$y_A = y_{max} = \frac{wl^4}{8EI} - \frac{7wl^4}{384EI}$$

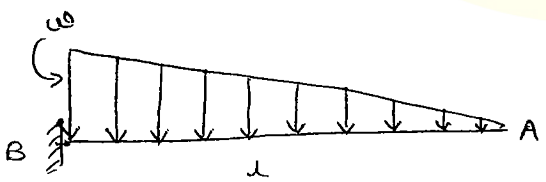
$$= \frac{48wl^4 - 7wl^4}{384EI}$$

$$= \frac{41wl^4}{384EI}$$

$$\theta_A = \theta_{max} = \frac{wl^3}{6EI} - \frac{wl^3}{48EI}$$

$$= \frac{7wl^3}{48EI}$$

7.

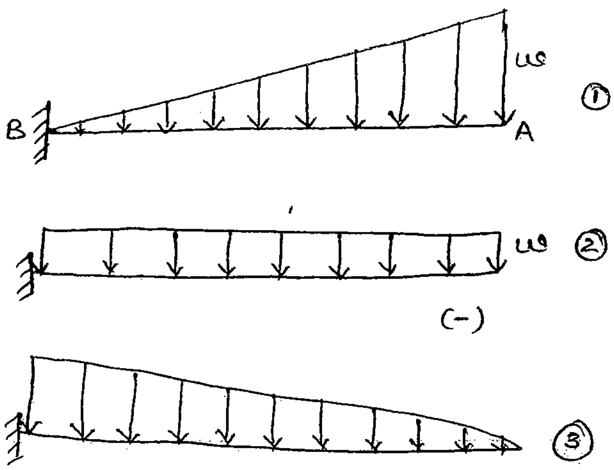


$$\theta_{max} = \theta_A = \frac{wl^3}{24EI}$$

$$y_{max} = y_A = \frac{wl^4}{30EI}$$

Total load, $W = \frac{1}{2} w \cdot l$

8.



① = ② − ③

$$y_A = y_{max} = \frac{wl^4}{8EI} - \frac{wl^4}{30EI}$$

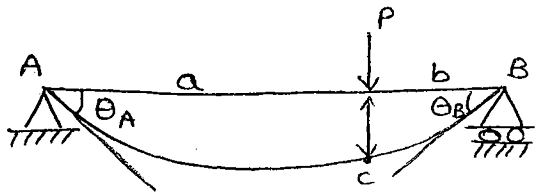
$$= \frac{11wl^4}{120EI}$$

$$\theta_A = \theta_{max} = \frac{wl^3}{6EI} - \frac{wl^3}{24EI}$$

$$= \frac{3wl^3}{24EI} \Rightarrow \frac{wl^3}{8EI}$$

Simply supported beam:-

1.



Max. slope $\theta_B = \dfrac{Pa(L^2-a^2)}{6EIL}$, $\theta_A = \dfrac{Pb(L^2-a^2)}{6EIL}$

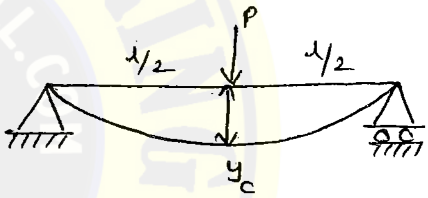
Not y_{max} , $y_c = \dfrac{Pb}{48EI}(3L^2-4b^2)$

y_{max} = not under load

$= \dfrac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}\,EIL}$

Occurs @ 'x' $= \sqrt{\dfrac{L^2-b^2}{3}}$ (or) $\simeq \dfrac{L}{13}$ from centre line towards load

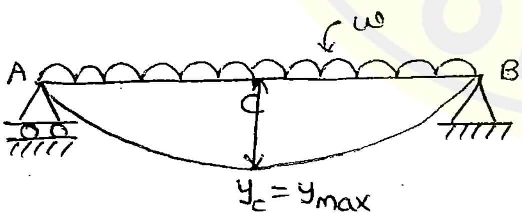
2. If $a = b = \dfrac{l}{2}$



$\theta_{max} = \theta_A = \theta_B = \dfrac{PL^2}{16EI}$

$y_{max} = y_c = \dfrac{PL^3}{48EI}$

3.

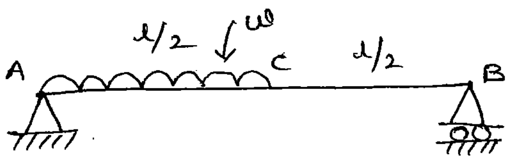


$\theta_A = \theta_B = \theta_{max} = \dfrac{wl^3}{24EI}$

$y_{max} = y_c = \dfrac{5wl^4}{384EI}$

Use $W = wl \rightarrow$ total load

4.

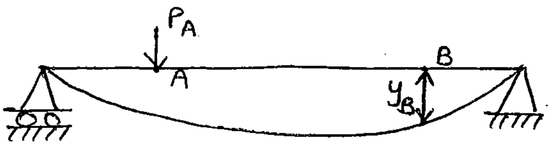


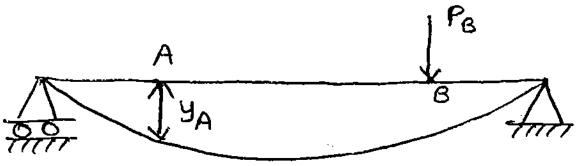
$y_c \rightarrow$ Here y_c is not max.

$y_c = \frac{1}{2}[y_{total\ udl}]$

$= \frac{1}{2}\left[\dfrac{5wl^4}{384EI}\right]$

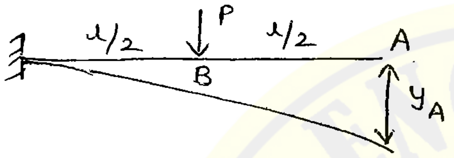
Maxwell's Reciprocal theorem:-



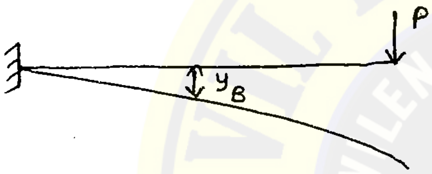


$$\boxed{P_A(y_A) = P_B(y_B)}$$

EX:-



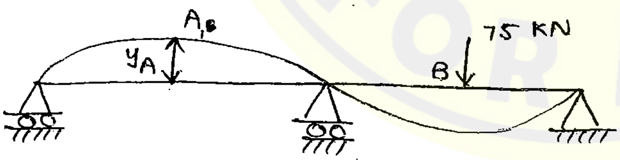
$$y_A = \frac{5Pl^3}{48EI}$$



$$\cancel{P}(y_B) = \cancel{P}(y_A)$$

$$\boxed{y_A = y_B = \frac{5Pl^3}{48EI}}$$

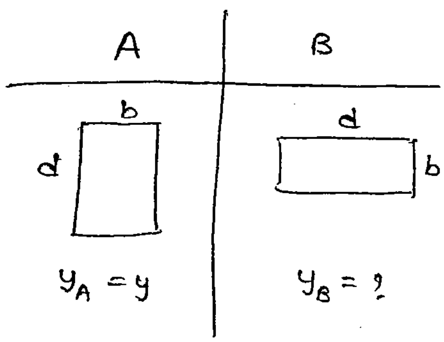
EX:-



$$40(y_A) = 75(5)$$

$$y_A = 9.375 \text{ mm}$$

P.9 No:- 110.



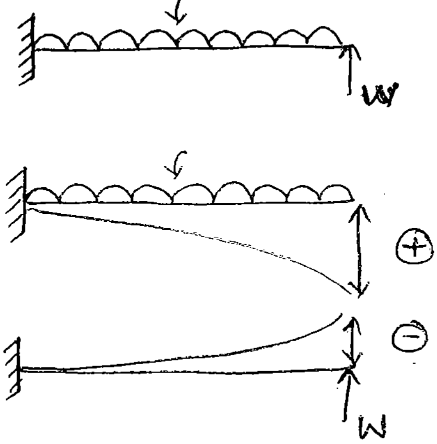
$$y \propto \frac{1}{I}$$

$$\frac{y_B}{y_A} = \frac{I_A}{I_B} = \frac{\left(\frac{bd^3}{12}\right)}{\left(\frac{db^3}{12}\right)} = \left(\frac{d}{b}\right)^2$$

$$y_B = \left(\frac{d}{b}\right)^2 y_A$$

$$= \left(\frac{d}{b}\right)^2 y$$

2. Total load, $W = wl$



$$\text{Net deflection} = +\downarrow y_{udl} - \uparrow y_W$$

$$= +\frac{wl^4}{8EI} - \frac{Wl^3}{3EI}$$

$$= \frac{(wl)l^3}{8EI} - \frac{Wl^3}{3EI} \qquad \therefore wl = W$$

$$= -\frac{5Wl^3}{24EI} \quad (-\text{ve upwards})$$

3.

A	B
d	2d
W	W/2

$$\frac{y_B}{y_A} = \frac{W_B}{W_A} \cdot \frac{d_A^3}{d_B^3}$$

$$= \frac{(W/2)}{W} \cdot \frac{d^3}{(2d)^3}$$

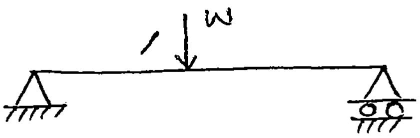
$$= \frac{1}{16}$$

$y = \frac{W}{I}$

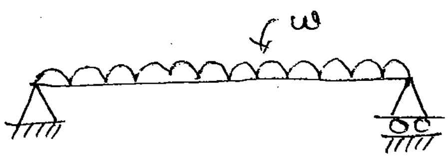
$I = \frac{bd^3}{12}$

$y = \frac{W}{d^3}$

4.



(1)

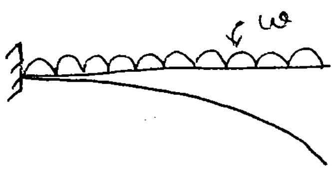


(2)

$$\frac{y_1}{y_2} = \frac{Wl^3/48EI}{5wl^4/384EI}$$

$$= \frac{8}{5} \qquad \text{Use } W = wl$$

5.



$$\theta_{max} = 0.02 \text{ radians} = \frac{wl^3}{6EI}$$

$$\boxed{\frac{wl^3}{EI} = 0.02 \times 6}$$

$$y_{max} = 18 \text{ mm} = \frac{wl^4}{8EI}$$

$$18 = \frac{wl^3}{EI}\left(\frac{l}{8}\right) \Rightarrow 18 = (0.12)\frac{l}{8}$$

$l = 1.2$ m

6. $l = 2m = 2000\ mm$

$w = 10\ kN/m$

$= 10 \left(\frac{1000\,N}{1000\ mm}\right)$

$= 10\ \frac{N}{mm}$

$EI = 2 \times 10^{10}\ kN\text{-}mm^2$

$= 2 \times 10^{10}(1000)\ N\text{-}mm^2$

$= 2 \times 10^{13}\ N\text{-}mm^2$

$$y_{max} = \frac{wl^4}{8EI}$$

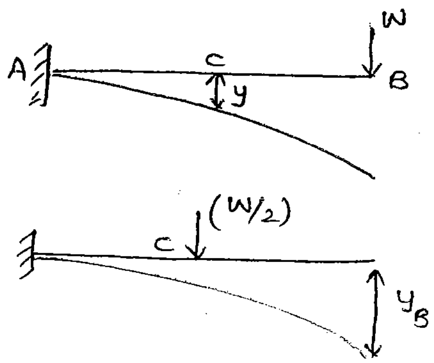
$$= \frac{10\,(2000)^4}{8 \times 2 \times 10^{13}}$$

$= 1\ mm$

7. $K = \frac{W}{y}$

$K \propto \frac{1}{y}$

9.



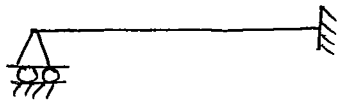
$\left(\frac{W}{2}\right) y = W (y_B)$

$y_B = \frac{y}{2}$

UNIT - 13

PROPPED AND FIXED BEAMS (8 to 10 M)

Propped cantilever:-



It is better to have indeterminate members which we have lesser values of design parameters (SF, BM, slope, deflection).

To analyse an Indeterminate member equilibrium equation and compatability are boundary conditions required.

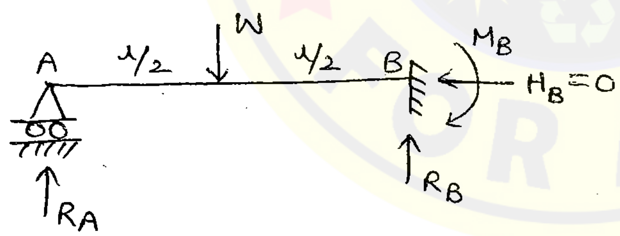
The number of compatability conditions required will be equal to the degree of static indeterminacy.

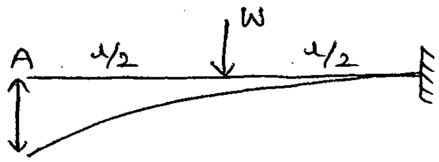
Available compatability condition

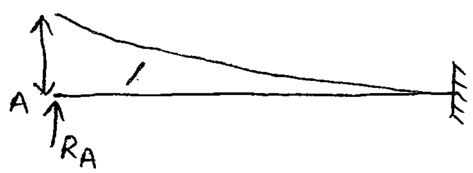
$$\boxed{\begin{matrix} y_A = 0 \\ y_B = 0 \\ \theta_B = 0 \end{matrix}}$$

** At Fixed support both slope and deflection are zero. If other supports deflection only zero and slope will be there.

1.







$y_A = 0$

$\oplus \downarrow y_w \ominus \uparrow y_{R_A} = 0$

$$+\frac{5Wl^3}{48EI} - \frac{R_A l^3}{3EI} = 0$$

$$\boxed{R_A = \frac{5W}{16}}$$

$\Sigma F_y = 0 \qquad R_A + R_B = W$

$$\boxed{R_B = \frac{11W}{16}}$$

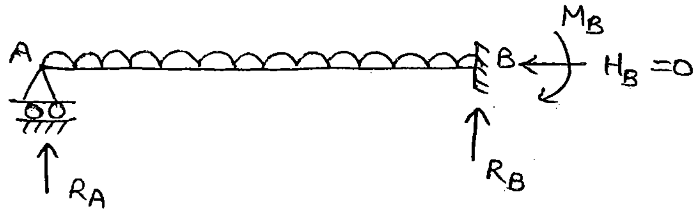
$$M_B = R_A(l) - W\left(\frac{l}{2}\right)$$

$$= -\frac{3Wl}{16} \text{ (hogging)}$$

Note:-

Over a fixed (or) continuous support there will be hoggin moment which creates tension on top fibre, and main reinforcement also on top fibre.

2.

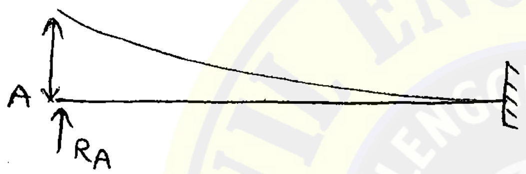


$y_A = 0$

$+ \downarrow y_{udl} - \uparrow y_{R_A} = 0$

$$+\frac{wl^4}{8EI} - \frac{(R_A)l^3}{3EI} = 0$$

$$\boxed{R_A = \frac{3wl}{8}}$$



$\Sigma F_y = 0 \qquad R_A + R_B = wl$

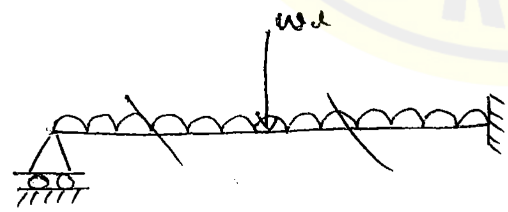
$$\boxed{R_B = \frac{5wl}{8}}$$

$$M_B = R_A(l) - (wl)\left(\frac{l}{2}\right)$$

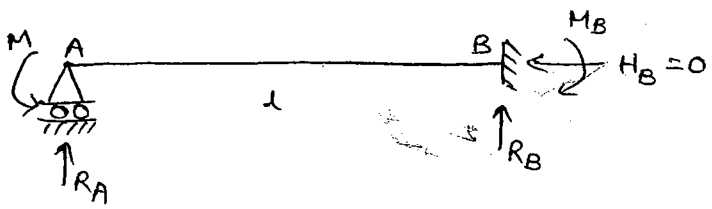
$$= \left(\frac{3wl}{8}\right)l - \frac{wl^2}{2}$$

$$\boxed{M_B = \frac{-wl^2}{8}} \text{ (Hogging)}$$

3.

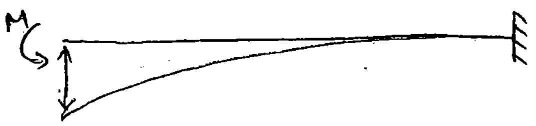


3.



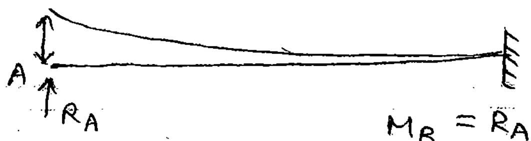
$y_A = 0$

$\oplus \downarrow y_M - \uparrow y_{R_A} = 0$



$$+\frac{Ml^2}{2EI} - \frac{R_A l^3}{3EI} = 0$$

$$\boxed{R_A = \frac{3M}{2l}}$$



$\Sigma F_y = 0, \quad R_A + R_B = 0$

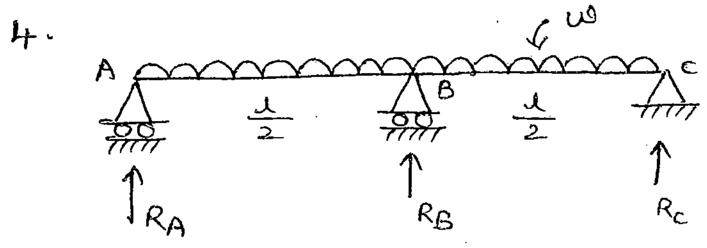
$$\boxed{R_B = \frac{-3M}{2l} (\downarrow)}$$

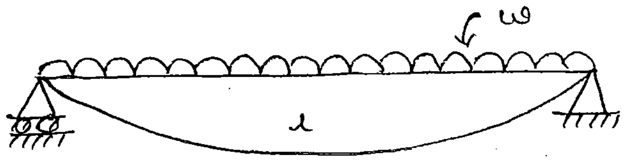
$M_B = R_A l - M$

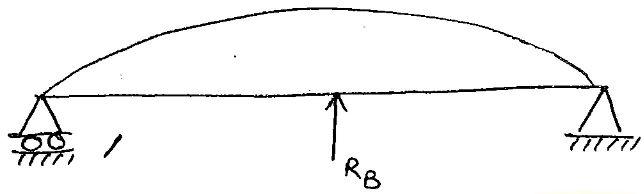
$\boxed{M_B = + M/2}$ (sagging)

Continuous beam:-

4.







$y_B = 0$

$\oplus \downarrow y_{udl} - \uparrow y_{R_B} = 0$

$$+\frac{5wl^4}{384EI} - \frac{R_B l^3}{48EI} = 0$$

$$\boxed{R_B = \frac{5lw}{8}}$$

Due to symmetry $R_A = R_C$

$\Sigma F_y = 0$

$R_A + R_B + R_C = wl$

$R_A + \frac{5wl}{8} + R_A = wl$

$$\boxed{R_A = \frac{3wl}{16}}$$

$M_B = R_A\left(\frac{l}{2}\right) - w\left(\frac{l}{2}\right)\left(\frac{l}{4}\right)$

$$\boxed{M_B = -\frac{wl^2}{32}}$$

Fixed beams:-

5.

verticle load:-

Indeterminate $= 4 - \underset{(\Sigma y,\ \Sigma M)}{2} = 2$

General load:-

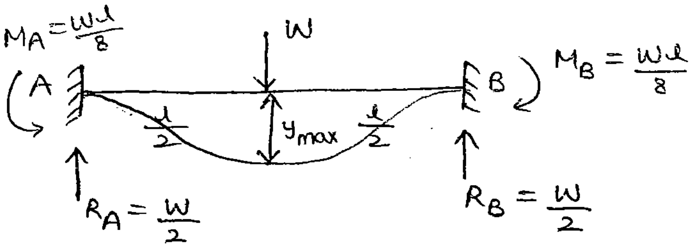
Indeterminate $= 6 - \underset{(\Sigma x,\ \Sigma y,\ \Sigma M)}{3} = 3$

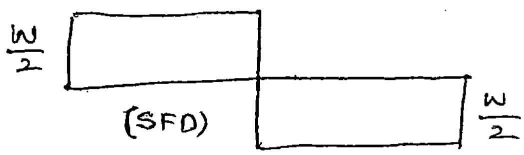
Available compatability condition:-

$\theta_A = 0 \qquad \theta_B = 0$

$y_A = 0 \qquad y_B = 0$

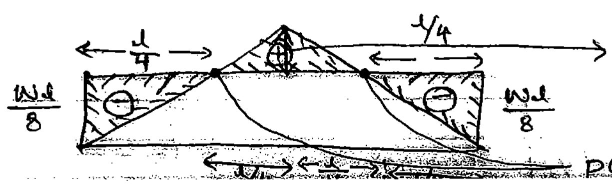
6.





Max. SF = Max. Reaction

$= \frac{W}{2}$

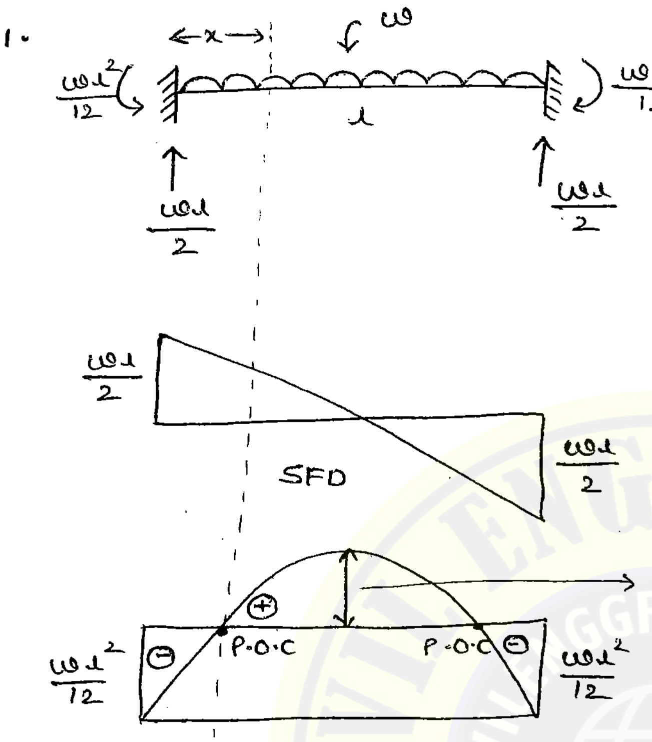


$\frac{Wl}{4} - \frac{Wl}{8} = \frac{Wl}{8}$

** Maximum deflection under load $= \frac{1}{4}\left[y_{\text{simply supported beam}} \right]$

$$= \frac{1}{4}\left[\frac{Wl^3}{48EI} \right]$$

Fixed beam :-

1. 

$$\frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24}$$

Maximum SF = Maximum reaction $= \frac{wl}{2}$

Maximum support (or) Hogging BM $= wl^2/12$

Maximum midspan (or) sagging BM $= wl^2/24$

No. of point of contraflexure = 2

* Distance of P.O.C from support $= 0.212\,l$

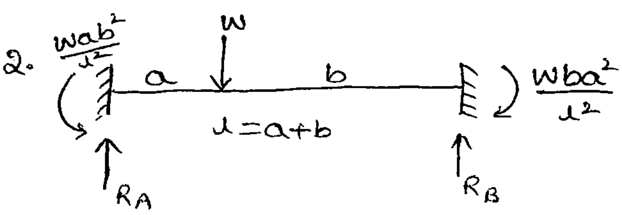
At P.O.C BM is zero

Assume distance of P.O.C is 'x' from left support

$$M_x = \frac{wl}{2}(x) - \frac{wl^2}{12} - w(x)\left(\frac{x}{2}\right)$$

$\therefore M_x = 0$

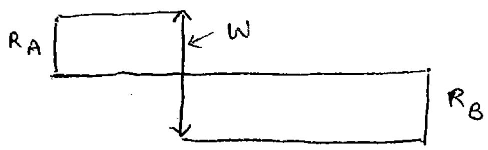
$$\boxed{x = 0.212\,l}$$

2. 

$\Sigma M_B = 0$

$$R_A(l) - W(b) - \frac{Wab^2}{l^2} + \frac{Wba^2}{l^2} = 0$$

$R_A =$

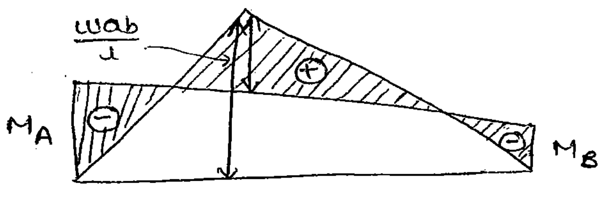


Max. SF = Max reaction = R_A =

Max support or Hogging BM = $\frac{wab^2}{l^2}$

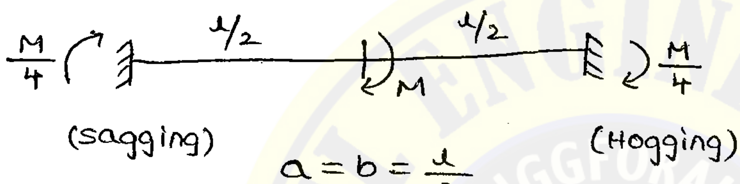
Max. mid span (or) sagging BM =

No. of P.O.C = 2

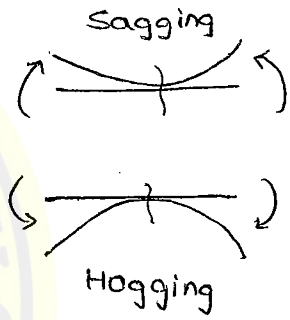


$M_A > M_B$

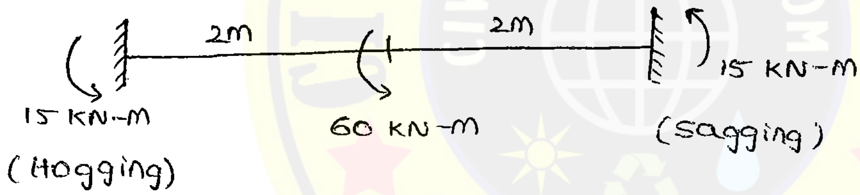
3.



$a = b = \frac{l}{2}$

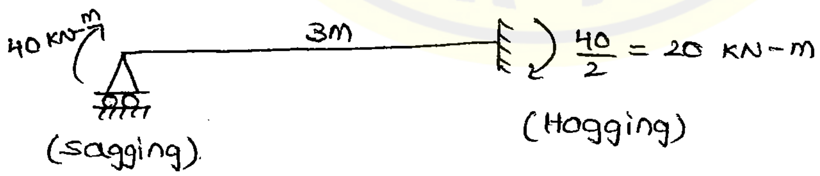


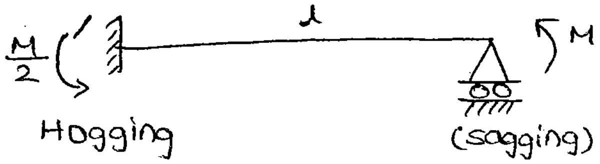
4.

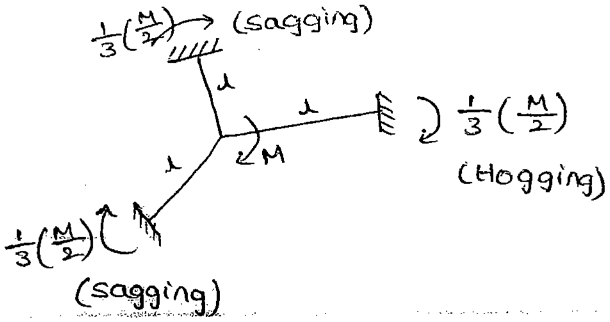


Tension top.

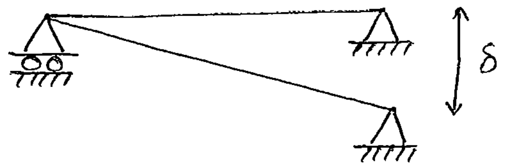
Ex:-





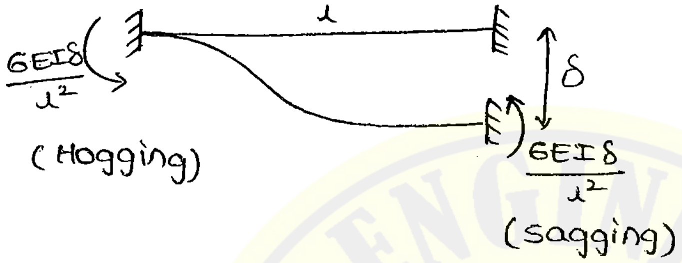


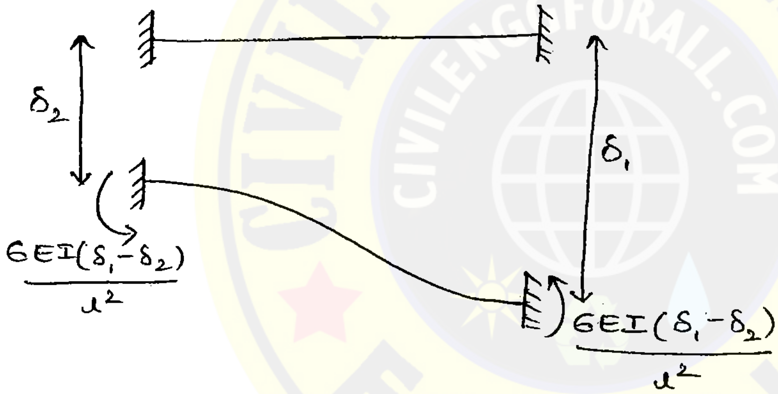
Sinking of supports:-



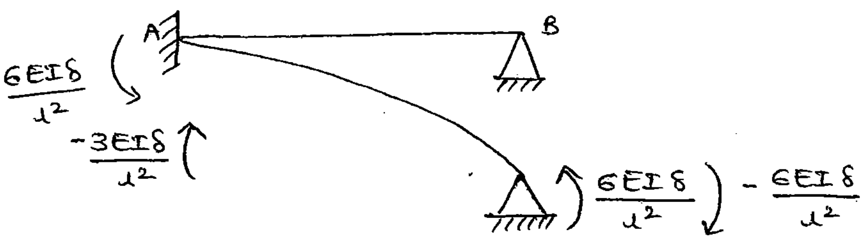
In determinate beams like simply supported, cantilever extra moments will not develop due to sinking of supports.

Ex:-



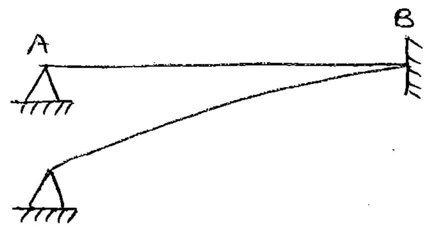


Support rotation:-



$M_A = \frac{3EI\delta}{l^2}$ (Hogging) $\quad M_B = 0$

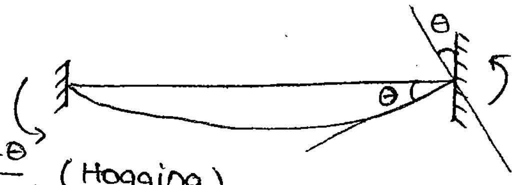
Roller (or) Hinge doesn't take any moment so 'B' point can make zero. so apply anticlockwise moment then that moment distributes 'half moment' to point A.



$M_B = \frac{3EI\delta}{l^2}$ (Hogging)

$M_A = 0$

EX:-



$\frac{2EI\theta}{l}$, (Hogging)

(Developed)

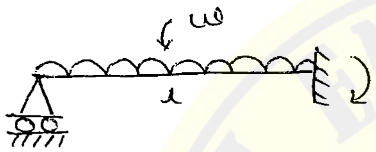
$\frac{4EI\theta}{l}$

(sagging)

Applied moment to cause 'θ' rotates @ B

Pg. No:- 146

2.



$\frac{\omega l^2}{8}$ (Hogging

3.

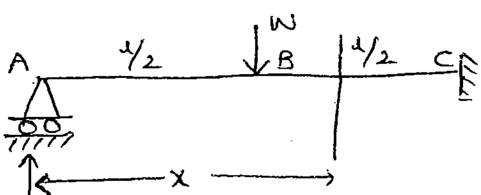
$W = \omega l$

$M_B = \frac{\omega l^2}{32}$ (Hogging) $\Rightarrow \frac{Wl}{32}$ (Hogging)

4.

$\frac{M}{2}$ (Hogging)

5.



$R_A = \frac{5}{16}W$

In propped cantilevers there will be one P.O.C possible.

In the above case assume P.O.C. lies in the zone BC, which is at a distance of 'x' from A.

$M_x = R_A(x) - W(x - \frac{l}{2})$

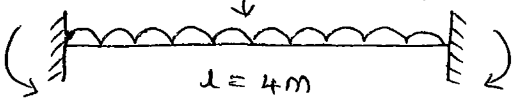
At P.O.C, BM = 0

$$0 = R_A x - W\left(x - \frac{l}{2}\right)$$

$x = \frac{8}{11} l$ from propped

$\boxed{x = \frac{3}{11} l}$ from fixed end

6.

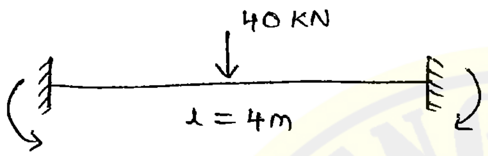


$$M = \frac{wl^2}{12}$$

$$= \frac{30(4m)}{12}$$

$= 40$ kN-m (Hogging)

7.

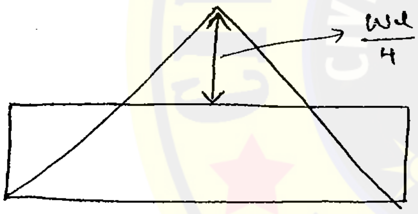


$\frac{Wl}{8}$ $\qquad$ $\frac{Wl}{8}$

$\frac{40 \times 4}{8}$

$= 20$ kN-m $\qquad$ $= 20$ kN-m (clock wise)

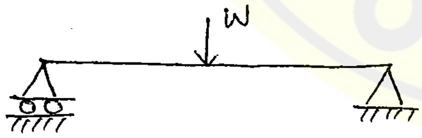
(Anticlockwise)



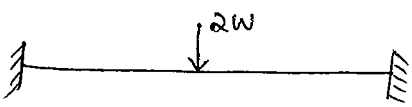
$$\frac{Wl}{4} - \frac{Wl}{8} = \frac{Wl}{8} = 20 \text{ kN-m}$$

$\frac{Wl}{8}$

8.



Max BM, $M = \frac{Wl}{4}$

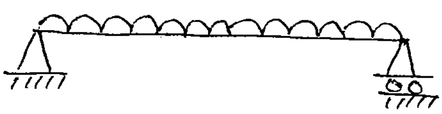


$$M = \frac{(2W)l}{8} = \frac{Wl}{4}$$

$\frac{Wl}{8}$

9.

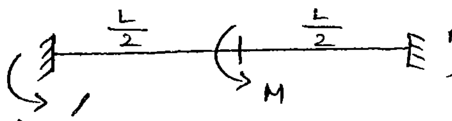
$$M = \frac{wl^2}{12} \longrightarrow w_{fix} = \frac{12M}{l^2}$$



$$M = \frac{wl^2}{8} \longrightarrow w_{s.s} = \frac{8M}{l^2}$$

$$\frac{w_{fix}}{w_{s.s}} = \frac{3}{2} = 1.5$$

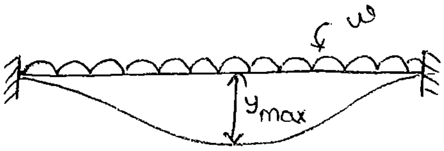
10.



$\frac{1}{2}\left(\frac{M}{2}\right) = \frac{M}{4}$ (sagging)

$\frac{M}{4}$ (hogging)

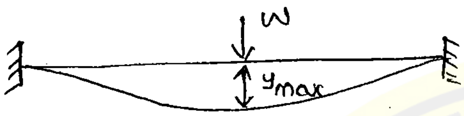
11.



$$y_{max} = \frac{1}{5}\left[y_{\text{simply supported}}\right]$$

$$= \frac{1}{5}\left[\frac{5wl^4}{384\,EI}\right]$$

12.



$$y_{max} = \frac{1}{4}\left[y_{s.s}\right]$$

$$= \frac{1}{4}\left[\frac{5Wl^4}{\cancel{384}\,48EI}\right] = \frac{1}{4}\left[\frac{Wl^3}{48\,EI}\right]$$

(51)

UNIT - II

COLUMNS AND STRUTS

Columns :-

Verticle compression member in a building (structure).

Strut :-

Verticle, Horizontal, Inclined compression member. (truss

Classification :-

1. Short column:-

Most of the practical column in masonry (or) R.C.C. (or) short column only.

Short column can be made safe by using higher factor of safety.

Short columns are sudden crushing failure.

2. Long columns:-

Fails by buckling (gradual failure)

Eg:- Steel structures.

For axially loaded column circular cross sections (solid and hollow) are better. For columns and moments (or) eccentrically loaded columns rectangular is better.

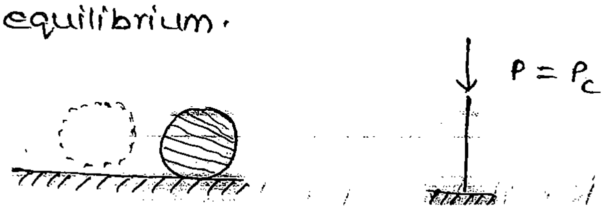
The main design criteria of a column is stability and strength.

Equilibrium conditions:-

1. Stable Equilibrium:-

2. Neutral equilibrium:-

The condition of a column just before the collapse is Neutral equilibrium.



3. Unstable Equilibrium:-

$P > P_c$ } Impossible in practice

** Euler's theory:-

, Applicable only for long columns fails by buckling

Axially loaded columns

$$P_e = \frac{\pi^2}{l^2} E I_{min}$$

Columns buckle about minor axis. Therefore I_{min} should be used.

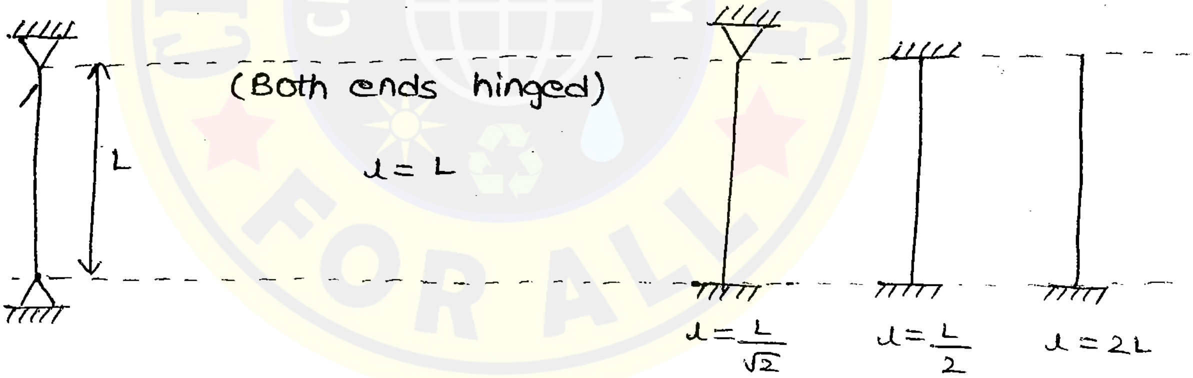
P_e = Euler's buckling load

E = young modulus of elasticity

l = effective length (centre to centre distance b/w two successive points of zero B.M)

L = Actual length of column.

1.



$\cancel{P_e \propto \frac{1}{L^2}}$ $\quad P_e \propto \frac{1}{l^2}$

→ Maximum load carrying capacity is $\boxed{\text{Fix - Fix}}$

→ Minimum load carrying capacity is $\boxed{\text{Fix - Free}}$

Eg:-

$$\frac{P_{Fix-Fix}}{P_{Fix-Free}} = \frac{(l_{Fix-Free})^2}{(l_{Fix-Fix})^2} = \frac{(2L)^2}{(\frac{L}{2})^2} = 16$$

$$\frac{P_{H-H}}{P_{H-Fix}} = \frac{(l_{H-F})^2}{(l_{H-H})^2} = \frac{1}{2}$$

Note:-

Minimum slenderness ratio (λ) for which Euler's theory is applicable is 80.

Eg:-

Rectangular cross section : 200 mm x 400 mm

Actual length of column, L = 4m (Fix - Hinge)

$E = 2 \times 10^5$ Mpa

A. Effective length, $l = \frac{L}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4000}{\sqrt{2}}$ mm

$$P_e = \frac{\pi^2}{l^2} EI_{min}$$

$$= \frac{\pi^2}{\left(\frac{4000}{\sqrt{2}}\right)^2} (2 \times 10^5) (400 \times 200^3 / 12)$$

$$= 65.7 \times 10^6 \text{ N}$$

$$= 65.7 \text{ MN.}$$

** Rankine's theory :-

Rankine's theory is applicable for all columns irrespective of slenderness ratio. Axially loaded columns

$$\boxed{\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}}$$

P_R = Rankine's Load

P_c = crushing load as a short column

P_e = Eulers load as a long column.

Eg:- $P_c = 50$ KN $P_e = 30$ KN

$$\frac{1}{P_R} = \frac{1}{50} + \frac{1}{30}$$

$$= \frac{30 + 50}{1500}$$

$$= \frac{8}{150}$$

$$P_R = 18.75 \text{ KN}$$

Rankine's theory (modified):-

$$\boxed{P_R = \frac{f_c A}{1+\alpha\lambda^2}}$$

f_c = maximum stress or strength of material

A = cross sectional area

α = Rankine's constant $= \frac{f_c}{\lambda^2 E}$

λ = slenderness ratio $= \frac{l}{r_{min}}$

Secant formula:-

Eccentrically Loaded columns.

** Note:-

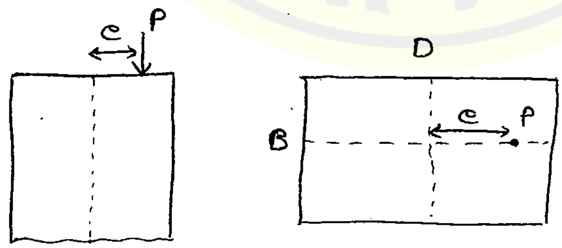
Old IS 800, column design is based on secant formula, the present IS 800 : 2007 is based on proffessor perry's formula.

$$\sigma_{max} = \frac{P}{A}\left[1+\frac{ey_c}{r^2}\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right] \rightarrow \text{secant formula}$$

$$\left(\frac{\sigma}{\sigma_0}-1\right]\left(1-\frac{\sigma_0}{\sigma_E}\right] = \frac{1.2ey_c}{r^2} \rightarrow \text{perry's formula.}$$

Perry's formula also eccentrically Loaded columns.

Short column with eccentric loading:-



c/s of column

$$\sigma_{min}, \sigma_{max} = \frac{P}{A} \pm \frac{M}{Z} \qquad M = Pe$$

$$= \sigma \pm (f_e)$$

Sign convention :

'+' compression

'-' Tension

Note:-

Most of the short columns are made of brittle materi[al] cannot resist tension. Therefore in the design tension in the column must be avoided.

* Minimum stress allowed in the column is zero.

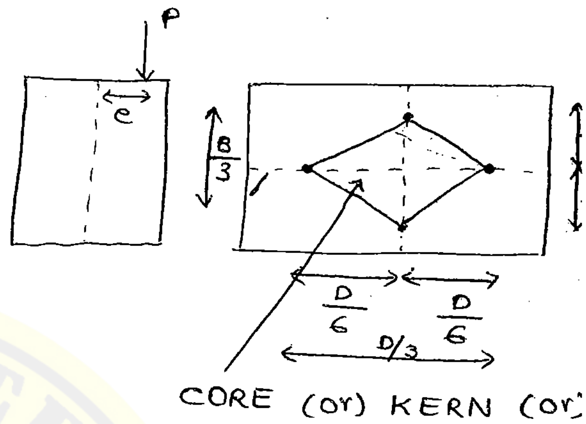
Rectangle (or) square:-

→ For No tension

$$\sigma_{min} = 0 = \frac{P}{A} - \frac{Pe}{Z}$$

$$= \frac{P}{BD} - \frac{P \cdot e}{\left(\frac{BD^2}{6}\right)}$$

$$\boxed{e = \frac{D}{6}}$$



Shape : Rhombus (or) Diamond

Area of core, $A_c = 2\left[\frac{1}{2} \cdot \frac{D}{3} \cdot \frac{B}{6}\right]$

$$= \frac{1}{18}[BD]$$

$$\boxed{A_c = \frac{1}{18}[A_{gross}]}$$

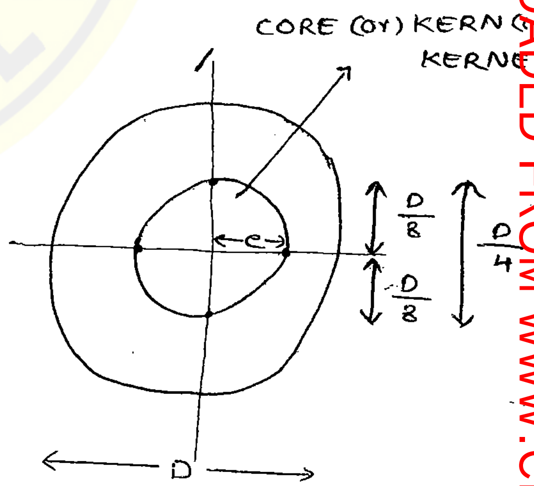
Solid circular section:-

→ For No tension

$$\sigma_{min} = 0 = \frac{P}{A} - \frac{Pe}{Z}$$

$$= \frac{P}{\frac{\pi D^2}{4}} - \frac{Pe}{\left(\frac{\pi D^3}{32}\right)}$$

$$\boxed{e = \frac{D}{8}}$$



Area of core, $A_c = \left[\frac{\pi}{4}\left(\frac{D}{4}\right)^2\right]$

$$= \frac{1}{16}\left[\frac{\pi}{4}D^2\right]$$

$$\boxed{A_c = \frac{1}{16}[A_{gross}]}$$

Note:-

Circular section is better for column.

Note:-

1. For solid circular cross sectional middle forth zone is used for No tension, in case of square (or) Rectangle middle third zone is used for No tension.

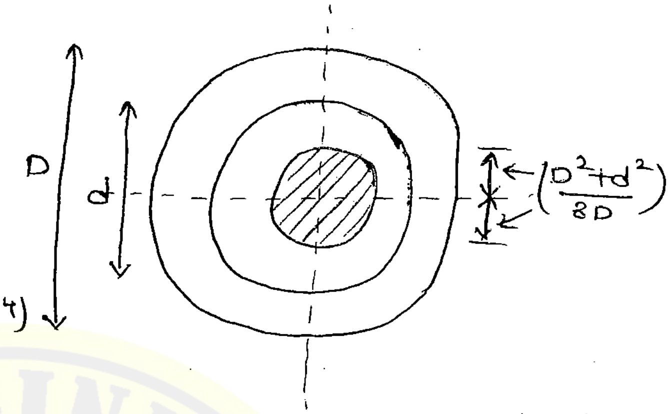
Hollow circular cross section:-

→ For No tension

$$\sigma_{min} = 0 = \frac{P}{A} - \frac{Pe}{Z}$$

$$0 = \frac{P}{\frac{\pi}{4}(D^2-d^2)} - \frac{Pe}{\frac{\pi}{32D}(D^4-d^4)}$$

$$\boxed{e = \frac{(D^2+d^2)}{8D}}$$

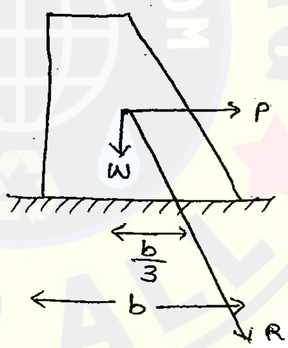


Dia of core $= \frac{D^2+d^2}{4D}$

Radius of core $= \frac{D^2+d^2}{8D}$

Dam:-

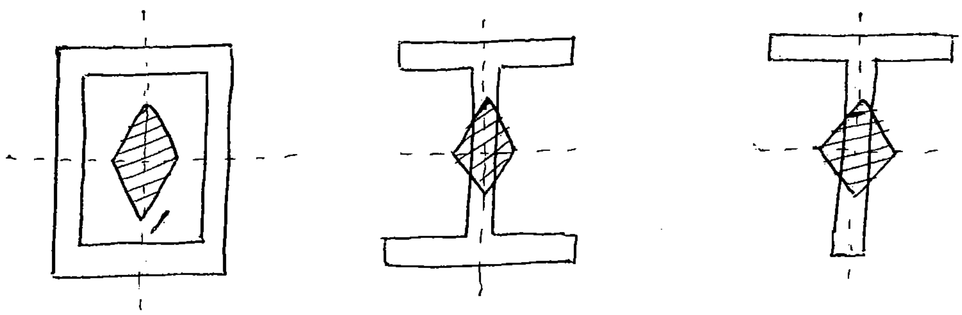
For no tension



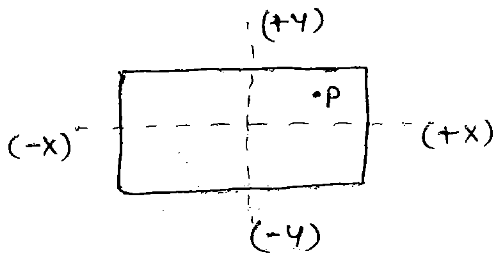
Note:-

In a dam (or) wall the resultant of all forces should intersect middle third portion of its base for no tension in any condition. (Middle third rule is applicable).

Ex:-

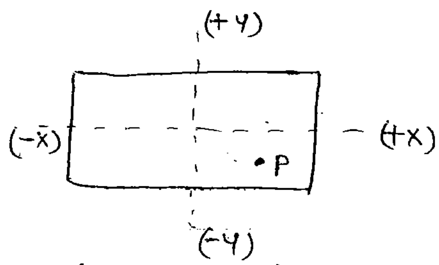


All are Rhombus (or) diamond shape core.



(+) compression

(-) Tension



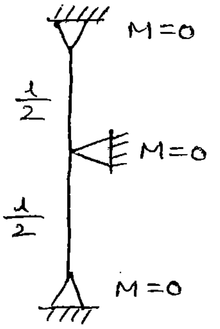
(+) compression, (-) Tension

Note:-

The quadrant in which load is applied will have always compressive stress.

P.g No:- 128

9.



$$\frac{l}{\sqrt{2}} = 0.7L$$

11. $$\lambda = \frac{\text{effective length}}{\text{least radius of gyration}}$$

$$= \frac{500}{4}$$

$$= \cancel{62.5\ cm}\ 125\ cm$$

$$r = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{\frac{\pi}{64}(16)^4}{\frac{\pi}{4}(16)^2}}$$

$$= \sqrt{\cancel{8\ cm}}\ 4\ cm$$

13. $$P_e = \frac{\pi^2}{l^2} EI$$

$L = 3m$ (Fix - Hinged)

effective length, $l = \frac{L}{\sqrt{2}} = \frac{300}{\sqrt{2}}$

$$I = \frac{\pi}{64}(6)^4$$

$$= \frac{\pi^2}{\left(\frac{300}{\sqrt{2}}\right)^2}(2\times10^6)\left(\frac{\pi}{64}(6)^4\right)$$

$$= 29289\ kg$$

Safe load, $P = \frac{P_e}{F.S}$

$$= 9763\ kg$$

14. $L = 6\ m$ (both ends Hinge)

$l = L = 6m$

$E = 0.8\times10^6\ kg/cm^2$

$I = 1766\ cm^4$

$$P_e = \frac{\pi^2}{l^2} EI$$

$$=$$

15. $l = 2L$ (Fixed - free)

$$P_e = \frac{\pi^2}{(2L)^2} EI$$

$$= \frac{\pi^2}{4L^2} EI$$

16.

$$\frac{P_{Fix-Fix}}{P_{Hinge-Hinge}} = \frac{(l_{H-H})^2}{(l_{F-F})^2}$$

$$\frac{P_{F-F}}{10} = \frac{(L)^2}{\left(\frac{L}{2}\right)^2} = 4$$

$$P_{F-F} = 40 \text{ KN}$$

UNIT – 8

SPRINGS

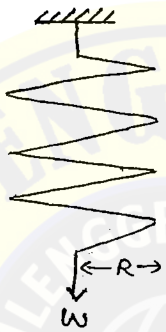
purpose:-

1. To absorb shock
2. To store energy (strain energy in the form of potential energ

Type of springs:-

1. Torsion spring:-

Closely coiled helical spring subjected to axial forc will undergo pure Torsion.



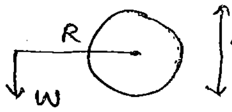
d = dia of spring wire

R = Mean radius of coil

n = no. of turns or coil

l = length of spring wire

$$\boxed{l = 2\pi R n}$$



Torsion in spring wire, $T = W \cdot R$

Torsional shear stress in the wire of spring

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau_{max} = \frac{T}{J}(r)$$

$$\boxed{\tau_{max} = \frac{T}{Z_p}} = \frac{16T}{\pi d^3}$$

$$\boxed{\tau_{max} = \frac{16(WR)}{\pi d^3}}$$

Angle of twist in spring wire

$$\frac{T}{J} = \frac{G\theta}{l} \Rightarrow \frac{WR}{\frac{\pi d^4}{32}} = \frac{G\theta}{2\pi R n} \Rightarrow \boxed{\theta = \frac{64 W R^2 n}{G d^4}}$$

Deflection at free end, $\tan\theta \approx \theta$

$$\theta = \frac{\delta}{R}$$

$$\delta = R \cdot \theta$$

$$\boxed{\delta = \frac{64WR^3n}{Gd^4}}$$

Stiffness of spring, $K = \frac{W}{\delta}$

$$\boxed{K = \frac{Gd^4}{64nR^3}}$$

Strain energy stored in spring, $\boxed{U = \frac{1}{2}W \cdot \delta}$

$$U = \frac{32W^2R^3n}{Gd^4}$$

Note :-

If a closely coiled helical spring is subjected to Torsion the wire of the spring will be subjected to Bending stress.

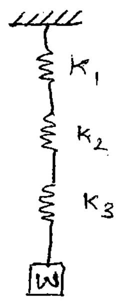
2. Bending spring :-

It is also called Leaf (or) laminated (or) carrying springs. In case of laminated springs plates can slide one over the other without friction and without shear stress. Therefore laminated spring can be called as pure bending spring.

The plates of the spring are bent into semi elliptical arcs

Eg :- Open coiled springs.

Springs in series :-



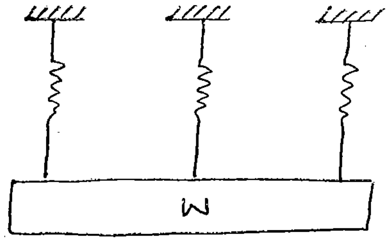
$$W_1 = W_2 = W_3 = W_4$$

Max deflection @ free end, $\delta = \delta_1 + \delta_2 + \delta_3$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

K_e = equivalent (or) ~~effective~~ stiffness (or) spring modulus

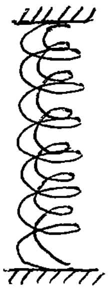
Springs in parallel :-



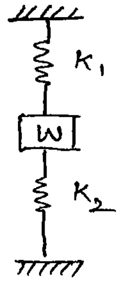
$W = W_1 + W_2 + W_3$

$\delta_1 = \delta_2 = \delta_3 = \delta$

$K_e = K_1 + K_2 + K_3$



(spring in spring)



P.g NO:-103.

2. $K = \dfrac{Gd^4}{64R^3n} = \dfrac{G(2r)^4}{64R^3n} = \dfrac{Gr^4}{4R^3n}$

Radius of wire, $r = \dfrac{d}{2}$, $d = 2r$

3.

A	B
d	2d

$\dfrac{K_B}{K_A} = \dfrac{(d_B)^4}{(d_A)^4} = \dfrac{(2d)^4}{(d)^4} = 16$

4.

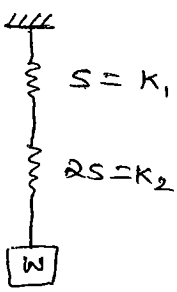
A	B
$(\frac{R}{2})$	R

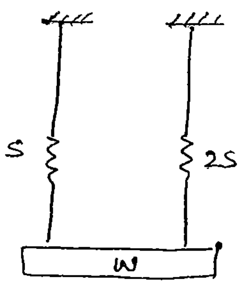
$\dfrac{\delta_B}{\delta_A} = \dfrac{R_B^3}{R_A^2} = \dfrac{R^3}{(\frac{R}{2})^3} = 8$

$\cancel{8}$. 5. $\dfrac{1}{K_e} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

$K_e = \dfrac{K_1 K_2}{K_1 + K_2}$

7.





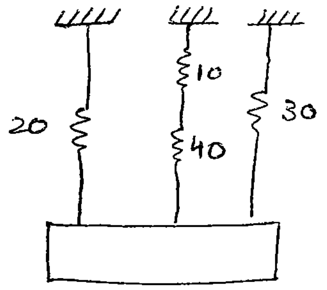
Parallel

$K_e = 2S + S$

$= 3S$

$\dfrac{(K_e)_{series}}{(K_e)_{parallel}} = \dfrac{2}{9}$

9.

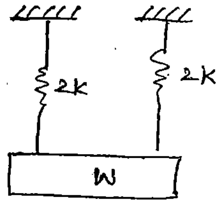


$$K_e = \frac{10(40)}{10+40} = 8 \text{ kN/m}$$

$$K_e = 20 + 8 + 30 = 58 \text{ kN/m}$$

10.

$$(K_e)_1 = K$$



$$(K_e)_2 = 4K$$

$$\frac{(K_e)_2}{(K_e)_1} = \frac{4K}{K} = 4$$

11.

$$K_e = 100 + 300$$

$$= 400 \text{ kN/m}$$

$$K_e = \frac{W}{\delta}$$

$$\delta = \frac{W}{K_e}$$

$$= \frac{400 \text{ kN}}{400 \text{ MN/m}}$$

$$= \frac{400 \times 10^3 \text{ N}}{\left(\frac{400 \times 10^6}{1000}\right) \frac{\text{N}}{\text{mm}}}$$

$$= 1 \text{ mm}$$

UNIT - 10

THIN CYLINDERS

Pressure vessel :-

Any vessel carrying internal pressure is a pressure vessel.

Classification :-

1. Thin vessels can be classified in two types

$$t \not> \frac{D}{20}$$

a. cylinders — boilers, storage tanks

b. spheres — baloon

2. Thick vessels $t > \frac{D}{20}$

a. cylinders — nozzles, jets.

b. Spheres

Thin cylinders :-

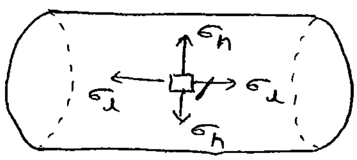
Stresses developed

$$\sigma_1 = \sigma_h = \frac{PD}{2t}$$ (compression) (or) Circumferential stress

$$\sigma_2 = \sigma_l = \frac{\sigma_h}{2} = \frac{PD}{4t}$$ (Tension) (or) Longitudinal stress.

Max. shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_h - \sigma_l}{2}$$



$$\boxed{\tau_{max} = \frac{PD}{8t}}$$

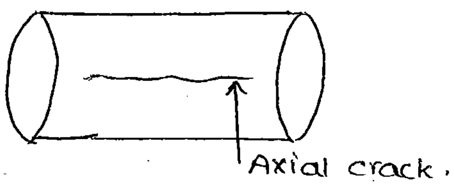
Strains :-

$$\frac{\delta D}{D} = \varepsilon_h = \frac{\sigma_h}{E} - \mu \cdot \frac{\sigma_l}{E}$$

$$\frac{\delta l}{l} = \varepsilon_l = \frac{\sigma_l}{E} - \mu \cdot \frac{\sigma_h}{E}$$

$$\frac{\delta V}{V} = \varepsilon_v = \varepsilon_l + 2 \cdot \varepsilon_h$$

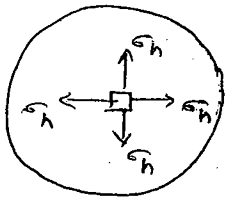
Failure criteria:-



In a thin (or) thick cylinder common failure is a crack along the axis of the cylinder.

Thin spheres:-

Stresses developed



$$\sigma_h = \frac{PD}{4t} \quad \text{(Tension)}$$

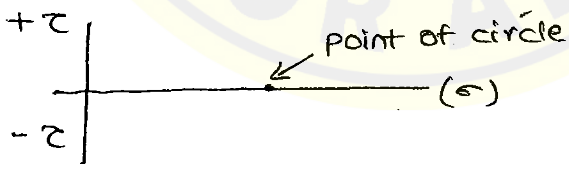
Note:-

For the same diameter, thickness and internal pressure, the thin cylinder is subjected to twice the hoop stress than a sphere.

Here principal stresses, $\sigma_1 = \sigma_2 = \sigma_h$

Maximum shear stress, $\tau_m = \frac{\sigma_1 - \sigma_2}{2} = 0$

Mohr circle for the state of stress on the surface of thin sphere.

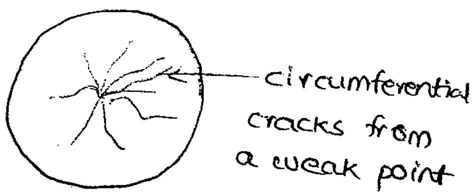


On the surface of thin sphere, shear stress developed is zero.

Strains:-

$$\frac{\delta D}{D} = \varepsilon_h = \frac{\sigma_1}{E} - \mu \cdot \frac{\sigma_2}{E}$$

$$= \frac{\sigma_h}{E} - \mu \cdot \frac{\sigma_h}{E}$$



$$\boxed{\frac{\delta V}{V} = \varepsilon_v = 3 \cdot \varepsilon_h}$$

P.g No:- 121

1. $\varepsilon_h = \varepsilon_1$

$\varepsilon_l = \varepsilon_2$

For a cylinder

$\varepsilon_v = \varepsilon_l + 2\varepsilon_h$

$= \varepsilon_2 + 2\varepsilon_1$

2. Given $P = 10\ kg/cm^2$

$\sigma_h = 200\ kg/cm^2$

$D = 100\ cm$

$$\sigma_h = \frac{PD}{2t}$$

$$200 = \frac{10(100)}{2t}$$

$t = 25\ mm$

3. $\sigma_h = 100\ M.pa$ $\quad E = 200\times10^3\ Mpa$

$\sigma_l = 50\ M.pa$ $\quad \mu = 0.3$

$$\varepsilon_h = \frac{\sigma_h}{E} - \mu\cdot\frac{\sigma_l}{E}$$

$$= \frac{100}{200\times10^3} - 0.3\cdot\frac{50}{2\times10^5}$$

$= 0.425\times10^{-3}$

4. $$\frac{\varepsilon_l}{\varepsilon_h} = \frac{\frac{\sigma_l}{E} - \mu\cdot\frac{\sigma_h}{E}}{\frac{\sigma_h}{E} - \mu\cdot\frac{\sigma_l}{E}}$$

$\sigma_h = 2\sigma_l$, $\quad \mu = \frac{1}{m}$

$$= \frac{\frac{\sigma_l}{E} - \frac{1}{m}\left(\frac{2\sigma_l}{E}\right)}{\frac{2\sigma_l}{E} - \frac{1}{m}\left(\frac{\sigma_l}{E}\right)}$$

$$= \frac{m-2}{2m-1}$$

5. $$\sigma_h = \frac{PD}{2t} = 80\ Mpa$$

$$\sigma_l = \frac{\sigma_h}{2} = 40\ Mpa$$

$$\varepsilon_h = \frac{\sigma_h}{E} - \mu\cdot\frac{\sigma_l}{E}$$

$$= \frac{80}{2\times10^5} - 0.28\ \frac{40}{2\times10^5}$$

$= 3.44\times10^{-4}$

7. $\sigma_l = \sigma_o$

$\sigma_h = 2\sigma_o$

$$\tau_{max} = \frac{\sigma_h - \sigma_l}{2}$$

$= 0.5\ \sigma_o$

9. $$\sigma_h = \frac{PD}{2t}$$

$$3600 = \frac{P(6)}{2\times0.2}$$

$P = 240\ kg/cm^2$

8. principal strain, $\varepsilon_1 = \varepsilon_h = ?$

$$\sigma_h = \frac{PD}{2t} = \frac{(1)(500mm)}{2\times5}$$

$= 50\ Mpa$

$$\sigma_l = \frac{\sigma_h}{2} = 25\ Mpa$$

$$\varepsilon_h = \frac{\sigma_h}{E} - \mu\cdot\frac{\sigma_l}{E}$$

$= 2.25\times10^{-4}$

10. $P = \gamma_w \cdot H$

$= (10\ kN/m^3)(200\ m)$

$= 2000\ kN/m^2$

$P = 2000 \times \frac{1000}{(1000)^2} \frac{N}{mm^2}$

$= 2\ N/mm^2$

$\sigma_h = \frac{PD}{2t}$

$20 = \frac{2 \times 1000\ mm}{2 \times t}$

$t = 50\ mm$

12. $\frac{\delta D}{D} = \varepsilon_h = \frac{\sigma_h}{E} - \mu \cdot \frac{\sigma_l}{E}$

$= \frac{PD}{2tE} - \mu \cdot \frac{PD}{4tE}$

$\delta D = \frac{PD^2}{2tE}\left(1 - \frac{\mu}{2}\right)$

UNIT - 12

STRAIN ENERGY RESILIENCE

Strain energy:-

The energy stored in a member due to external workdone is the strain energy.

$$\boxed{U = \frac{1}{2} W \cdot \delta}$$

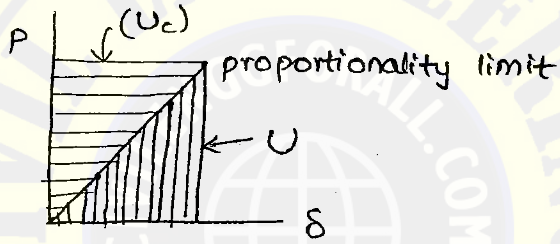
Unit :- N-m (or) Joule

Energy :- Scalar

Resilience:-(U)

The energy stored in a member within proportionality limit is Resilience.

The recoverable strain energy is Resilience



The area under load and deformation curve upto proportionality limit is also Resilience.

$$\boxed{U = \frac{1}{2} P \cdot \delta} \rightarrow ①$$

U_c - complimentary Resilience

P = gradual loading (or) slowly increased load.

The area above load deformation curve is complementary resilience & which is equal to actual Resilience ~~upto~~ upto P.L

$$\varepsilon = \frac{\delta}{l} \rightarrow \delta = \varepsilon l$$

$$\sigma = \frac{P}{A}$$

$$P = \sigma \cdot A$$

Sub in eq ①

$$U = \frac{1}{2}(\sigma A)(\varepsilon l)$$

$$\boxed{U = \frac{1}{2} \sigma \cdot \varepsilon \cdot V} \rightarrow ②$$

$$E = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = \frac{\sigma}{E}$$

Sub in eq (2)

$$U = \frac{1}{2}(\sigma)(\varepsilon)V$$

$$= \frac{1}{2}\sigma\left(\frac{\sigma}{E}\right)V$$

$$\boxed{U = \frac{\sigma^2}{2E}\cdot V} \longrightarrow (3)$$

$\delta = \frac{Pl}{AE}$ sub in (1)

$$U = \frac{1}{2}P\left(\frac{Pl}{AE}\right)$$

$$\boxed{U = \frac{P^2 l}{2AE}} \longrightarrow (4)$$

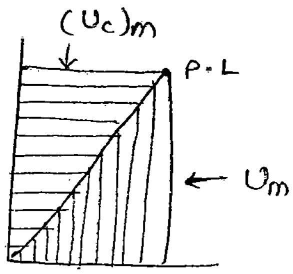
Proof Resilience:-

The maximum resilience stored in a member which can be obtain by loading upto proportionality limit

Modular Resilience:-

Resilience per unit volume (or) Area under stress strain curve upto proportionality limit is called Modulus of Resilience

$$\boxed{U_m = \frac{U}{V}}$$



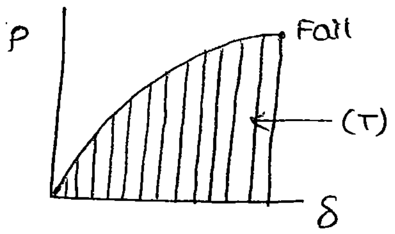
From (2) $U_m = \frac{1}{2}\sigma\cdot\varepsilon$

From (3) $U_m = \frac{\sigma^2}{2E}$

Units:- Unit of stress (or) M.pa

Toughness:-

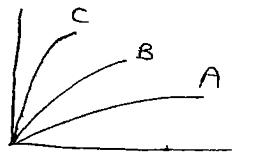
Maximum strain energy stored in a member till fail (or) Area under P-δ curve upto failure.



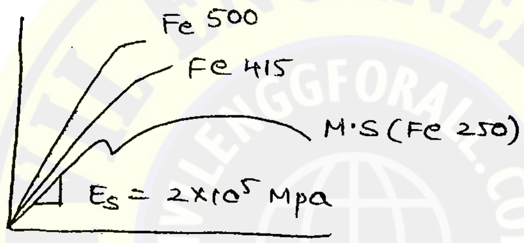
Complementory Toughness ≠ Toughness

Usually soft and Ductile material are tough, the absorb a lot of energy before failure.

↑ tough : ↑ area under the curve



↑ tough : (A)



$\boxed{\uparrow \text{tough}}$

Type of Loading:-

1. Gradual load:-

All the loads by default are gradual loads only

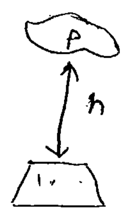
$$\left.\begin{aligned}\sigma &= \frac{P}{A}\\ \delta &= \frac{Pl}{AE}\end{aligned}\right\}\ \text{for gradual loads only}$$

2. Impact load:-

Workdone = strain energy stored

$$P\cdot h = \frac{\sigma^2}{2E}\cdot V$$

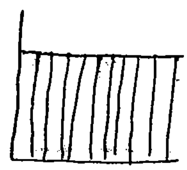
$$\sigma_{impact} = \sqrt{\frac{2Ph\cdot E}{V}}$$



3. Sudden load:- (Imaginary load):-

$$\sigma_{sudden} = 2(\sigma_{graduall}) = \frac{2P}{A}$$

$$\delta_{sudden} = 2(\delta_{graduall}) = \frac{2PL}{AE}$$



Forms of strain Energy:-

1. Axial Force:-

$$U = \frac{1}{2} P \cdot \delta$$

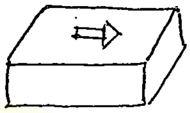
$$= \frac{1}{2} \cdot \sigma \cdot \varepsilon \cdot V$$

$$= \frac{\sigma^2}{2E} \cdot V$$

$$U = \frac{P^2 L}{2AE}$$

2. Shear Force:-

$$U = \frac{\tau^2}{2G} \cdot V$$



τ = shear stress due to SF.

3. Bending Moment:-

$$U = \frac{f^2}{2E} \cdot V$$

4. Torsion:-

$$U = \frac{1}{2} \cdot T \cdot \theta$$

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{GJ}$$

τ = Torsional shear stress.

$$U = \frac{1}{2} T \cdot \left(\frac{Tl}{GJ}\right)$$

$$U = \frac{1}{2} \frac{T^2 l}{GJ}$$

$$\boxed{U = \frac{\tau^2}{4G} \cdot V}$$

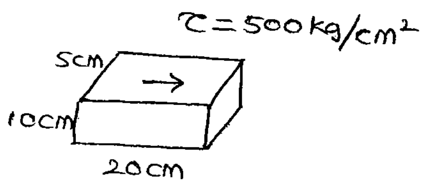
P.g No:- 141.

12. $\delta l_{\text{sudden (or) instantaneous}} = \frac{2Pl}{AE}$

$$= \frac{2 \times 80 \times 040}{\frac{\pi (4)^2}{4} \times 2 \times 10^5}$$

$$= 2.5 \text{ mm}$$

~~≃ 3 mm~~



13. $U = \frac{\tau^2}{2G} \cdot V$

$= \frac{500^2}{2 \times (1 \times 10^6)} \times (20 \times 10 \times 5)$

$= 125$ kg-cm

14. $\sigma_{sudden} = 2\,(\sigma_{gradual})$

$= \frac{2P}{A}$

$= \frac{2(20)}{20}$

$= 2\ t/cm^2$

$= 2 \times 10^{-4}\ t/m^2$

15. $U_m = \frac{1}{2} \sigma \cdot \varepsilon$

$= \frac{1}{2} \times 200 \times \frac{\delta l}{l}$

$= 100 \times \frac{2}{2000}$

$= 0.1$ units.

16. Given $d = 20$ mm $l = 1000$ mm $E = 200 \times 10^3$ Mpa $P = 100 \times 10^3$ N

$U = \frac{P^2 l}{2AE}$

$= \frac{(10^4)^2 (1000)}{2 \times \frac{\pi (20)^2}{4} \times (2 \times 10^5)}$

$= 0.796$ kN m