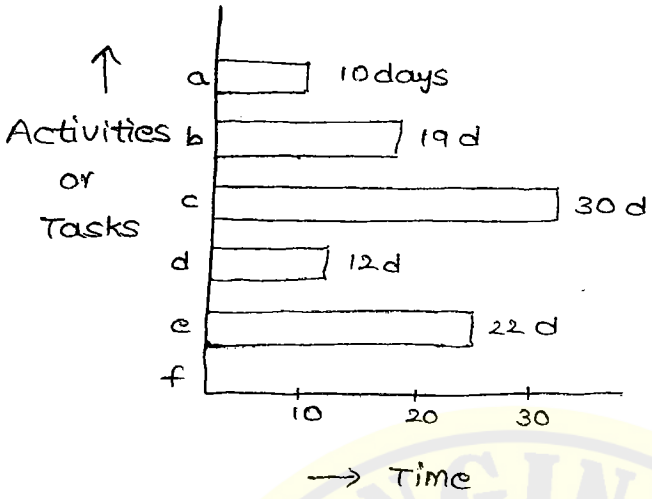
B. Sai kumar
A3 - civil

PERT & CPM

PERT - Program Evaluation Review Technique

CPM - Critical path method.



R. M. Walker and J. E. Kelly developed PERT & CPM

Fig:- Bar chart (or) Gantt chart

It is a simple graphical representation of various activities

Major drawbacks of a bar chart:-

1. It fails to represent inter-dependencies between various activities.
2. Probability of completion of various activities cannot be estimated from a bar chart.
3. The progress of the project at any point of time during execution cannot be determined from bar chart.

Terminology:-

1. ○ Event / node
2. → Activity / process / Task
3. ---→ Dummy .....

Event or node:-

It is a symbol of starting point and ending point of an activity. It always refers at a point of time.

Activity (or) process (or) Task:-

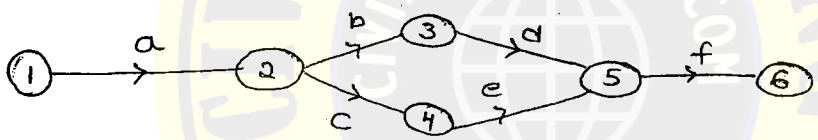
A process is be a represented by a Arrow and is always quantifies an amount of time. Every activity consists of one starting event and one final event.

Dummy activity:-

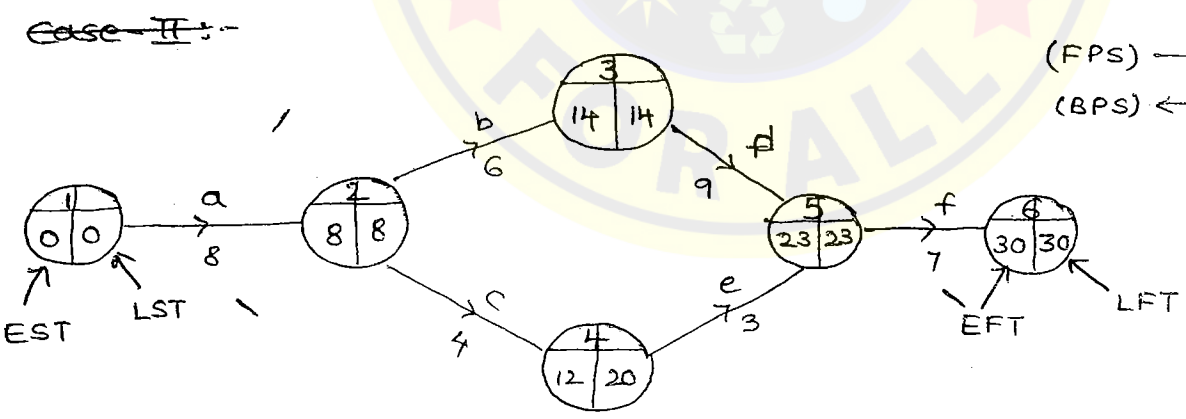
It is almost similar to an activity with a difference that it does not consume any resources.

Case - I:

Activities	pre. activities	Time (days)
a	–	8
b	a	6
c	a	4
d	b	9
e	b,c	3
f	d,e	7



~~Case - II:-~~



Earliest Finishing time of the project (EFT)

Earliest starting time of the project (EST)

Forward pass scheduling:-

The process of time calculations from first event to the last event is known as Forward pass scheduling

Backward Pass Scheduling:-

The process of time calculation from last event to the first event is known as Backward pass scheduling.

pass (P_1) ⟶ a - b - d - f = 30 days

pass (P_2) ⟶ a - c - e - f = 22 days

Critical activities:-

Critical activities are those activities which when delayed beyond their expected durations delay the project.

Non critical activities:-

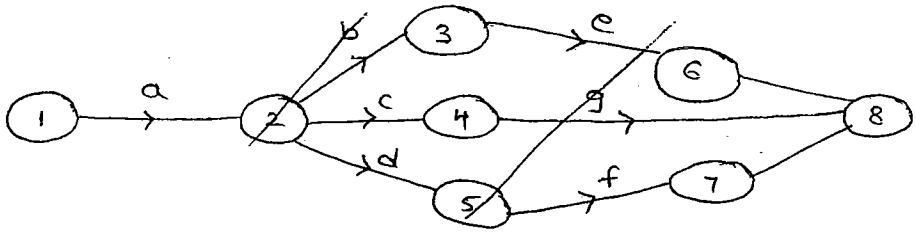
Those activities which can be delayed beyond their expected duration (to some extent) without delaying the project.

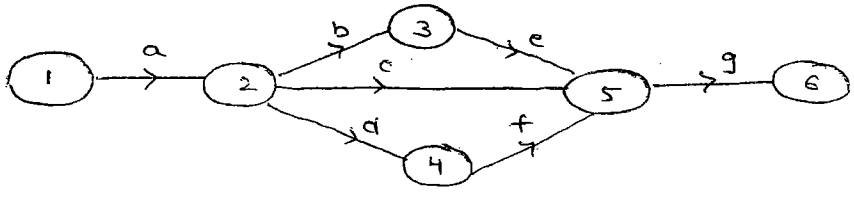
Critical path:-

It is a longest path of a network of a project. The significance of the critical path is that it gives us earliest Finishing time of a project.

Case - II :-

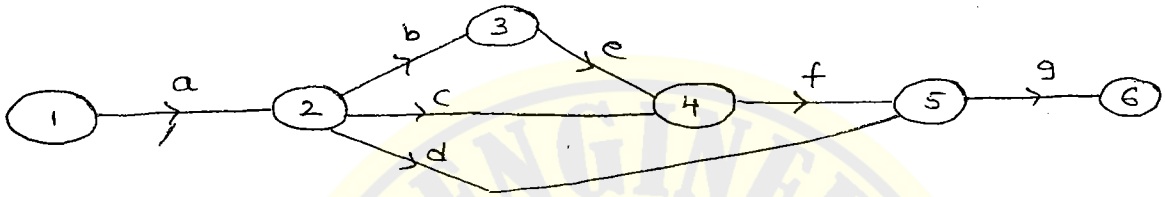
Activities	pre. activities	Time
a	-	
b	a	
c	a	
d	a	
e	b	
f	d	
g	e, c, f	

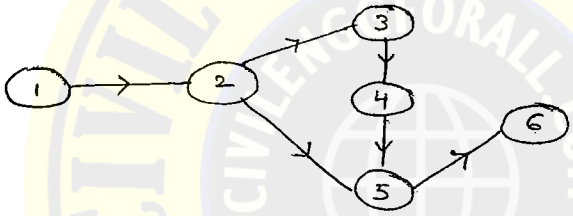




Case - III :-

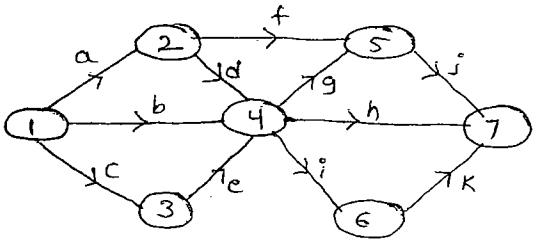
Activity	pre. activity	Time
a	–	
succedor ← b	a → precessedor	
c	a	
d	a	
e	b	
f	c, e	
g	d, f	





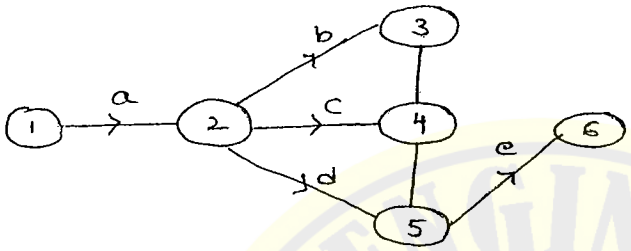
Case - IV :-

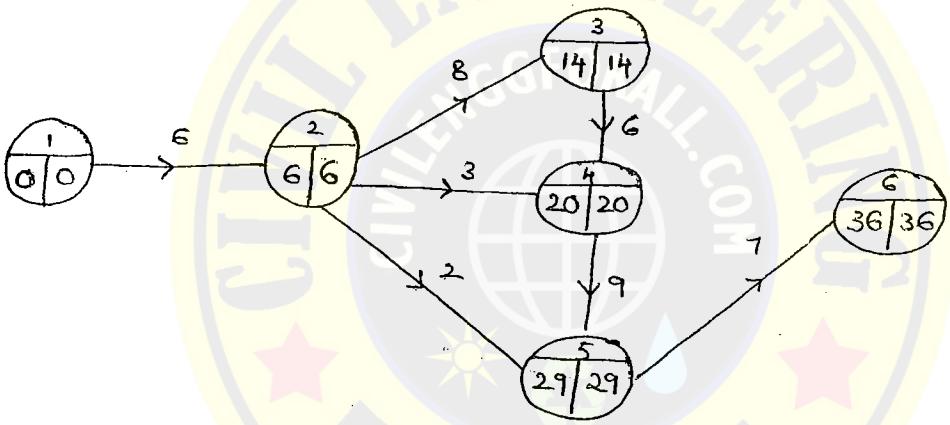
Activities	pre. activities
a	–
b	–
c	–
d	a
e	c
f	a
g	b, d, e
h	b, d, e
i	b, d, e
j	f, g
K	i



Case : 6

Activities	Pre. Activities	Time
1 - 2		6
2 - 3		8
2 - 4		3
2 - 5		2
3 - 4		6
4 - 5		9
5 - 6		7





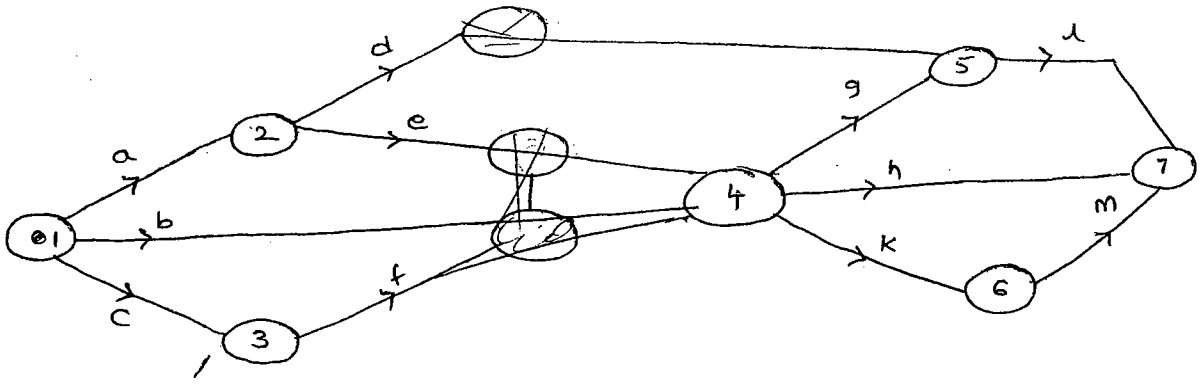
Critical path :-

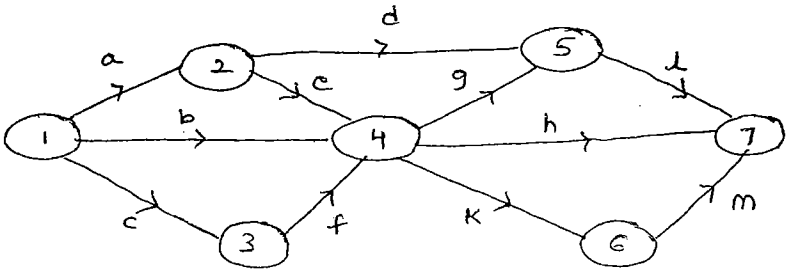
1 - 2 - 3 - 4 - 5 - 6

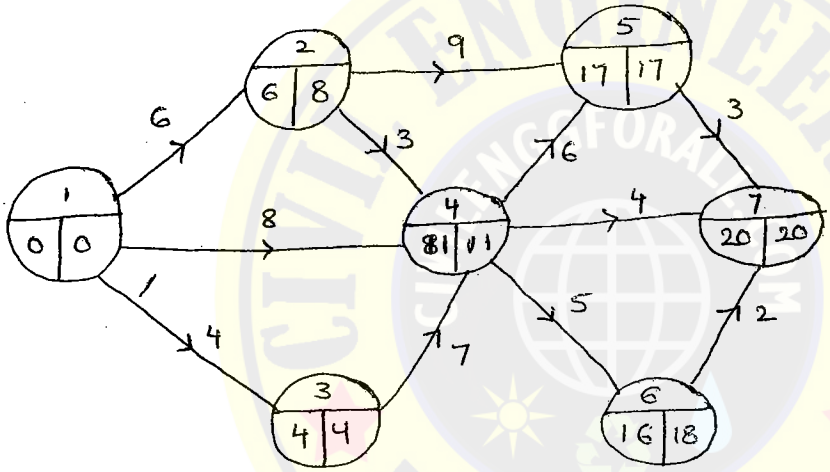
Earliest Finishing Time (EFT) = 36 days

Case :- 7

Activities	pre. Activities	Time
a	—	6
b	—	8
c	—	4
d	a	9
e	a	3
f	c	7
g	e,b,f	6
h	e, b, f	4
k	e, b, f	5
l	d, g	3
m		







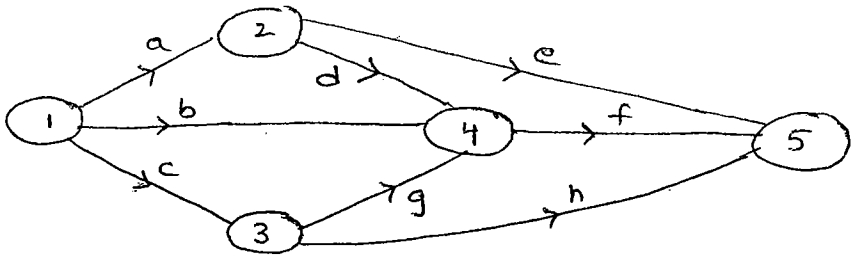
Critical path:-

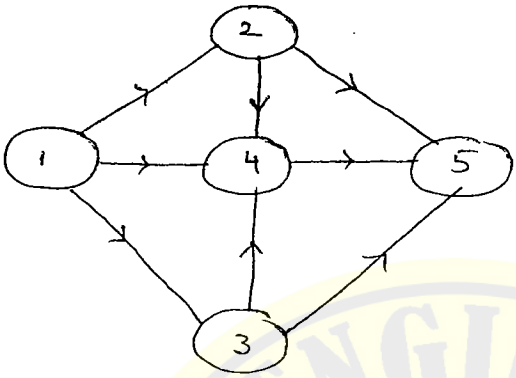
1 - 3 - 4 - 5 - 7

EFT = 20 days

case - 8:-

Activities	Pre. activities (Predecessor)	Time
a	–	8
b	–	6
c	–	9
d	a	10
e	a	7
f	b, d, g	6
g	c	12
h	c	6



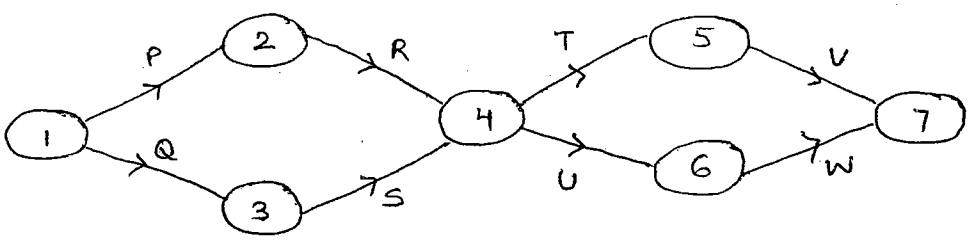


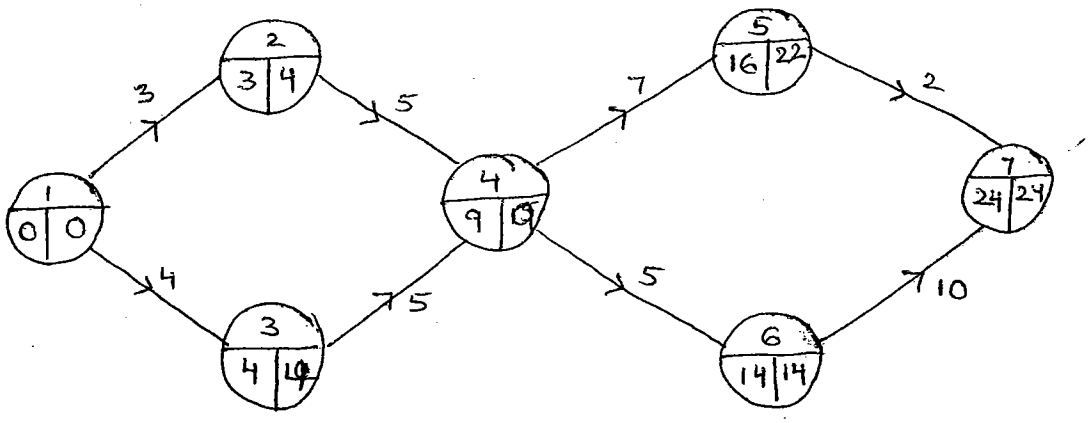
Critical path:-

1 - 3 - 4 - 5

Case:-9

Activities	pre. activities	Time
P	–	3
Q	–	4
R	P, Q	5
S	R, Q	5
T	R, S	7
U	R, S	5
V	T	2
W	U	10





Critical path:-

1 - 3 - 4 - 6 - 7 (or) Q - S - U - W

EFT = 24 days

Difference between PERT and CPM:-

Activities	Predecessor activities	Time
a	—	8
b	a	6
c	a	10
d	b	8
e	c	8
f	d, e	8

Activities	Pre. activities t_o	t_m	t_p	t_e
1-2	2	4	6	4
2-3	3	6	9	6
2-4	5	10	15	10
3-5	4	8	12	8
4-5	6	8	12	8.3
5-6	4	8	16	8.6

1. PERT is event oriented while CPM is activity oriented
2. PERT is concerned only with time, while CPM is concerned both time and also cost.
3. Time estimates of the activities are probabalistic in PERT while there are deterministic in CPM
4. PERT is applied in all research applications (or) areas where no past data is available. While CPM is applied in all commercial business areas.

Time estimates in PERT:-

- PERT assumes three probabalistic time estimate for every activity.

1. Optimistic time
2. Pessimistic time
3. Most likely time

Optimistic time (t_o):-

It is a time estimate of an activity under the assumption that every thing goes as per schedule. It is the smallest of the three estimates.

Pessimistic time (t_p):-

It is a time estimate of an activity under the assumption that ~~every thing~~ nothing goes as per schedule. It is the longest duration of the three estimates.

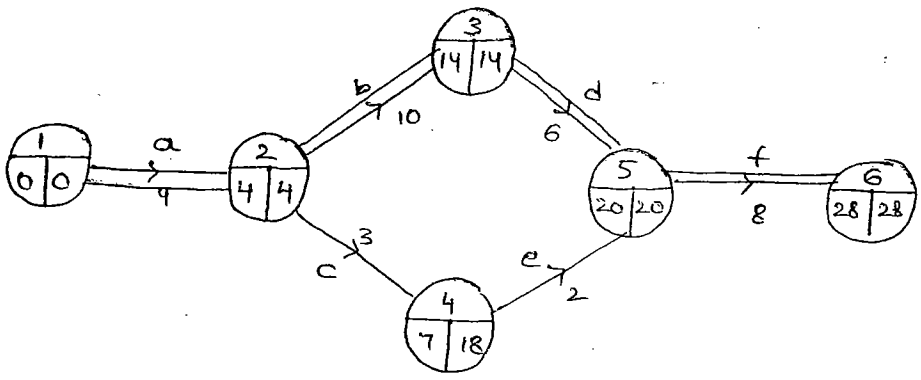
Most likely time (t_m):-

It is a time estimate of an activity which is a compramise between the two extremes t_o and t_p. therefore its value lies in between t_o and t_p.

t_e = expected time of an activity

$$t_e = \left[\frac{t_o + 4t_m + t_p}{6}\right]$$

Slack time of an activity:-



Critical path = a-b-d-f

EFT = 28 days

Slack time

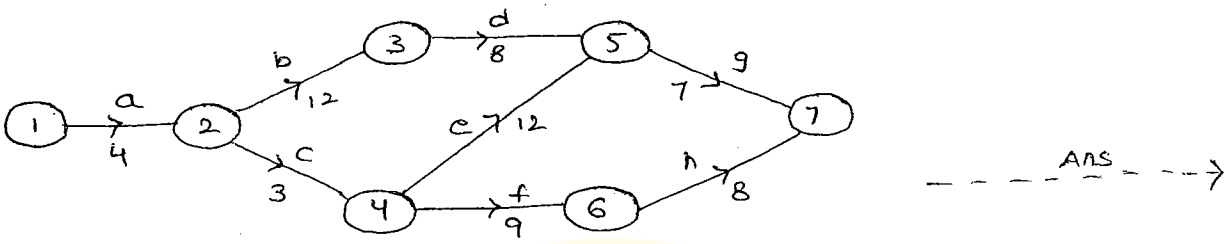
e = (0+4+3+2 = 9)

= 20 - 9 = 11 days

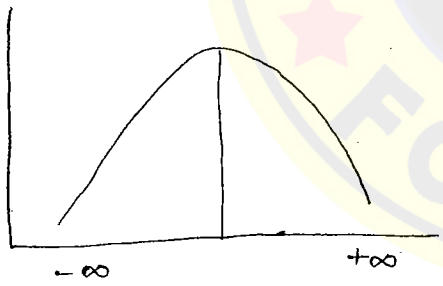
The extend of time duration by which a non critical activity can be delayed beyond its expected duration is known as slack time of an activity.

Critical path can also be defined as that path of a network having zero slack time.

Example:-



Beta distributions



Normal distribution

Normal distribution characteristics:-

1. Normal distribution is symmetrical about y-axis
2. Normal curve meets x-axis at '∞' on both sides
3. The total area in the normal distribution is assumed to be 1 unit with equal spread on both the sides.
4. Measures of central tendency ~~are equ~~ for normal distributions are equal and occur at mid points.
5. Activities in PERT followed Beta distribution.
6. Every project is expected to follow normal distributions.

7. probability that any project can be completed by or before its EFT duration. (i.e., 0.5)
8. similarly probability that a project needs more than EFT duration (i.e., 0.5)

Calculation of critical path in terms of slack time :-

Ans:-

Activity	t	EST	EFT	LST	LFT	S = LFT - EFT	LS = EST
a	4	0	4	0	4	0	0
b	12	4	16	4	16	0	0
c	3	4	7	9	12	5	5
d	8	16	24	16	24	0	0
e	12	7	19	12	24	5	5
f	9	7	16	14	23	7	7
g	7	24	31 (EFT)	24	31	0	0
h	8	16	24	23	31	7	7

Fundamental assumptions in network construction:-

1. It is assumed that for every project Earliest start time is equal to Latest start time.
2. It is assumed that for every project earliest finishing time is always equal to Latest finishing time.

Time components of an activity :-

1. Event time = t
2. EST
3. EFT = EST + t
4. LFT
5. LST = (LFT - t)
6. S = (LFT - EFT) = (LST - EST)

$LS_b = 4$

$LS_c = 9$

∴ since activities a, b, d, g having zero(0) slacks

∴ critical path = a - b - d - g

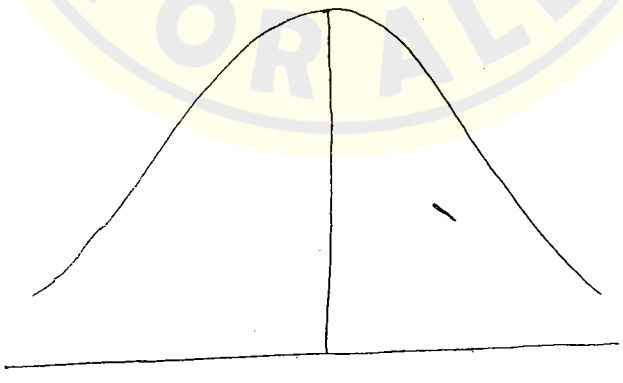
Calculation of probability of completion of project:-

Activity	t_o	t_m	t_p	t_e	$t_o - t_p$	$\frac{t_o - t_p}{6}$	$\left(\frac{t_o - t_p}{6}\right)^2$
1 - 2	2	4	6	4			0.44
2 - 3	3	6	9	6			1.0
2 - 4	4	6	8	6			0.44
2 - 5	5	10	15	10			2.78
3 - 4	6	8	12	8.33			1.0
4 - 5	3	6	9	6			1.0
5 - 6	4	6	12	6.67			1.76

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

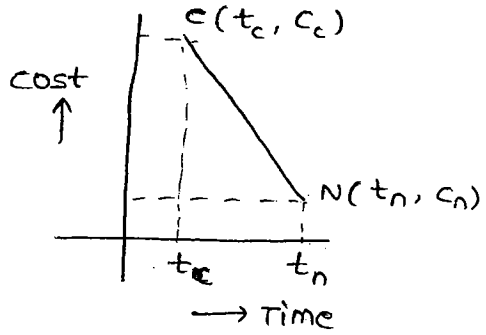
EFT $\cong$ 30.99 = 31 days

1. Determine the probability that these project can be completed within 25 days.
2. Probability can be completed with in 37 days
3. Determine the probability that these project needs or take more than 31 days.

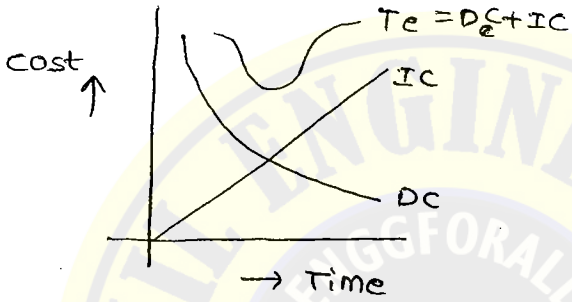


Standard Normal deviation $z = \left(\frac{t_{sh} - t_{cp}}{\sqrt{\sigma_{cp}^2}}\right)$

Crashing (or) optimisation of project parameters (or) Time and cost trade off :-



Cost slope of an activity $= \frac{\Delta C}{\Delta t} = \frac{(C_c - C_n)}{(t_n - t_c)}$



$\Delta C = C_c - C_n$

$\Delta t = t_n - t_c$

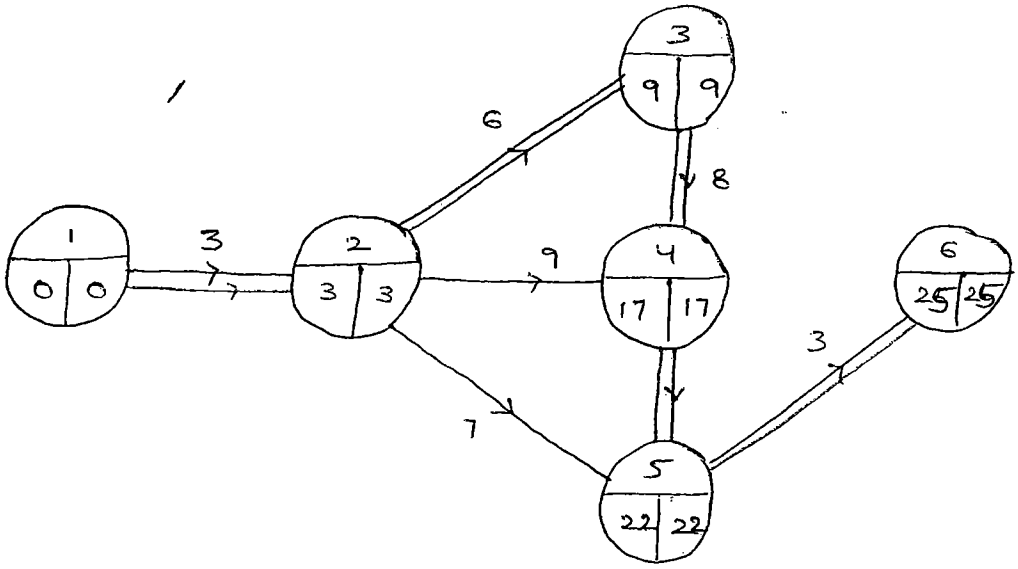
Activities	Normal		Crash		ΔC	Δt	$\Delta C/\Delta t$
	t_n	C_n	t_c	C_c			
1-2	3	360	2	400	40	1	40 ①
2-3	6	1440	4	1620	180	2	90 ③
2-4	9	2160	5	2380	220	4	55
2-5	7	1120	5	1600	480	2	240
3-4	8	400	4	800	400	4	100 ④
4-5	5	1600	3	1770	170	2	85 ②
5-6	3	480	2	760	280	1	280 ⑤

$\Sigma = 7560$

I.C = 160/d

Cost slope of an activity :-

It is an additional direct cost to be spent on a activity a day.



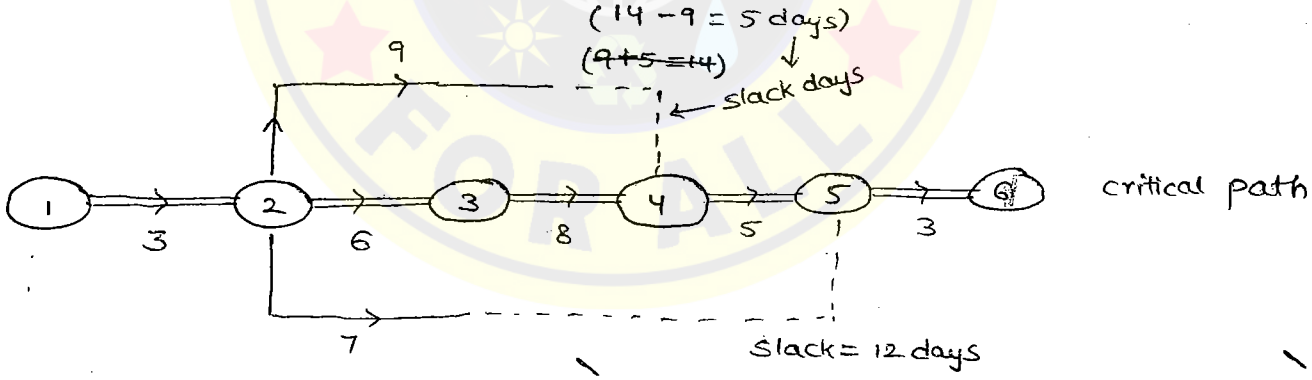
Critical path : 1-2-3-4-5-6 (completely)

EFT : 25 days

Slack time for ② - ④ = (3+9) - 17 = 5 days

② - ⑤ = (3+7) - 22 = 18 days

Time scale version of a project :-



← critical path (25 days) →

1. Only critical activity need to be crash
2. Crash critical activities in an increasing cost slope order

Total cost$_{25}$ = Direct cost + Initial cost

= 7560 + 25 (160)

= 11,560

Crashing :-

1st crash (1-2) = 2 days

$T.C_{24}$ = DC + IC

= (7566 + 1X40) + (24 + 160)

DC IC

= 11,440

2nd crash (4-5) = 22

TC_{22} = DC + IC

= (7560 + 1X40) + 2X85 + 22X160.

= 11,290.

3rd crash

TC_{20} = DC + IC

= 7560 + 1X40 + 2X85 + 2X90 + 20X160

= 11150

Note:-

During crashing a non critical activity can become a critical activity if it loses its slack time.

A network of a project can always have more than one critical path.

4th crash (2-4 & 3-4) :-

TC_{16} = DC + IC

= 2560 + 1X40 + 2X85 + 2X90 + 4X100 + 1X55 + (16X160)

IC

= 10965

5th crash (5-6) :-

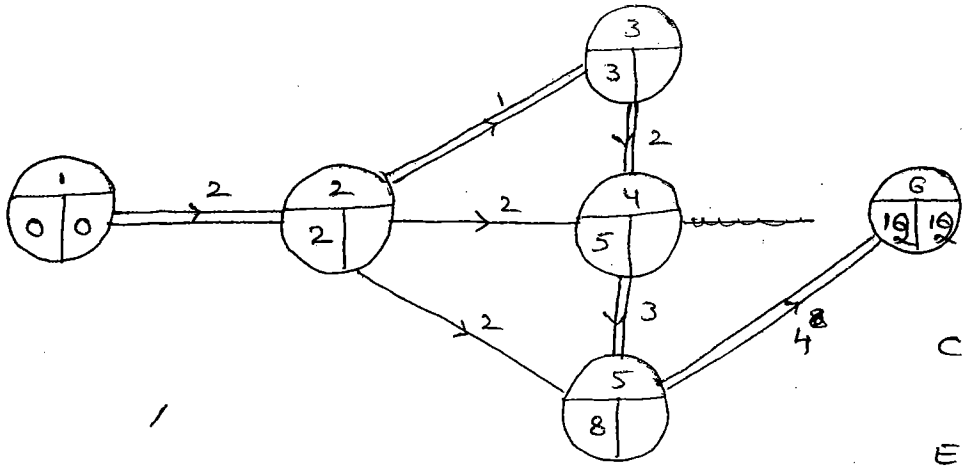
T_{C15} = 7560 + 1X40 + 2X85 + 2X90 + 4X100 + 1X55 + 1X280 + 15X16

= 11,085

Observation	T.T	T.C
1.	25 days	11560
2.	24 days	11440
3.	22	11290
4.	20	11150
5.	16	10965 *
6.	15	11085

∴ Total min time of the project are = 10965

Resources Allocations:-



Critical path:-

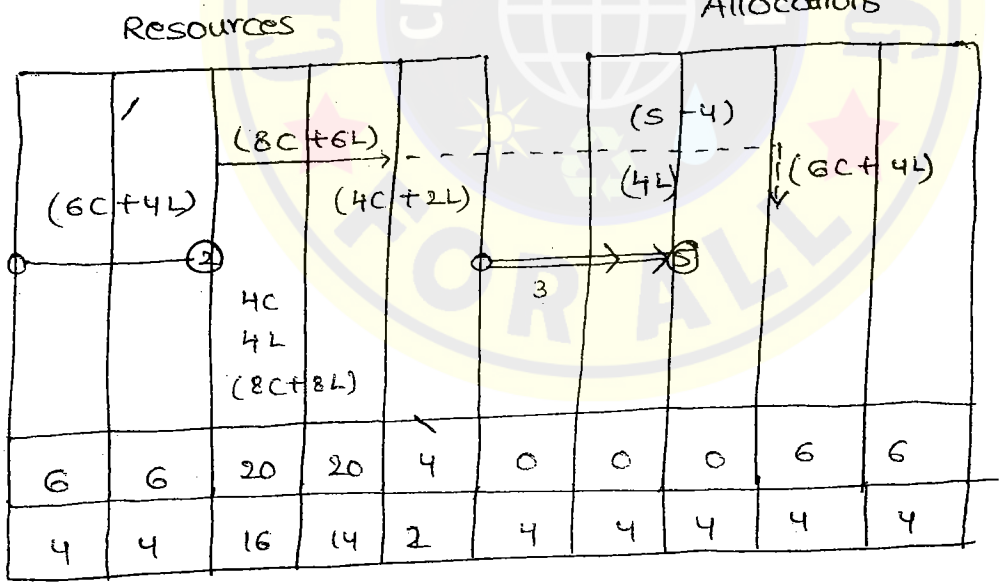
1-2-3-4-5-6

EFT = 12 days

Activity	time	C	L
1-2	2	6	4
2-3	1	4	4
2-4	2	8	6
2-5	2	8	6
3-4	2	4	2
4-5	3	–	4
5-6	4	6	4

C – carpenters

L – Labourers



Defination of Resources allocation:-

The main goal or objective of resources allocation is effective utilization of man power. To achelve this goal there are two techniques

1. Resources smoothing
2. Resources Levelling

Resources smoothing:-

In this method total project duration is maintaine at minimum level. Activity is with slacks time are being shifted so that uniform demand is achieved. In other wor the main constraint in this case is the project duration

Resources levelling:-

Sometime the availability of the resources is limited however the resources cannot go below the maximum amount leaded for any activity in the project.

Unless the duration of any activity increased, the minim amount of any resources required for the project will be equa to the maximum amount of resources needed. The main constrain in this method is the resources. If the maximum demand on any resources is not to be exceed a certain limit, the activi will have to be rescheduled. to so that the total demand on the resources at any time will be within the limit unde this conditions the project duration may exceed.