FLUID MECHANICS (45 M)

E

A_3 - civil

Introduction :-

Nature

- Living matter
  - → plants
  - → animals
  - → human beings
- Non living matter
  - solid
  - fluid
    - liquid
    - gases

Fluid mechanics:-

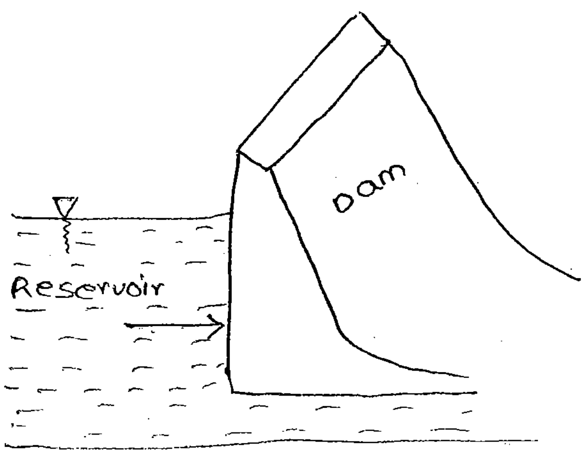
Fluid mechanics is that branch of engineering sciene which deals fluid behaviour under influence of, forces and energy at rest or under motio.

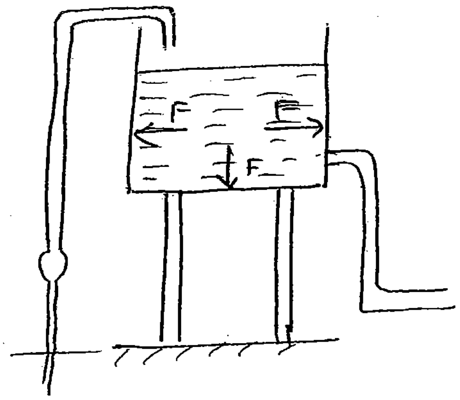
Fluid branches :-

1. Fluid statics
2. Fluid kinematics
3. Fluid dynamics

Eg of fluid statics :-

1. Water thrust force on the concrete wall of dams, pressu forces on the overhead tank, ship design (buoyancy)

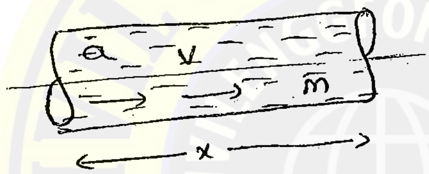




Fluid kinematics:-

1. Fluid kinematics deals motion of the fluid without forces causing motion.

Eg:- Estimation of displacement (x), velocity (v), acceleration (a), volumetric flow rate ($\vartheta = m^3/sec = Q$), Mass flow rate.

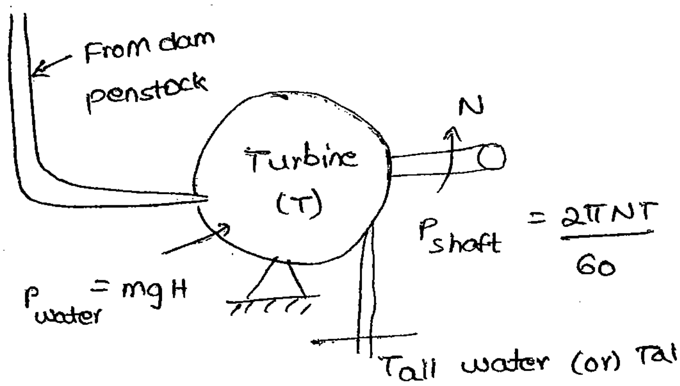


Fluid dynamics:-

study the motion of the fluid under the influence of forces, energy etc.

Ex:- Flow through a turbine, a pump, flow measuring devices like venturimeter, orifice meter, pitot tube, current meter, rotor meter, venturimeter,

~~Defination~~ ~~of~~



6. weight of the matter :-

weight of the matter = Force of that matter due to gravity

units :- kg(f) (or) N

$F = ma$

$N = kg\text{-}m/s^2$

$M^1 L^1 T^{-2}$

$W = mg$

$kg(f) = 1\,kg \times 9.81\ m/s^2$

$= 9.81\ \frac{kg-m}{s^2}$

$kg(f) = 9.81\ N$

Ex:-1. A mass of a block is 100 N. → Given data, W = 100

The weight of the block is 10 kg → Given data, M = 10 kg

pg No:- 15)

$g_{earth} = 9.81$

$g_{moon} = \frac{9.81}{6}$

18.) $W = Mg$

$392 = M \times 9.8$

$M = 40\ kg$

$F = ma$

$800 = 40 \times a$

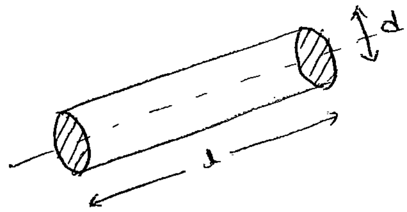
$a = 20\ m/s^2$

acceleration is same on earth and moon

a = 20, 20

7. Area occupied by the matter :-

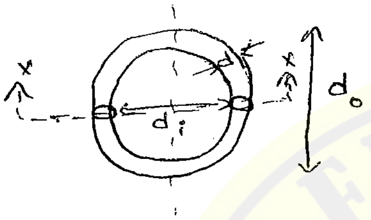
Area is the two dimensional space occupied by the matter.



$$A_{c/s} = \frac{\pi d^2}{4}$$

$$A_{surface} = \pi d \cdot l = A_{shear\ contact\ area}$$

$$A_{projected} = l \cdot d$$

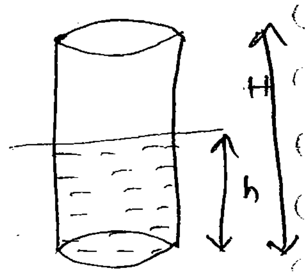


$$A_{plan} = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$A_{c/s_{xx}} = \frac{\pi}{4} d^2 \times 2 \qquad A_{c/s} = \frac{\pi d^2}{4}$$

It is the volume occupied by the matter in 3-D.

Two types of volume (V):



Total volume

$$V = A_{c/s} \cdot H$$

$$= \frac{\pi D^2}{4} \times H$$

wetted volume

$$V = A_{c/s} \cdot h$$

$$= \frac{\pi D^2}{4} \times h$$

unit :- mm^3, cm^3, m^3, In^3, ft^3, $yard^3$

$$\boxed{1\ m^3 = 1000\ lit}$$

~~1 m = 100 cm~~ ^{3}

$100\ cm^3 = 1000$ lit

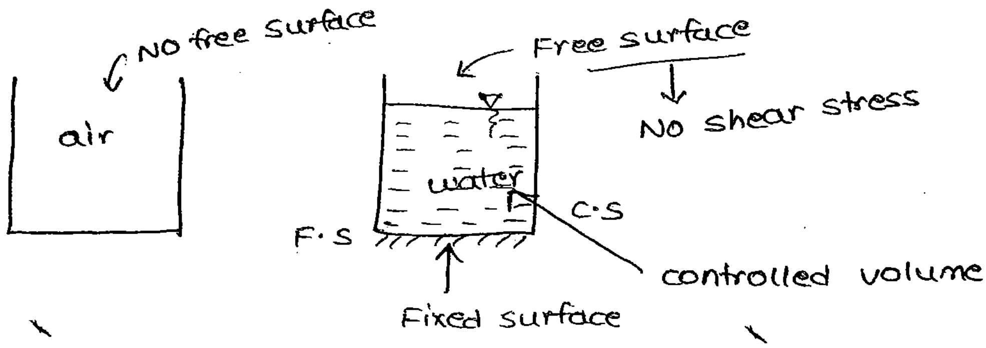
$100\ 0000\ cm^3 = 1000$ lit

$$\boxed{1000\ cm^3 = 1\ lit}$$

$1\ lit = 10 \times 10 \times 10\ cm^3$

Defination of the fluid:-

Fluid is a matter which indefinately deform under the influence of "shear stress" whatever be the magnitude so small



controlled volume = Fixed surface + controlled surface + Free surface.

controlled volume:-

It is the region in space upon which attention is focused for a particular analysis. It can be fixed (or) extensable

Continuous and continuum concept:-

$$K_n = \text{Knudson Number} = \frac{\text{Free (or) Mean depth}}{\text{characteristic length}}$$

$$K_n = \frac{\lambda}{L} \le 0.01$$

$$\lambda \le \frac{L}{100}$$

Different properties of matter:-

M → solid matter

m → fluid matter

1. Free surface
2. Controlled surface → walls exerts force due to pressure
3. Fixed surface → rigid wall
4. controlled volume → a fixed quantity of matter in space or region
5. Mass of the matter → Matter is a certain quantity in controlled volume which remains same every where in universe
6. Weight of the matter → It is the gravitational force of that matter

7. Area occupied by the matter

8. volume occupied by the matter

9. Specific mass of the matter (or) Density (or) Mass density of a matter.

10. Specific weight (or) weight density (or) unit weight

11. specific volume

12. specific gravity (or) Relative density

13. Bulk modulus of elasticity of a liquid

14. compressibility of a fluid

15. Dynamic viscosity (or) coefficity of viscosity (or) Absolute viscosity (or) viscosity coefficient.

16. Kinematic viscosity.

17. pressure intensity of fluid

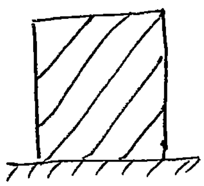
18. Vapour pressure

19. Surface tension (or) Energy

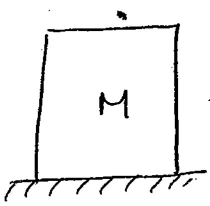
20. cavitation factor of a fluid.

Free surface

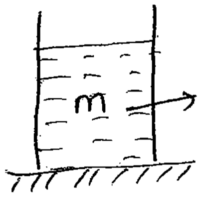
→ No shear stress



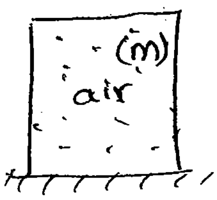
(or)



= solid matter



→ liquid



8. Mass density of a matter :-

It is the certain quantity of matter occupied in one unit volume

$$(\rho) \text{ Mass density} = \frac{x \text{ Mass of water}}{1 \text{ unit volume}}$$

$$\rho = \frac{m}{v}$$

$$m = \rho v$$

$$m = \rho A_{c/s} \cdot H$$

$\rho_{air} = 1.2 \text{ kg/m}^3$

$\rho_{water} = 1000 \text{ kg/m}^3$

$\rho_{mercury} = \rho_{Hg} = 13600 \text{ kg/m}^3$

$\rho_{concrete} = 2400 \text{ kg/m}^3$

$\rho_{steel} = 7850 \text{ kg/m}^3$

$\rho_{Aluminium} = 2800 \text{ kg/m}^3$

$\rho_{wood} = 600 \text{ kg/m}^3$

$\rho_{gold} = 19600 \text{ kg/m}^3$

$\rho_{platinum} = 24600 \text{ kg/m}^3$

$\rho_{diamond} \approx 3000 \text{ kg/m}^3$

10. Weight density :-

$$\gamma = w = \text{weight density} = \frac{\text{weight of the mater}}{\text{unit volume}}$$

$$= \frac{W}{V} \left(\frac{N}{m^3} \text{ (or) } \frac{Kgf}{m^3} \right)$$

$$\gamma = \frac{Mg \text{ (or) } mg}{V}$$

$$w = \gamma = \rho g$$

$$W_{water} = \gamma_{water} = 1000 \times g$$

$$= 1000 \times 9.81 \quad \left[kg \cdot \frac{m}{s^2} = N/m^3 \right]$$

$$= 9810 \ N/m^3$$

$$= 9.81 \ KN/m^3$$

~~9. Spe~~

11. Specific volume :- (v_s)

$$v_s = \frac{1}{R}$$

$$= \frac{\text{volume occupied}}{\text{unit mass}}$$

$$= \frac{v}{m} = \frac{1}{m/v} = \frac{1}{\rho} \left(\frac{m^3}{kg} \right)$$

$$v_{water} = \frac{1}{1000} = 0.001 \ m^3/kg$$

12. Specific gravity (S (or) S.G) :-

(or)

$$\text{Relative density} = \frac{\rho_{matter}}{\rho_{water}}$$

$$S_{matter} = \frac{\rho_{matter}}{\rho_{water}}$$

$$S = \frac{\rho_{mercury}}{\rho_{water}} = \frac{13600}{1000} = 13.6$$

$$S_{gold} = 19.6$$

$$S_{water} = 1$$

$$S_{air} = \frac{1.2}{1000} = 0.0012$$

ISRO 2015

Ex:-) A liquid of 3000 kg occupied in 4 m^3 volume (5)

One of its volume is 0.75 then it is called

a) specific mass b) specific weight.

c) specific volume d) specific gravity

$$\rho = \frac{m}{v} = \frac{3000}{4} = 750\ kg/m^3$$

$$S = \frac{\rho}{\rho_{water}} = \frac{750}{1000} = 0.75$$

Pg. No:- 17

13) specific wt: (or) wt. density $= \frac{W}{V} = \frac{N}{m^3} = F^1 L^{-3} T^0$

$\boxed{\text{specific wt} = F L^{-3}}$

$$\text{Density} = \frac{\text{Mass}}{\text{volume}} = \frac{kg}{m^3}$$

$$= M^1 L^{-3} T^0$$

$$\gamma = \rho g$$

$$\frac{N}{m^3} = \rho \cdot \frac{m}{s^2}$$

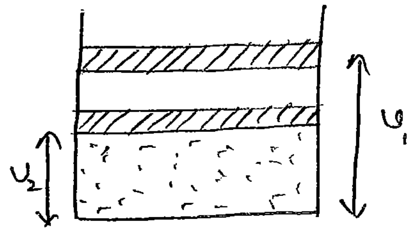
$$\rho = \frac{N\text{-}s^2}{m^4}$$

$\boxed{\text{Density; } \rho = F^1 L^{-4} T^2}$

13. Bulk modulus of elasticity of a liquid (K):-

Of a matter which represents the whole matter (volume) subject at to external pressure or stress. It is observed that the fluids have tendency to get compress by application of external force for the given range of pressure the given mass of the fluid gets compressed due to which volume change which indicates there is change in volumetric change.

$$K = \frac{+dp}{-\left(\frac{dv}{v_1}\right)} \quad N/m^2$$



$$K = \frac{(P_2 - P_1)}{\left(\frac{v_2 - v_1}{v_1}\right)} \quad N/m^2$$

$$= \frac{\text{Increase in pressure}}{\text{Decrease in volumetric strain}}$$

(or)

Increase in mass densities ratio

Ex:-

$K_{water} = 2 \text{ G Pa}$

$= 2 \times 10^9 \ N/m^2$

$= 20000 \ kgf/cm^2$

$K_{air} = 1 \text{ Atm}$

$= 1 \times 10^5 \ N/m^2$

$= 100 \ k \cdot Pa$

$2 \times 10^9 \ N/m^2$

$2 \times 10^8 \ \frac{kgf}{(100 cm)^2}$

$= \frac{2 \times 10^8 \ kgf}{10^4 \ cm^2}$

$= 2 \times 10^4 \ kg/cm^2$

$= 20 \ T/cm^2$

Pg No:- 16

25. $+dp = 200 \ N/cm^2$

$= 200 \frac{N}{\left(\frac{m}{100}\right)^2}$

$dp = 2 \times 10^6 \ N/m^2$

$$\frac{+dP}{P_1} = 0.1\% \Rightarrow \frac{0.1}{100} = \frac{1}{1000}$$

$$K = \frac{dP}{\left(\frac{+dP}{P_1}\right)} = \frac{2\times10^6}{\frac{1}{1000}} \Rightarrow 2\times10^9 N/m^2 \Rightarrow 2\ GPa$$

$$\Rightarrow 2\ GN/m^2$$

Pg NO:-19

23. As) 20,000

Pg NO:- 17

10.) $dp = +18 \frac{MN}{m^2}$

$$= +18\times10^6\ N/m^2$$

$$\frac{dv}{v_1} = -1\%$$

$$K = ?\ \frac{MN}{m^2}$$

$$K = \frac{dP}{\frac{-dv}{v_1}} = \frac{+18\times10^6}{-(-1\%)} = \frac{+18\times10^6}{\frac{1}{100}}$$

$$= 18\times10^8\ N/m^2$$

$$= 1800\times10^6\ N/m^2$$

$$= 1800\ MN/m^2$$

14. compressibility of a fluid:- (β)

It is the reciprocal of the bulk modulus of elasticity whose units (m^2/N) $\beta = \frac{1}{K}$. More the compressibility lesser the bulk modulus.

Ex:- Air, which is easily compressible compare to water. It is 20000 times easy to compress.

Expective questions:-

Ex:- Explain K_{steel} with K_{water}

Ans:- $K_{air} = 1\ kgf/cm^2 = 1\ bar$

$= 1\times10^5\ N/m^2$

$K_{water} = 20000\ kgf/cm^2$

$= 2\times10^9\ N/m^2$

$K_{steel} = ?$

$$\boxed{E = 3K(1-2\mu)}$$

$$2\times10^{11} = 3K_{steel}(1-2\times0.28)$$

$$K_{steel} = 1.6\times10^{11}\ N/m^2$$

$$= 160\times10^9\ N/m^2$$

$$\frac{K_{steel}}{K_{water}} = \frac{160\times10^9}{2\times10^9}$$

$$= 80$$

Relation between Elastic constants:-

$$E = 2G(1+\mu)$$

$$E = \frac{9KG}{3K+G}$$

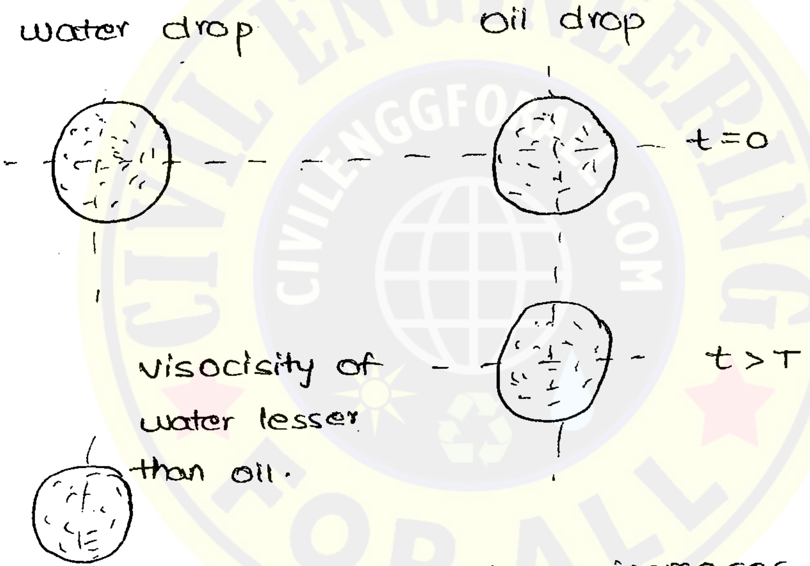
15. Viscosity:-

Every fluid constitute are made up of number of molecules and molecules attract each other by certain forces named as cohesive forces (same molecules at tracting forces) there is another force called Adhesive force (attraction of one molecule to another molecule) water ⟶ both Adhesive and cohesive water wetted the glass.

Mercury → only cohesive force

viscosity is due to cohesive forces which represents resistance to motion of the fluid. More the cohesive forces more the viscosity. viscosity property by changing temperature of the fluid

Ex:-) For liquids like water, oil etc. temperature increase viscosity decreases i.e., easy to flow



* For gases like air, temperature increases viscosity also Increases

→ For the given controlled volume, the gas molecules temp increases due to which the momentum exchange more or dominates the cohesive forces there by molecules have more closely pack and showing high viscous in nature.

** Newtons law of viscosity:-

It states that shear stress required to produce motion to the given fluid is directly proportional to rate of shear strain (or) rate of angular deformation (or) velocity gradient (or) rate of change of velocity with respect to fluid film.

Shear stress $\propto$ Rate of shear strain

(or)

$\propto$ Rate of angular deformation

(or)

$\propto$ velocity gradient

(or)

$\propto$ Rate of change of velocity w.r.t fluid film thickness.

$$\tau \propto \left(\frac{d\phi}{dt}\right) \Rightarrow \text{Rate of shear strain (or) Rate of angular deformation}$$

(or)

$$\tau \propto \left(\frac{dv}{dy}\right)$$

$$\tau = \mu \frac{dv}{dy}$$

$\mu \downarrow$ coefficient of viscosity (or) Dynamic viscosity (or) Absolute viscosity

$$\frac{\text{shear force}}{\text{shear area}} = \mu \cdot \frac{\text{velocity}}{\text{Thickness of fluid}}$$

$$\frac{F_s}{A_s} = \mu \frac{V}{y}$$

$$\frac{N}{m^2} = \mu \cdot \left(\frac{m}{sec \cdot m} = sec^{-1}\right)$$

$$\boxed{\mu = \frac{N-S}{m^2}}$$

***Q. Reason for naming coefficient of viscosity as Dynamic viscosity.

A. Force required per unit surface area over a period of time (t) study the motion of fluid and influence of force called dynamics. Based on this it is name as Dynamic viscosity.

$$\mu = \frac{N\text{-}sec}{m^2} = \text{pascal-sec} = Pa \cdot s = F^1 L^{-2} T^1$$

$$= \frac{kg\text{-}m/sec^2}{m^2} = \frac{kg}{m\text{-}sec} = M^1 L^{-1} T^{-1}$$

Kinematic viscosity (γ) :-

It is defined as the ratio of dynamic viscosity to that fluid mass density

$$\gamma = \frac{\mu}{\rho} = \frac{kg/m\text{-}sec}{kg/m^3} = m^2/sec = M^0 L^2 T^{-1}$$

→ In C.G.S. units dynamic viscosity units "poise"

→ In C.G.S. units kinematic viscosity units "stoke"

S.I	C.G.S.
$\mu = Pa \cdot sec$ $= \frac{N\text{-}sec}{m^2}$ $= \frac{kg}{m\text{-}sec}$	$\mu = \frac{Dyne\text{-}sec}{cm^2}$ $= poise$
$\gamma = \frac{m^2}{sec}$	$\gamma = \frac{cm^2}{sec} = stoke$

** Q) Why viscosity is called kinematic viscosity?

Ans) The viscosity is so called kinematic, it deals motion kinematics (displacement velocity) without ~~forces~~ forces ~~motion~~ involvement

Relation b/w S.I & C.G.S units for viscosity:-

$N = kg\text{-}m/sec^2$

$Dyne = gram\text{-}cm/sec^2$

$$N = kg - \frac{m}{sec^2}$$

$$= 1000\,gm - \frac{100\,cm}{sec^2}$$

$$= 10^5\ \frac{gm-cm}{sec^2}$$

*** $\boxed{N = 10^5 \text{ dyne}}$

→ $1 \text{ poise} = \frac{\text{Dyne-sec}}{cm^2} = \frac{gm}{cm\text{-}sec}$

$$= \frac{kg/1000}{\frac{m}{100}\text{-sec}}$$

$$= \frac{kg}{10\ m\text{-sec}}$$

$$1 \text{ poise} = 0.1\ \frac{kg}{m\text{-sec}}$$

$$10 \text{ poise} = 1\ kg/m\text{-sec}$$

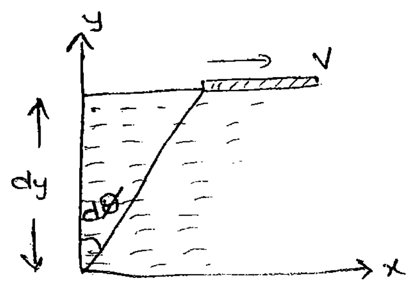
→ 10 C.G.S = 1 S.I of dynamic viscosity.

→ $\text{stoke} = \frac{cm^2}{sec}$

$$= \frac{\left(\frac{m}{100}\right)^2}{sec}$$

$$1 \text{ stoke} = 10^{-4}\ \frac{m^2}{sec}$$

~~10,000 C.G.S viscosity = 1 S.I of~~



$$\tan d\phi = \frac{dx}{dy}$$

$$d\phi = \frac{dx}{dy}$$

$$\frac{dx}{dt} = \frac{d\phi}{dt} \cdot dy$$

$$dv = \frac{d\phi}{dt} \cdot dy$$

$$= \frac{d\phi}{dt} \cdot \frac{dv}{dy}$$

$$\tau \propto \frac{d\phi}{dt}$$

$$\tau \propto \frac{dv}{dy}$$

C.G.S	S.I
$\rho_w = 1\ g/cc$	$\rho_w = 1000\ kg/$
$\mu_w = 1\ CP$	$\mu_w = 10^{-3}\ N\text{-}/m^2$
$\gamma_w = 1\ C.S$	$\gamma_w = 10^{-6}\ m^2/sec$

$$\rho_{water} = 1000\ kg/m^3$$

$$\mu_{water} = 1\ C.P$$

1 centi poise

$$= 10^{-2}\ \text{poise}$$

$$= 10^{-2} \times 0.1\ \frac{N\text{-}sec}{m^2}$$

$$= 10^{-3}\ \frac{N\text{-}sec}{m^2}$$

$$\mu = \frac{1}{1000}\ \frac{N\text{-}sec}{m^2}$$

$$\gamma_{water} = \frac{\mu_{water}}{\rho_{water}}$$

$$= \frac{\frac{1}{1000}\ \frac{N\text{-}sec}{m^2}}{1000\ kg/m^3}$$

$$= 1 \times 10^{-6}\ m^2/sec$$

$$= 1\ C.S$$

$$= 1\ \text{centi stoke}$$

$$\rho_{air} = 1.2\ kg/m^3$$

$$\mu_{air} = 18 \times 10^{-6}\ \frac{N\text{-}S}{m^2}$$

$$\gamma_{air} = 15 \times 10^{-6}\ m^2/sec$$

$$= 15\ C.S$$

$$\frac{\mu_{water}}{\mu_{air}} = \frac{1 \times 10^{-3}}{18 \times 10^{-6}}\ \frac{N\text{-}S/m^2}{N\text{-}S/m^2}$$

$$= 55$$

$$\frac{\gamma_{air}}{\gamma_{water}} = \frac{15\ CS}{1\ CS} = 15$$

$$\mu_{water} > \mu_{air}$$

$$\gamma_{air} > \gamma_{water}$$

Pg No:- 17

32. $\gamma = 6$ stokes

$= 6 \frac{cm^2}{sec}$

$= 6\times10^{-4} \frac{m^2}{sec}$

$S = 2.0$

$S = \frac{\rho}{\rho_{water}}$

$\rho = 2000\ kg/m^3$

$\gamma = \frac{\mu}{\rho}$

$\mu = \gamma\cdot\rho$

$= 2000\times 6\times10^{-4}$

$= 12\times10^{-1}$

$= 1.2\ \frac{N\text{-}s}{m^2}$

Pg No:- 15

21. $\tau = \mu\cdot\frac{V}{y}$

$F = \mu\cdot A\cdot\frac{V}{y}$

$F \propto V$

$\frac{F_1}{F_2} = \frac{V_1}{V_2}$

$\frac{800}{2.4\times1000} = \frac{1.5}{V_2}$

$V_2 = 4.5\ cm/sec$

22. $y = 3\ mm = 0.003\ m$

$\mu = 2$ poise

$\mu = 2\times0.1 = 0.2\ \frac{N\text{-}s}{m^2}$

$V = 1.5\ m/s$

$\tau = \mu\cdot\frac{V}{y}$

$= 0.2\times\frac{1.5}{0.003}$

$\boxed{\tau = 100\ N/m^2}$

Note:- $\mu = x$ poise

$S = x$ No unit

$\gamma = 1$ stoke

24. $\mu_{water} = 1\times10^{-3}$ pa-sec $\gamma_w = 1\times10^{-6}$ m²/sec

$\mu_{air} = 10\times10^{-6}$ pa-sec $\gamma_{air} = 15\times10^{-6}$ m²/sec

$\mu_{water} > \mu_{air}$ $\gamma_{air} > \gamma_{water}$

$\gamma_{water} < \gamma_{air}$

23. $\mu = 0.5$ poise $= 0.5\times0.1$ kg/m-sec

$\mu = 0.05$ kg/m-sec

$\rho = s\times \rho_w$

$= 0.5\times1000 = 500$ kg/m³

$$\alpha = \frac{\mu}{\rho} = \frac{0.05}{500} = 1\times10^{-4}\ \text{m}^2/\text{sec}$$

$= 1$ cm²/sec

$\alpha = 1$ stoke

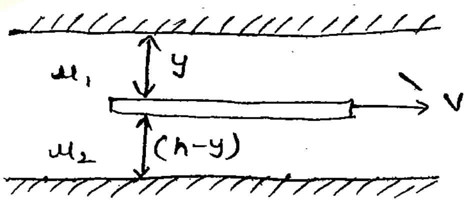
$\mu = x$ poise

$s = x$ No unit

$\alpha = 1$ stoke

Eg:- A thin plate is placed in between two fixed flat surfaces by distance 'h' apart. The viscosity of liquids on the top and bottom of the plate are μ_1 and μ_2 respectively. The position of the thin plate such that the viscous resistance (external effort required) for uniform velocity 'v' of the thin plates is minimum (Assume 'h' is to be very small).

A. $\tau = \mu\cdot\frac{du}{dy}$



$$\frac{F_1}{A_1} = \mu_1\frac{du}{dy}$$

$$F_1 = \mu_1 A_1 \frac{V}{y} \longrightarrow (1)$$

$$F_2 = \mu_2 A_2 \frac{V}{(h-y)}$$

$$\mu_1 A_1 \frac{V}{y} = \mu_2 A_2 \frac{V}{(h-y)}$$

$$y = \frac{\mu_1 (h-y)}{\mu_2}$$

pull required to drag the plate is minimum. The force required to drag the plate is minimum if.

$$\frac{dF}{dy} = 0 ; \quad F_{net} = F_1 + F_2$$

$$= \mu_1 \left(\frac{V}{y}\right) A + \mu_2 \left(\frac{V}{h-y}\right) A$$

$$\frac{dF_{net}}{dy} = \mu_1 V A \left(\frac{-1}{y^2}\right) + \mu_2 V A \left(\frac{1}{h-y}\right) - 1 = 0$$

$$= VA \left[-\frac{\mu}{y^2} + \frac{\mu^2}{(h-y)^2} \right] = 0$$

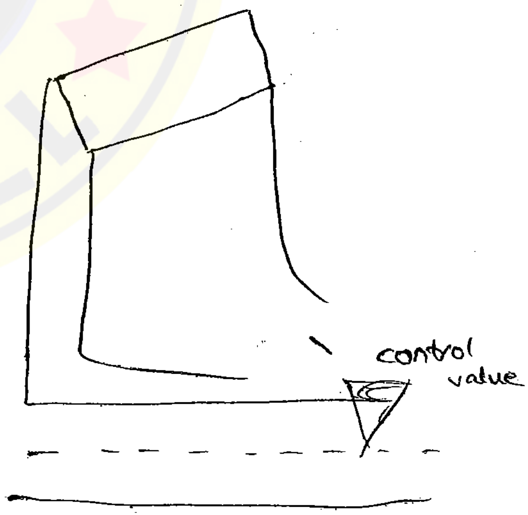
$$\frac{\mu_1}{y^2} = \frac{\mu_2}{(h-y)^2}$$

$$\frac{y}{h-y} = \sqrt{\frac{\mu_1}{\mu_2}}.$$

Velocity of pressure waves :-

→ small pressure disturbance travel through the fluid medium at a velocity equal to the velocity of sound (c) which depends on Bulk modulus and mass density.

→ The compressibility of water is considered in the case of water hammer problems. Due the sudden closure of valves in pipe lines, a high pressure wave is generated.



$$P = \rho v c \ N/m^2$$

c = velocity of pressure waves due to sudden closure of values in pipe lines.

= wave speed

c = velocity of sound in that fluid

$$c = \sqrt{\frac{kg\text{-}m/sec^2}{kg/m^3}}$$

$$c = \sqrt{\frac{m^2}{sec^2}} = \frac{m}{sec}$$

$$c = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{Kg}{\gamma}} = \sqrt{\frac{Kg}{\omega}}$$

$\omega = \gamma =$ specific weight.

Ex:-) The bulk modulus and density of a liquid are given as $2\ KN/mm^2$ and $8000\ kg/m^3$. What is the velocity of sound through the liquid?

A) $K = 2\ KN/mm^2$

$\rho = 8000\ kg/m^3$

$$c = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{2\times10^3\times1000^2}{8000}} = 500\ m/s$$

$C_{air} = 320\ m/s \qquad C_{water} = 1414\ m/s$

$$C_{air} = \sqrt{\frac{K_{air}}{\rho_{air}}} = \sqrt{\frac{1\ bar}{1.2\ kg/m^3}} = \sqrt{\frac{1\times10^5\ N/m^2}{1.2\ kg/m^3}} = 320\ m/s$$

$$C_{water} = \sqrt{\frac{K_\omega}{\rho_\omega}} = \sqrt{\frac{2\times10^9\ N/m^2}{1000\ kg/m^3}} = 1414\ m/sec$$

Velocity of sound in water is 4 to 5 times of velocity of sound in Air.

Fluid classification

Ideal fluid
(or)
Theoritical fluid
(or)
Super fluid

Real Fluid
↓
possess viscosity
(Having)

Ideal Fluid:-

→ zero viscosity

→ No shear stress required to move

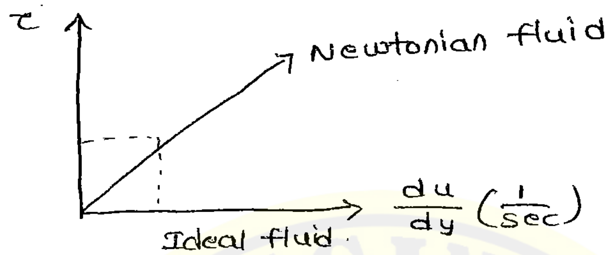
→ velocity gradient not exist

→ Incompressibility.

Real Fluid.

- Newtonian fluid
- Non-Newtonian fluid
  - pseudo plastic
  - Dilatent fluid
- Binghams plastic/ Ideal plastic fluid
- Thexotropic fluid.

→ Fluid power law (or) Rhelogical fluid equation.



$$T = \tau_{yield} + \mu\left(\frac{du}{dy}\right)^n$$

$$\boxed{\tau = B + A\left(\frac{du}{dy}\right)^n}$$ → Rhelogical fluid equation.

(i.) Ideal fluid :-

$A = \mu = 0$

$B = 0$

$\tau = 0 + 0\left(\frac{du}{dy}\right)$

$\tau = 0$

Not available in nature

No such fluid in nature

(ii). Newtonian Fluid :-

$B = 0$; $A = \mu$, $n = 1$

$\tau = B + A\left(\frac{du}{dy}\right)^n$

$= 0 + \mu\left(\frac{du}{dy}\right)^1$

$\tau = \mu\left(\frac{du}{dy}\right)$

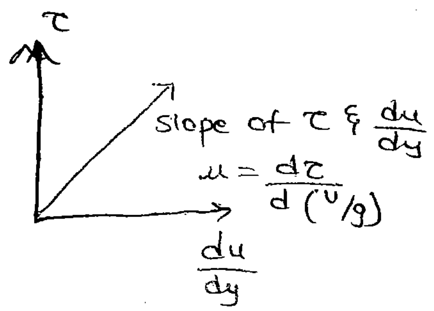
$\tau \propto du/dy$

** → A fluid obeys Newtons law of viscosity is known as Newtonian law.

Ex:- Air, water, mercury

→ viscosity is independent of shear stress

$du/dy\ (s^{-1})$	0	1	2	3	4	5
$\tau\ (N/mm^2)$	0	3	6	9	12	15



* slope is constant throughout

Ex:-) The following shear stress - shear rate relationship was obtained for a fluid. The fluid is classified as. (12)

du/dy	0	1	3	5
τ	0	6	18	30

A) $\text{slope} = \dfrac{\tau}{\left(\frac{du}{dy}\right)} = \dfrac{6}{1} = 6$

$= \dfrac{18}{3} = 6$

$= \dfrac{30}{5} = 6.$

slope constant. Hence Newtonian

Non-Newtonian Fluid:-

$B = 0 \; ; \; A = \mu \; ; \; n \neq 1$

Non Newtonian fluid is one fluid which does not obeys Newton law of viscosity.

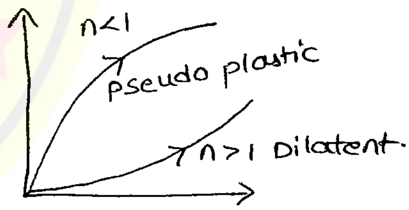
$B = 0; \; A = \mu; \; n \neq 1$ then $n < 1$ (or) $n > 1$

If $n > 1$ then fluid is called Dilatent fluid.

If $n < 1$ then fluid is called pseudo plastic fluid.

Dilatent fluid — $n > 1$ — Eg:- syrup, fruit juice — shear thick

pseudo Plastic — $n < 1$ — Ex:- Milk, Blood — shear thin



Bingham plastic fluid:-

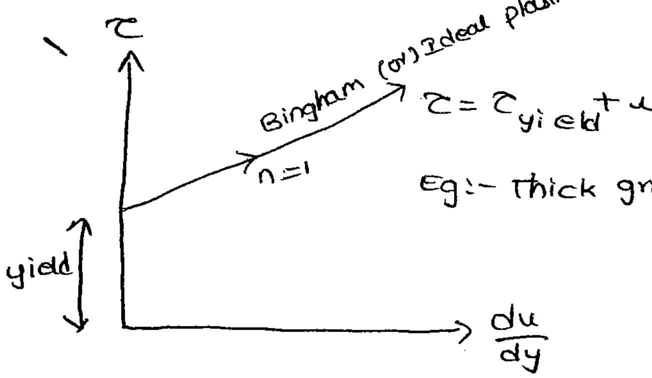
$B = \tau_{yield}$

$A = \mu$

$n = 1$

$\tau = B + A\left(\frac{du}{dy}\right)^n$

$\tau = \tau_{yield} + \mu\left(\frac{du}{dy}\right)^{n=1}$



$\tau = \tau_{yield} + \mu\left(\frac{du}{dy}\right)$

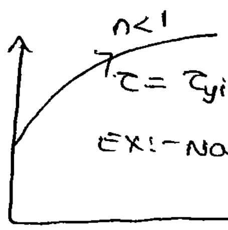
Eg:- Thick grease

Thexiotropic fluid:-

$B = \tau_{yield}$

$A = \mu$

$n < 1$



$\tau = \tau_{yield} + \mu\left(\frac{du}{dy}\right)^{n<}$

Ex:- Nail polish

$\tau = B + A\,(du/dy)^n \implies \tau = \tau_{yield} + \mu\,(du/dy)^{n<1}$

Pressure and pressure Intensity of fluid:-

Pressure is the force per normal area

$$P = \frac{F}{A} = N/m^2 = pa. \qquad kgf/cm^2$$

$$1 \text{ bar} = 10^5 \, N/m^2 = 10^5 \, pa = 0.1 \, Mpa.$$

$$10 \text{ bar} = 1 \, Mpa.$$

Pressure is a scalar quantity.

Pressure:-

Energy stored (Joules) per unit volume.

$$P = \frac{\text{energy}}{\text{volume}} = \frac{J}{m^3} = \frac{N-m}{m^3} = N/m^2 \qquad \text{Joule} = N-m$$

$$P = \frac{F}{A} = \frac{w}{A} = \frac{mg}{A} = \frac{\rho v g}{A}$$

$$\frac{\rho A H g}{A} = \rho g H$$

$$\boxed{P = \rho g H = \gamma H}$$

Potential Energy $= P.E = mgH$

Pressure Energy $= \rho g H \quad \because$ Energy / unit volume

$$P = \frac{mgH}{V} = \frac{m}{v} \cdot g \cdot H$$

$$\boxed{P = \rho \cdot g \cdot H}$$

$$1 \text{ Atm. pressure} = \frac{1 \, kg \cdot f}{cm^2} = \frac{10 \, T}{m^2} = \frac{10 \times 1000 \, kg(f)}{m^2}$$

$$= \frac{10000 \times 10 \, N}{m^2} = 100000 \, N/m^2$$

$$= 1 \times 10^5 = 100 \, kN/m^2$$

$$\boxed{1 \text{ Atm} \cdot p = 100 \, kpa = 1 \text{ bar}.}$$

$$H_{water_{Atm}} = \frac{100000}{1000 \times 9.81} = 10.1 \, m$$

$$H_{mercury} = \frac{100000}{13600 \times 9.81} = 0.76 \, m = 76 \, cm$$

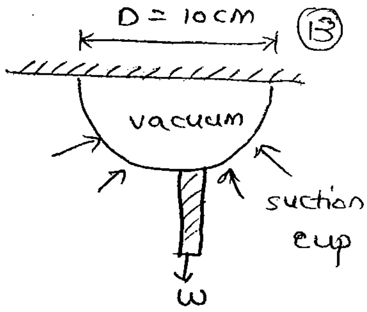
$$H_{air} = \frac{100000}{1.2 \times 9.81} = 8332 \, m$$

$$1 \text{ Atm} = 1 \text{ kg(f)}/\text{cm}^2$$

$$P = W/A$$

$$\frac{1 \text{ kg(f)}}{\text{cm}^2} = \frac{W \text{ (kg f)}}{\frac{\pi}{4}(10^2) \text{ cm}^2}$$

$$W = \frac{1 \times \pi \times 10^2}{4} = 78 \text{ kg·f}$$



Ex:-) A 70 kg person walks on snow with a total foot impla area of 500 cm². what pressure does he exert on snow?

A. $$P = \frac{F}{A} = \frac{70 \times 9.81}{500 \times 10^{-4}} = 13.73 \times 10^3 \text{ N/m}^2$$

Surface Tension (or) Surface Energy of a liquid:-

Every free surface of a fluid exhibits certain surface tensile force due to cohesive forces of that fluid. It is simply a tensile force exerted per unit length exposed to surroundings.

$$\sigma = \frac{F_t}{L} = \frac{N}{m} = \frac{N\text{-}m}{m^2} = \text{Joules}/m^2$$

$$\sigma = \text{Energy}/\text{unit surface Area}$$

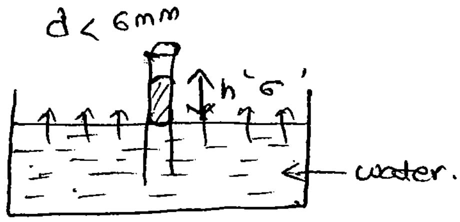
$$\sigma = \frac{F_t}{\text{unit length}} = \frac{N}{m} = \frac{J}{m^2}$$

$$\sigma_{\text{water Air}} = 0.072 \text{ N/m}$$
$$= 0.072 \text{ J/m}^2$$

Significance of surface Tension and Application:-

water is liquid which shows a tendency of raising due to upward (tensile) force when water is in contact with other surface this is known as capillarity Rise observed in plants.

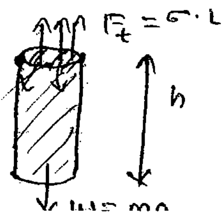
Ex:- SAP, biological phenomenon.



$$h = \frac{4\sigma}{\rho g d} = \frac{4\sigma\cos\theta}{4 \cdot g \cdot d}$$

θ = Negligible around 10°

Derivation:-



$$\sigma = \frac{F_t}{L}$$

$$F_t = \sigma L$$

$$W \downarrow = F_t \uparrow$$

$$mg = \sigma . L$$

$$\rho v g = \sigma . \pi d$$

$$\rho . \frac{\pi}{4} d^2 . h . g = \sigma . \pi d$$

$$h = \frac{4\sigma}{\rho g h}$$

$$h \propto \frac{1}{d}$$

$$\boxed{\frac{h_1}{h_2} = \frac{d_2}{d_1}}$$

$$h = \frac{4\sigma}{\rho g d} = \frac{4\sigma}{\gamma d}$$

$$\boxed{h = \frac{4\sigma}{\gamma d}}$$

$$\boxed{h = \frac{4\sigma}{w d}}$$

→ capillarity is a phenomenon due to difference of cohesion and adhesion forces.

→ surface tension is due to cohesion only.

Ex:- spherical shape of rain (or) water droplet during fall

P.g NO:- 16

27.
$$\frac{h_1}{h_2} = \frac{d_2}{d_1}$$

$$\frac{15}{h_2} = \frac{4}{3}$$

$$h_2 = 4$$

If it is mercury then capillarity is not rise instead it falls (depression)

$$\boxed{h_{hg} = \frac{4\sigma\cos\theta}{\rho \cdot g \cdot d}}$$

where $\theta > 90°$ (Non wetted)

$\theta_{mercury} = 140°$

Note:-

Mercury posses only cohesive forces and 'o' adhesive nes. due to this it always tends to depress or falls in the narrow table.

Other effects of surface Tension:-

1. Rise of kerosene (or) oil in wick "Due to capillarity".

2. Insects can walk on liquid surfaces. Brick or flowers absorb water when they immersed due to surface Tension Spherical shape due to ~~shear~~ surface tension.

pressure ~~due~~ exerted due to surface Tension:-

1. Liquid Drop

2. Liquid Bubble

3. Liquid Jet

Liquid Drop:-

$$\text{pressure} = \frac{F_c}{A_N} = \frac{F_c}{\frac{\pi d^2}{4}}$$

$F_c = F_t$ (under Equilibrium)

$P A_n = \sigma . L$

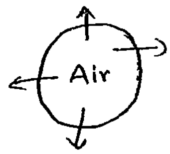
$P . \frac{\pi}{4} d^2 = \sigma . \pi D$

$$\boxed{P_{droplet} = \frac{4\sigma}{D}} \quad N/m^2$$

$$\boxed{P_{droplet} = \frac{2\sigma}{r}}$$

Liquid Bubble :-

$$F_c = F_t$$



$$P \cdot A_n = \sigma \cdot L$$

$$P \frac{\pi d^2}{4} = \sigma \cdot \pi \times D \times 2$$ (Two surfaces exposed to air)

$$\boxed{P_{bubble} = \frac{8\sigma}{D}}$$

$$\boxed{P_{bubble} = \frac{4\sigma}{R}}$$

$$\boxed{\therefore \frac{P_{bubble}}{P_{droplet}} = 2}$$

P.g No:- 17

12. $P = \frac{4\sigma}{d} = \frac{4\times 0.073}{0.001} = \frac{4\times 73}{1} = 292\ N/m^2$

26. $P = \frac{8\sigma}{d}$

$$= \frac{8\times 0.04}{100}$$

$$= \frac{8\times 4}{100\times 100}$$

$$=$$

CHAPTER - II

PRESSURE MEASUREMENT AND FLUID STATICS

$P = \rho g h$

$P = \gamma h$

$dp = -\gamma \cdot dh$

$$\frac{dp}{dh} = -\gamma$$

$dp = -\gamma(-dh)$

$= \gamma dh$.

Hydro static law:-

It states that the rate of pressure change w.r.t height specific weight of the fluid.

Height increases → pressure decreases

Depth increases → pressure increases

$dp = -\gamma \cdot dh$ → Hydro static law.

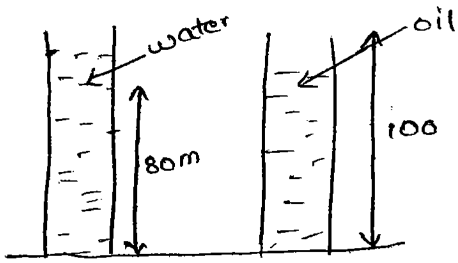
$$\frac{dp}{dh} = -\gamma$$

$$P = \rho_x g h_x = \rho_y g h_y$$

Ex:-

$$P = \rho_{oil} \times g \times h_{oil} = \rho_{water} \times g \times h_{water}$$

$$\left(\frac{N}{m^2}\right) = \left(\frac{kg}{m^3}\right) \times \left(\frac{m}{s^2}\right) \times m$$



$P = \rho_{oil} \times g \times h_{oil} = \rho_w \times g \times h_w$

$= (0.8 \times 1000) \times 10 \times h_{oil}$

$= 1000 \times 10 \times 80$

$$h_{oil} = \frac{80}{0.8} = 100 \text{ m}$$

Units :-

pressure : $P_{SI\ unit} = \frac{N}{m^2} = Pa$

$1\ bar = 10^5\ N/m^2 = 100\ k\cdot pa$. Head of a fluid.

1 ~~atomp~~ Atmospheric pressure $= \frac{1\ kg(f)}{cm^2} = \frac{10\ T(f)}{m^2}$

$= \frac{10000\ kg(f)}{m^2}$

$= 100000 N/m^2$

$= 100\ kN/m^2$

$= 100\ k\ pa$

$= 1\ bar$

P.g NO:- 41

30. $P = \rho g h$

$= 1000 \times 9.81 \times 25$

$= 245 \times 10^3\ N/m^2$

$= 245\ KN/m^2$

23. $P = \rho_{Hg} \times g \times h_{Hg}$

$P = 13600 \times 9.81 \times h_{Hg}$

$100000 = 13600 \times 9.81 \times h_{Hg}$

$h_{Hg} = 0.76\ m$

$= 76\ cm$ of mercury

$= 760\ mm$ of mercury.

10. $P_\omega = \rho_\omega \times g \times h_\omega$

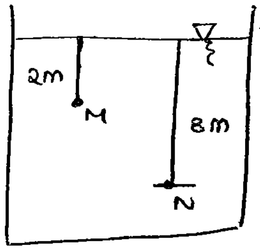
$= 1000 \times 10 \times 10$

$= 100\ kN/m^2$

27. $$\frac{P_M}{P_N} = \frac{\cancel{\rho g}\, h_m}{\cancel{\rho g}\, h_N}$$

$$= \frac{2}{8}$$

$$= 1 : 4$$



49. $$\frac{P_A}{P_B} = \frac{\rho g h_A}{\rho g h_B}$$

$$= \frac{h_A}{h_B} = \frac{0.5}{2.0} = 1 : 4$$

45. 1 atm pressure $= \rho_\omega g h_\omega$

$$100000 = 1000 \times 10 \times h_\omega$$

$$h_\omega = 10 \text{ m of water}$$

$$= 10.33 \text{ m of water.}$$

Pg No:- 39

9. $$P = \rho g h$$

$$= 1000 \times 9.81 \times 1$$

$$= 9810\ N/m^2 \text{ (or) pa}$$

$$= \cancel{9}.$$

8. $$P = \rho g h$$

$$P = \gamma h \quad \left[\frac{kgf}{m^3} \times m\right]$$

$$4.8\ \frac{kg(f)}{cm^2} = (S_{oil} \times \rho_{water}) \times h_{oil}$$

$$\frac{4.8 \times 9.81\,(N)}{\left(\frac{m}{100}\right)^2} = (0.8 \times 1000) \times h_{oil}$$

$$h_{oil} = \frac{4.8 \times 9.81 \times 10000}{800} \left(\frac{N}{m^2}\right)$$

$$h_{oil} = 6000\,cm \Rightarrow 60\ m$$

P.g. NO:- 38

19. $\frac{P_A}{P_B} = \frac{\cancel{\rho g} h_A}{\cancel{\rho g} h_B}$

$= \frac{0.5}{2}$

$= 1:4$

18. $P = \rho g h$

$3.6 \times 9.81 \times 10^4 \left(\frac{\cancel{kg} N}{m^2}\right) = 900 \times h_{oil} \times 9.81$

$h_{oil} = 40 m$

P.g NO:- 35

**25. deci $= 10^{-1}$ deca $= 10^{1}$

centi $= 10^{-2}$ Quntal $= 10^{2}$

milli $= 10^{-3}$ kilo $= 10^{3}$

micro $= 10^{-6}$ Mega $= 10^{6}$

nano $= 10^{-9}$ Giga $= 10^{9}$

picco $= 10^{-12}$

a. 1 milli bar $= 10^{-3} \times 10^{5}$ N/m²

$= 100$ N/m² (or) pa

$= 100$ S.I units.

b. 1 mm of mercury = 1 tor

1 Atm = 760 mm of mercury

= 760 tor

$P = \rho g h$

$= 13600 \times 10 \times 0.001$

$= 136$ N/m² (or) pa (or) SI unit

c. $\frac{N}{mm^2} = 1 \times 10^6\ N/m^2 = 10$ lakh SI units

d. $\frac{1\ kgf}{cm^2} = \frac{9.81\ N}{\left(\frac{m}{100}\right)^2}$

$= 10^5\ N/m^2$

$= 1$ lakh SI units.

Pg.No:-33

8. All of the above

Types of pressure:-

1. Atmospheric pressure
2. Gauge pressure
   a. positive pressure
   b. Negative (or) vacuum pressure (or) suction pressure.
3. Absolute pressure

Units:-

1 Atm pressure = 1 kgf/cm², 100 k·pa, 1 bar, 76 cm of Hg, 10·3 m of water.

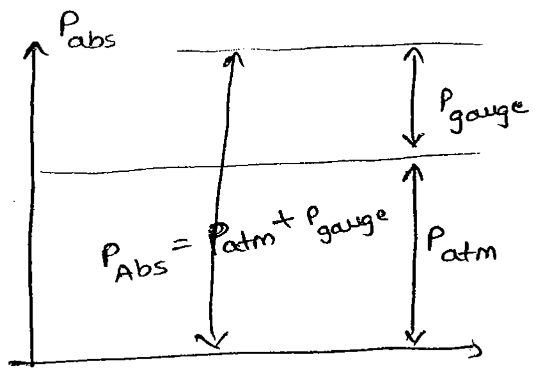
Gauge pressure:-

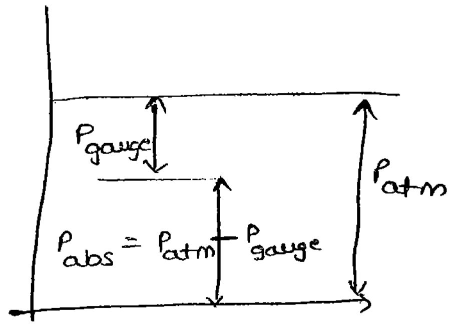
Gauge pressure is measured by pressure gauge if reading value is more than the atmospheric pressure is called positive gauge pressure. If the pressure gauge below the atmospheric is called Negative (or) vacuum (or) Suction pressu

Absolute pressure:-

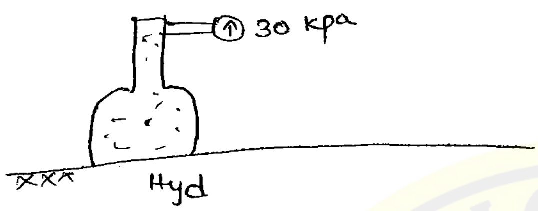
It is the sum of the local atmospheric pressure and gauge pressure.

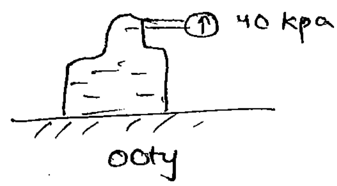
$$P_{absolute} = P_{atm.\ local} \pm P_{gauge}$$





Ex:-)





$P_{Atm} = 100$ kpa

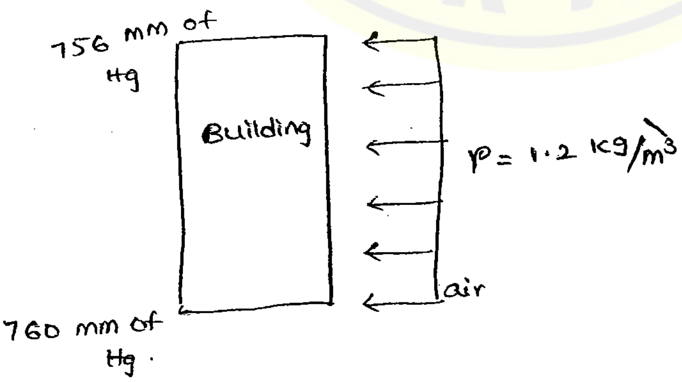
$P_{abs} =$ constant

$$P_{abs} = \underset{Hyd}{P_{Atm}} + \underset{Hyd}{P_{gauge}} = \underset{ooty}{P_{Atm}} + \underset{ooty}{P_{gauge}}$$

$$= 100 + 30 = \underset{ooty}{P_{atm}} + 40$$

$$P_{atm} = 90 \text{ kpa.}$$

Ex:-)



What is the height of building?

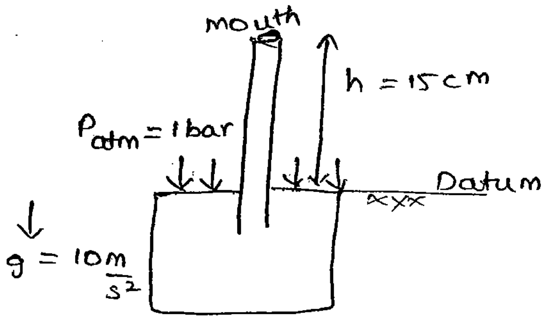
$$\Delta p = \rho_{air} \times g \times h_{air}$$

$$13600 \times 9.81 \times 0.004 = 1.2 \times 9.81 \times h_{air}$$

$$h_{air} = 45.33 \text{ m}$$

Height of building $= 45.33$ m

Ex:-) Find Absolute pressure in mouth of a person



A. $P_{datum} = P_{mouth} + \rho g h$

$1\ Atm = \cancel{P}\ P_{mouth} + 1000 \times 10 \times 0{\cdot}15$

$100000 = P_{mouth} + 1500$

$P_{mouth} = 98500\ N/m^2$

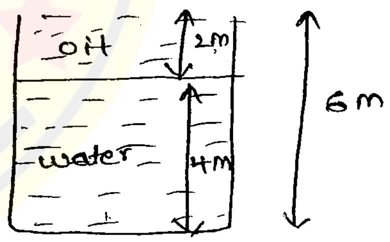
$= 98{\cdot}5\ K{\cdot}pa.$

Note:-

Maximum suction pressure possible to lift water is 10·33 m. (Theoritically). But practically 6–7 m suction height only handle by pipes.

P·g No:- 35

24.



$P_{Bottom} = \rho_w g h_w + \rho_{oil} g h_{oil}$

$= \gamma_w h_w + \gamma_{oil} h_{oil}.$

$= 10(4) + (0{\cdot}9 \times 10) \times 2$

$= 40 + 18\ KN/m^2$

$= 58\ KN/m^2$

10. $P_{standard} = 760$ mm of Hg

$P_{local} = 710$ mm of mercury

$P_{absolute} = 360$ mm of Hg

$P_{abs} = P_{atm\ local} + P_{gauge}$

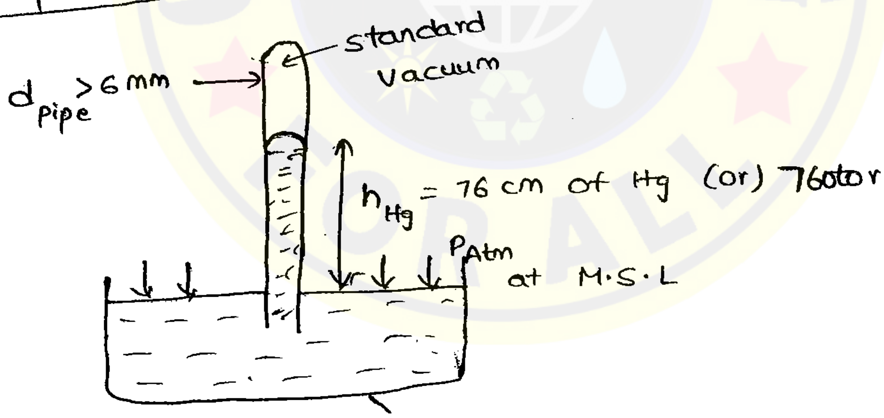
$P_{gauge} = \cancel{710}\ 360 - 710 = 350$ mm of Hg.

Pressure measuring devices:-

1. Barometer ~~(or)~~ and Aneroid.
2. Piezometer
3. simple Manometers (U-tube manometer)
4. Differential U-tube manometer.
5. Inverted differential manometers
6. Inclined Manometers
7. Mechanical pressure gauge (Bourdon)
8. Tranducer
9. Dead weight pressure gauge

Barometer:-

It is a pressure gauge used for measuring the local atmospheric pressure.



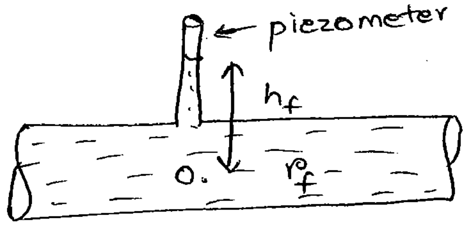
→ sudden fall in Barometer reading indicates that a chance of cyclone.

Aneroid meter:-

It is also a pressure gauge used to measure local atmospheric pressure.

piezometer :-

It is a pressure gauge which measures pressure of the fluids at given point (positive or negative pressure)

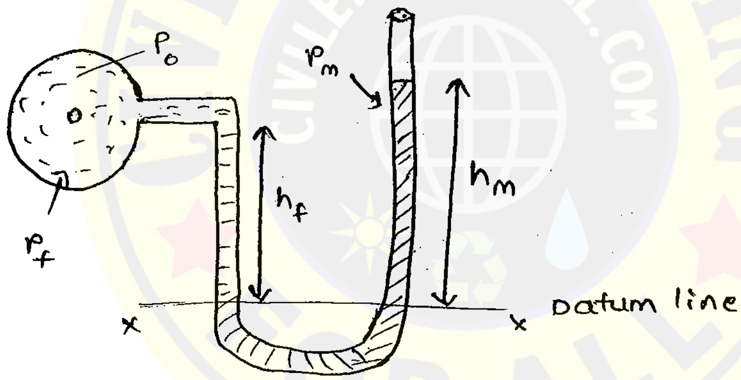


$$P_o = \rho_f g h_f$$

Note :-

piezometer measures static pressures not dynamic.

** U-tube manometers :-



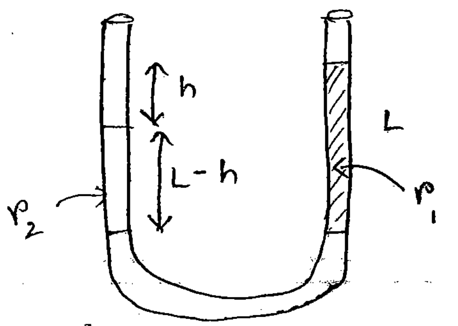
$$P_x = P_{x'}$$

$$P_o + \rho_f g h_f = \rho_m g h_m$$

$$P_o = (\rho_m g h_m - \rho_f g h_f) \; (N/m^2)$$

Pg No :- 34

14.

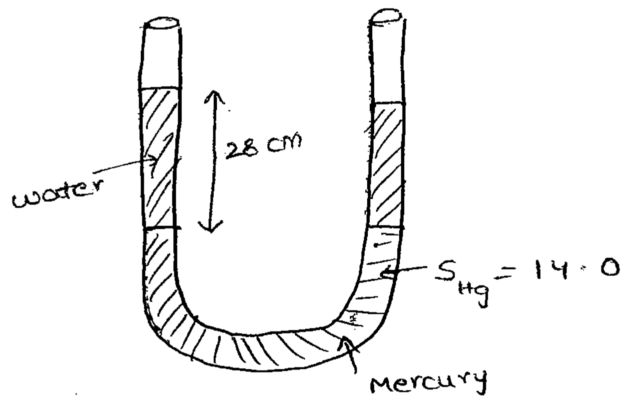


$$\rho_2 g (L-h) = \rho_1 g L$$

$$\rho_2 (L-h) = \rho_1 L$$

$$h = L\left(1 - \frac{\rho_1}{\rho_2}\right)$$

EX:-)



There is a U-tube filled with mercury and both ends are open to air. An amount of water filled in left arm of height 28 cm. ($S_{water} = 1 \cdot 0$). Determine the height of the mercury raised in right arm.

A) 2 cm ✓B) 1 cm c) 4 cm D) 0.5 cm

Sol:-

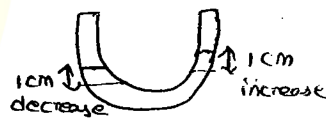
$S_w \cdot h_w = S_{Hg} \cdot h_{Hg}$

$1 \cdot 0 \times 28 = 14 \times h_{Hg}$

$h_{Hg} = \underline{2\ cm}$ (of total mercury)

Total mercury $= x$

After $= x + 2$



But it distribute right and left arm because mercury level is same on both sides. So mercury increase by $\underline{1m}$

Note:-

$W = Mg$

$1\,kgf = kg \times 9.81\ m/s^2$

$1\,kgf = 9.81\ \frac{kg\text{-}m}{s^2}$

$1\,kg(f) = 9.81\ N$

$\cong 10\ N$

Energy = Joules

$= N\text{-}m$

$= \frac{kg\text{-}m}{s^2} \cdot m \Rightarrow M^1 L^2 T^{-2}$

power = watt

$= \frac{J}{sec}$

15. Momentum = mass x velocity

I (or) $M = mV$

Angular momentum $= I \cdot \omega$

$= mL^2 \cdot \omega$

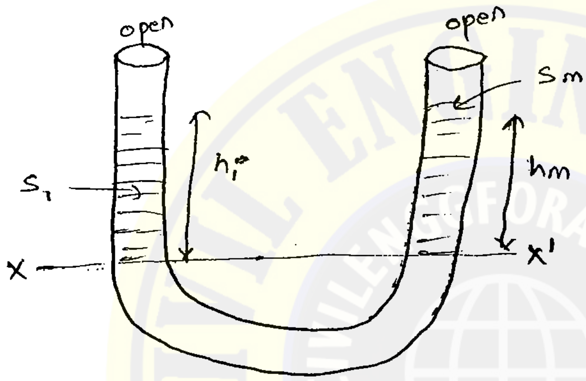
$= kg \cdot m^2 \cdot \frac{rad}{sec}$

$= M \cdot L^2 T^{-1}$

14. Ex:-) U-tube Manometer

$h_m = 30$

$h_1 = 20$



$P_x = P_{x'}$

$\rho_1 g h_1 = \rho_2 g h_2 \; (N/m^2)$

$= \frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m \quad [N/m^2]$

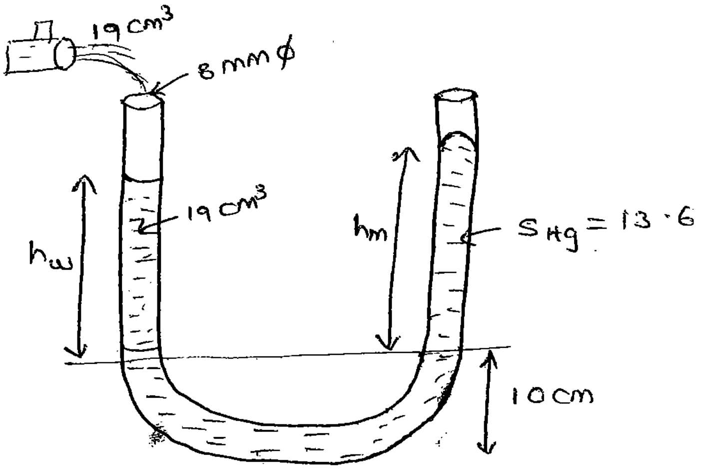
$\frac{\rho_1}{\rho_w} g h_1 = \frac{\rho_2}{\rho_w} \cdot g h_2$

$s_1 h_1 = s_2 h_2$

$\frac{S_1}{S_2} = \frac{h_2}{h_1}$

$\frac{S_1}{S_m} = \frac{h_m}{h_1} = \frac{30}{20} = 1.5$

EX:-)



Find height of mercury.

A. $S_w \cdot h_w = S_{Hg} \cdot h_{Hg}$ ⟶ This formula valid for $P_1 = P_2$ (equal) both sides pressure equal

$1.0\ h_w = 13.6 (h_{Hg})$

$V_w = A \cdot h_w$

$19 = \frac{\pi (0.8)^2}{4} \times h_w$

$cm^3 \quad cm^2 \quad cm$

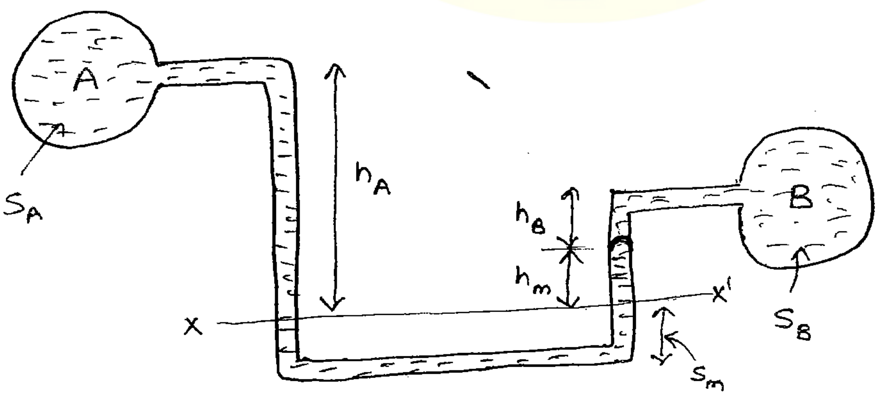
$\boxed{h_w = 37.8\ cm}$

$1.0 (37.8) = 13.6\ h_{Hg}$

$\boxed{h_{Hg} = 2.8\ cm}$

Differential pressure between two points:-

$P_A - P_B = ?$



$P_x = P_{x'}$

$P_A + \rho_A g h_A = P_B + \rho_B g h_B + \rho_m g h_m$

$S_A = \frac{\rho_A}{\rho_w}$

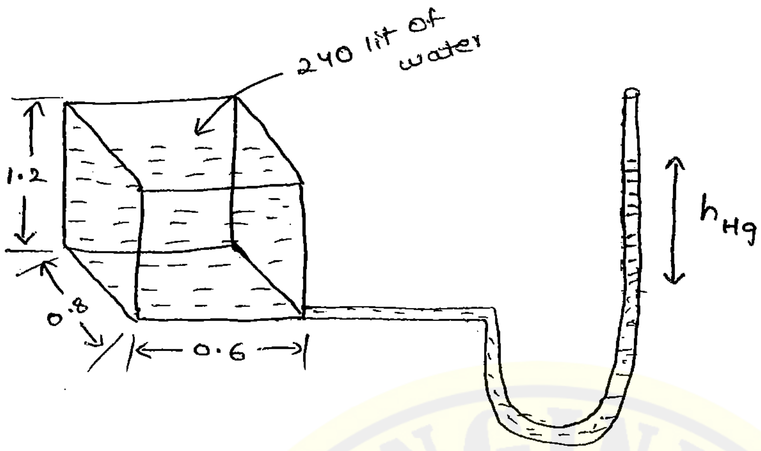
$S_B = \frac{\rho_B}{\rho_w}$

$S_m = \frac{\rho_m}{\rho_w}$

$$P_A - P_B = \rho_B g h_B + \rho_m g h_m - \rho_A g h_A$$

P.g NO:-39

1)



$$S_w h_w = S_{Hg} h_{Hg}$$

$$1.0 \; h_w = 13.6 \; h_{Hg}$$

$$v_w = 240 \text{ lit} \qquad 1 \; m^3 = 1000 \text{ lit}$$

$$= \frac{240}{1000} \; m^3$$

$$v_w = 0.24 \; m^3$$

$$A \times h_w = 0.24$$

$$L \times b \times h = 0.24$$

$$h_w = \frac{0.24}{0.6 \times 0.8} = 0.5 \; m$$

$$1.0 \, (0.5) = 13.6 \; h_{Hg}$$

$$h_{Hg} = \frac{0.5}{13.6} \; m$$

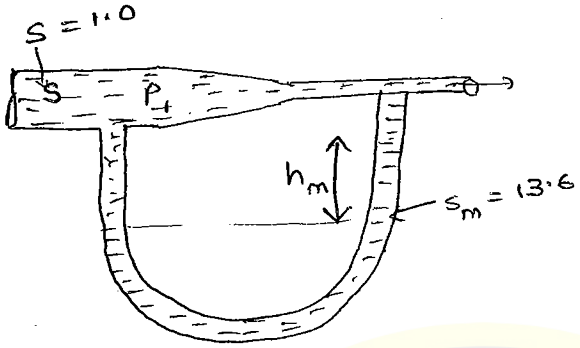
$$= 37 \; mm$$

28. absolute pressure = atm. press + gauge pressure

$= 100 + 200$

$= 300$ k.pa

EX:-)



Differential U-tube manometer

$$P_1 - P_2 = \rho g(\Delta h)_{1-2}$$

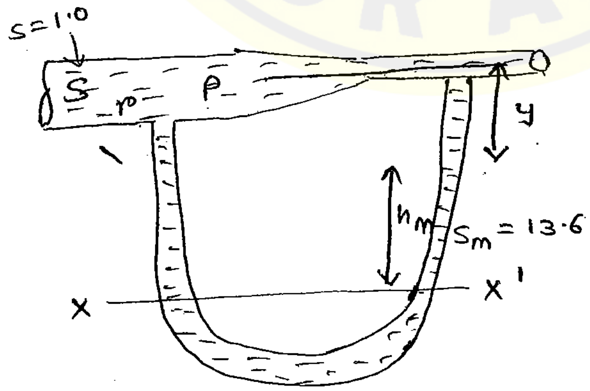
$$\boxed{(\Delta h)_{(1-2)} = h_m\left(\frac{S_m}{S} - 1\right)}$$ → Diff. U-tube manometer pressure b/w two points.

$$\Delta h_{1-2} = h_{mercury}\left(\frac{S_{mercury}}{S_{water}} - 1\right)$$

$$\Delta h_{1-2} = 0.5\ (12.6)$$

$$= 6.3\ m$$

EX:-)



$$P_x = P_{x'}$$

$$P_1 + \rho_1 g(y + h_m) = P_2 + \rho_m g h_m + \rho g y$$

$$P_1 - P_2 = \rho_m g h_m + \rho g y - \rho g y - \rho g h_m$$

$$P_1 - P_2 = g h_m(\rho_m - \rho)$$

$$\rho g(\Delta h_{1-2}) = g h_m(\rho_m - \rho)$$

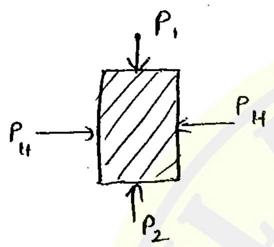
$$\Delta h_{1-2} = h_m\left(\frac{\rho_m}{\rho} - 1\right)$$

$$= h_m\left(\frac{S_m}{S} - 1\right)$$

Pascal's Law:-

It states that for any fluid (liquid or gas) under rest then the fluid pressure at that point is same in all directions

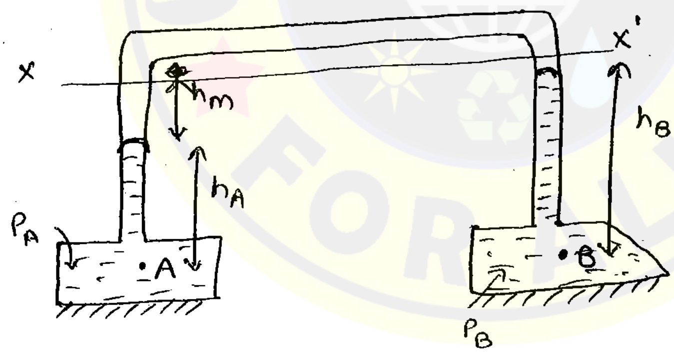
Note:-



$P_{Resultant} = (P_2 - P_1) \uparrow$

Net pressure always Upward

Inverted differential Manometer:- (small differential ~~properties~~ pressure measures)



$\rho_m << \rho_A$ (or) ρ_B

$S_m << S_A$ (or) S_B

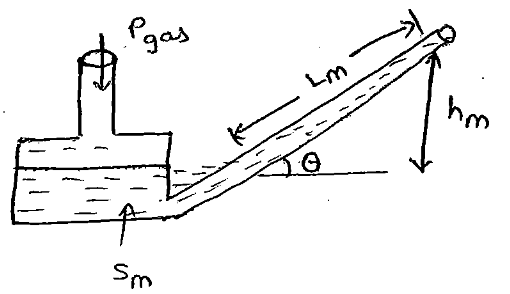
$$P_x = P_{x'}$$

$$P_A - \rho_m g h_m - \rho_A g h_A = P_B - \rho_B g h_B$$

$$P_A - P_B = \rho_m g h_m + \rho_A g h_A - \rho_B g h_B$$

$$= \rho_m g (\Delta h)_{AB}$$

Inclined Manometer:-



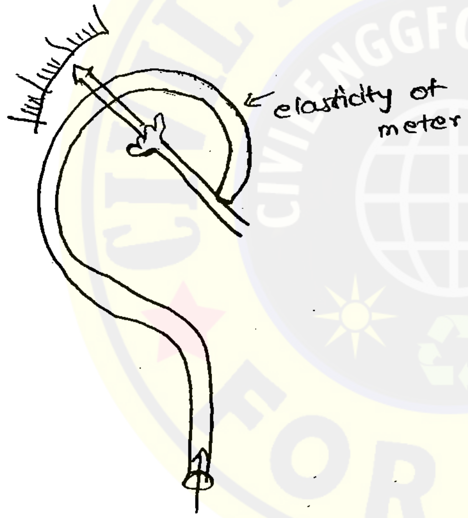
$$P_{gas} = \rho_m g h_m \qquad \sin\theta = \frac{h_m}{L_m}$$

$$P_{gas} = \rho_m g L_m \cdot \sin\theta \qquad h_m = L_m \sin\theta$$

Note:-

~~Me~~ Sensivity of Inclined manometer $= \frac{1}{\sin\theta}$

Mechanical pressure gauge (Bourdan) :-



30 psI → front wheel

40 psI → back wheel

$2.2 \ lb(f) = 1 \ kgf$

1 pound = 0.453 kg

or

(lb(f))

1 atm = kgf/cm^2

~~30~~ psI → $\frac{pound}{Inch^2}$

$= \frac{lb(f)}{In^2}$

$= \frac{0.453 \ kgf}{(2.54)^2 cm^2}$

~~30~~ 1 psI = 0.07 atm

30 psI = $30 \times 0.07 = 2.1$ Atm

40 psI = $40 \times 0.07 = 2.8$ Atm

Transducers:-

→ It converts pressure signals into other signals

→ Transducers are pressure measurement devices.

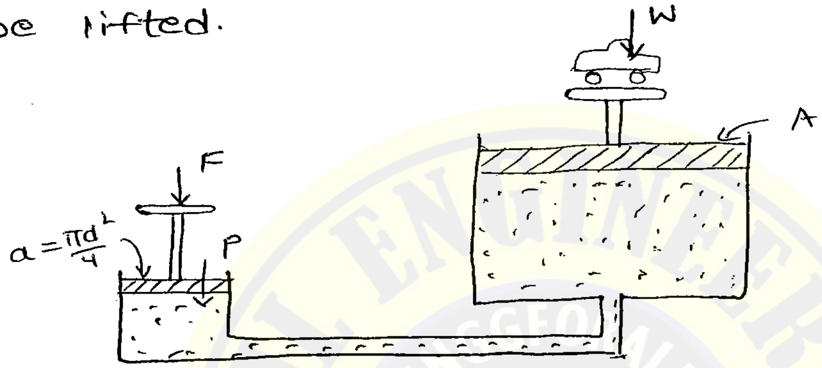
Ex:-) Bellows

Dead weight pressure gauge:-

It is used to check or calloborate the existing or working pressure gauge.

**Hydraulic Jack:-

It is a simple machine which works on pascals law (Hydro static law). It ~~so~~ multiplies the forces. It has mechanica advantage with minimum effort maximum or heavy load can be lifted.



$$P = \frac{F}{a} = \frac{W}{A}$$

$$P = \frac{F}{\frac{\pi d^2}{4}} = \frac{W}{\frac{\pi D^2}{4}}$$

$$P = \frac{F}{d^2} = \frac{W}{D^2}$$

$$\boxed{F = W\left(\frac{d}{D}\right)^2}$$

P.g No:- 35

22. $F = W\left(\frac{d}{D}\right)^2$

$= 100\left(\frac{1}{10}\right)^2$

$= 1$ KN

26. $h = h_{Hg}\left(\frac{S_m}{S} - 1\right)$

$= 8\left(\frac{13.6}{0.8} - 1\right)$

$= 128$ cm of oil

$= 1.28$ m of oil.

28. $(\Delta h)_{A-B} = 0 \cdot 6 \left(\frac{13 \cdot 6}{1 \cdot 0} - 1 \right)$

$= 6 \times 12 \cdot 6$

$= 7 \cdot 56$ m of water.

29.

$P_{vessel} = 50$ cm of Hg vaccuum $= \frac{P_{vessel}}{\rho_{Hg} \cdot g}$

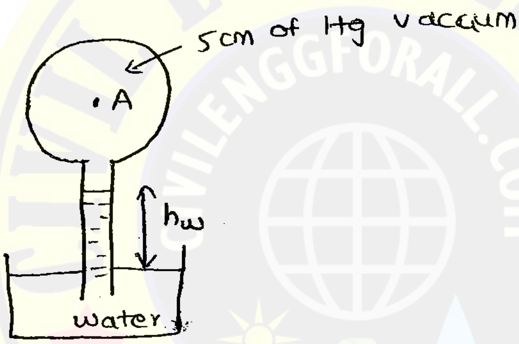
$= -50$ cm of Hg

$P_{vessel} = \rho_{Hg}\, g\, h_{Hg}$

$\frac{P_{vessel}}{\rho_{Hg}\, g} = h_{Hg}$

50 cm of Hg $= h_{Hg}$

30. Ex:-)



$P_A = \rho_{Hg}\, g\, h_{Hg} = \rho_w \cdot g \cdot h_w$

$S_{Hg} \cdot h_{Hg} = S_w \cdot h_w$

$13 \cdot 6 \times 5 = 1 \cdot 0 \times h_w$

$h_w = 68$ cm

$\therefore$ 1 cm of Hg $= 13 \cdot 6$ cm

30. $P_p = 26$ cm of Hg vacuum

$= -26$ cm of Hg (gauge).

$P_{p_{abs}} = P_{atm} + P_{gauge}$

$= 76 + (-26)$

$= 50$ cm of Hg (abs)

CHAPTER

FLUID STATICS (Hydrostatic force on different Surfaces)

Hydrostatic is a branch of fluid mechanics deals fluid behaviour under rest. There are two forces considered in fluid statics.

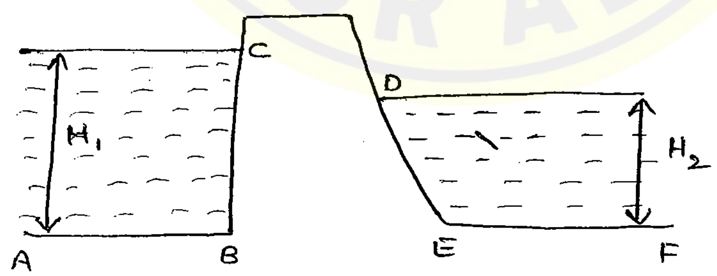
1. Gravity force ($W = mg$)
   (self wt. of fluid)
2. Force due to pressure ($F = P \times A$)
   Neglects viscous force compression effect.

Hydrostatic force on different surfaces :-

1. plain surfaces
2. Curved surfaces

Types of plain surfaces:-

1. Horizontal surface
2. verticle surface
3. Inclined Surface



Horizontal plane surface = AB

Verticle plane surface = BC

Inclined plane surface = DE

→ Hydrostatic force (or) pressure force (or) Thrust (or) Normal force of static fluid (or) Total pressure.

→ Units are 'N'

→ Force of rest fluid perpendicular or Normal to wetted surface.

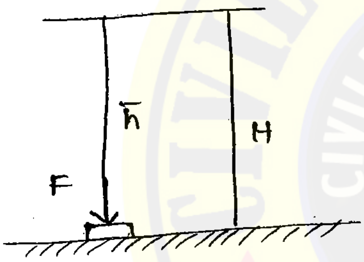
$$F = P\,A_{wetted}$$

$$F = \rho g \bar{h}\, A_{normal}$$

where

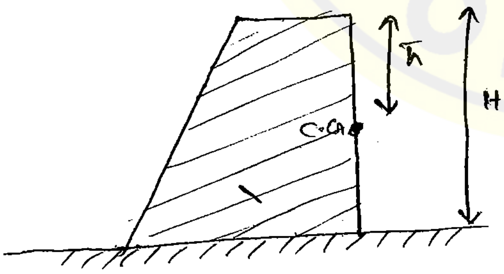
$\bar{h}$ =

Ex:- $\bar{h}$ for Horizontal plane surface.

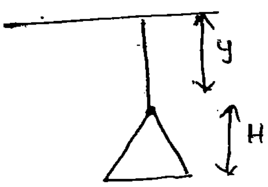


$H = \bar{h}$

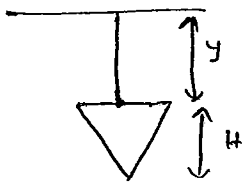
Ex:- $\bar{h}$ for verticle plane surface.



where $\frac{H}{2} = \bar{h}$.



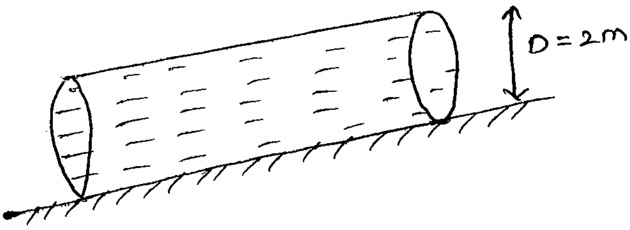
$\bar{h} = y + \frac{2}{3}H$

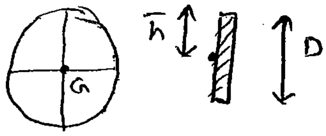


$\bar{h} = y + \frac{1}{3} \cdot H$

P.g No:- 40

14.





$F = \rho g \bar{h} A$

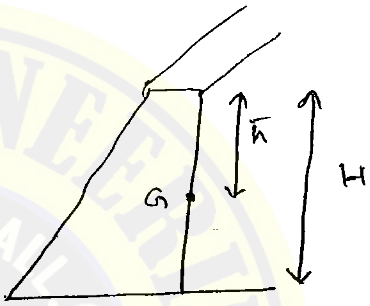
$= 1000 \times 9.81 \times \frac{D}{2} \times \frac{\pi}{4} D^2$

$= 1000 \times 9.81 \times 3.14$

$F_{end\ plate} = 30.76$ KN

P.g No:- 42

43. $F = \rho g \bar{h} A$

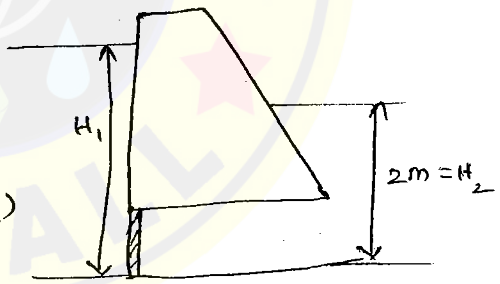


$= \rho g \cdot \frac{H}{2} (L \times H)$

$F = w \cdot \frac{H^2}{2}$

P.g No:- 39

4. $F_{net} = F_1 - F_2$



$= \rho g \frac{H_1}{2}(L \cdot H_1) - \rho g \frac{H_2}{2}(L H_2)$

$\rho g A \bar{h}_1 - \rho g A \bar{h}_2$

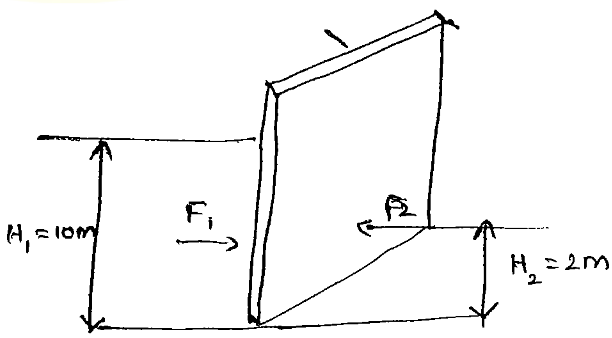
$1000 \times 10 \times (3\times2)(10+\frac{2}{2}) - 1000\times10\times(3\times2) \times (2+\frac{2}{2})$

$= \frac{\rho g L}{2}(H_1^2 - H_2^2)$

$= \frac{1000\times10\times}{2}(10^2 - 2^2)$

$= 480$

$= 480$ kN



P.g No:- 35

31. $F = \rho g \bar{h} A$

$= 1000 \times 9.81 \times H \times \frac{\pi D^2}{4}$

$= 9810 \times 1 \times \pi$

$= 3140$ kg

P.g No:- 38

20. $F = \rho g A \bar{h}$

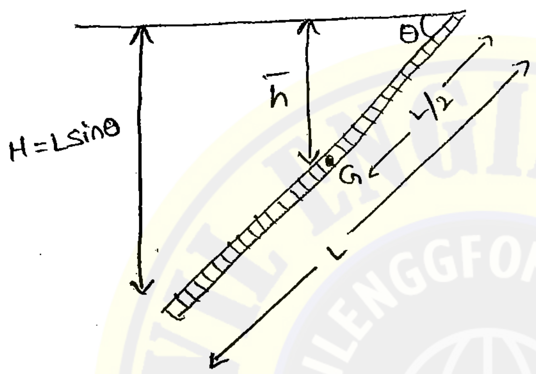
$= \rho g \bar{h} \frac{\pi(D_1^2 - D_2^2)}{4}$

$= \rho g \bar{h} \pi (R_1^2 - R_2^2)$

$= 1000 \times 4 \times 3.14 (2^2 - 1^2)$

$= 12000 \pi$

Hydrostatic force on Inclined surface:-



$H = L\sin\theta$, $H/2 = \bar{h}$

$\bar{h} = \frac{L}{2}\sin\theta$

$F = \rho g A \bar{h}$

$F = \rho g (L \times B)(\frac{L}{2}\sin\theta)$

P.g No:- 36

37. $F = \rho g A \bar{h}$

$F = 1000 \times 10 \times 1.625 \times \frac{\pi}{4} (1.2^2 - 0.8^2)$

$F = 10000\ N$

$F = 10\ kN$

13. $F = \rho g A \bar{h}$

$= \rho g (B \times L) (\frac{L}{2}\sin\theta)$

$= \rho g (1 \times 1) \left(\frac{1 \times \sin 45}{2}\right)$

$= \rho g \left(\frac{1}{\sqrt{2} \times 2}\right)$

$= \frac{\rho g}{2\sqrt{2}}$

Hydrostatic force characteristics:-

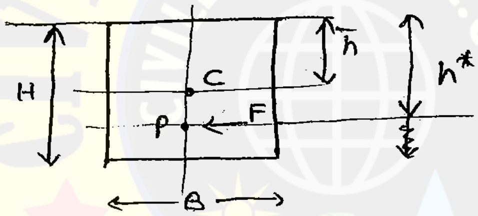
1. It has magnitude equal to $\rho g \bar{h} A$
2. It has a point of action (or) application. (or) point of concentration is known as centre of pressure (or) pressure centre. $(h^* = h_c = h_{cp})$
3. It is always at centroid of pressure diagram of the static fluid on the surface. (or) by analytical method

$$\boxed{h^* = \bar{h} + \frac{I \sin^2\theta}{\bar{h} A}}$$

For horizontal surface $\theta = 0$, $h^* = \bar{h}$

For vertical surface, $\theta = 90^\circ$ $h^* = \bar{h} + \frac{I}{\bar{h} A}$

Ex:-) Rectangle



$$A = B \times H$$

$$\bar{h} = \frac{H}{2}$$

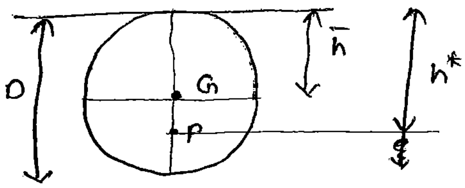
$$h^* = \bar{h} + \frac{I}{A\bar{h}}$$

$$= \frac{H}{2} + \frac{\frac{BH^3}{12}}{\frac{H}{2} \times B \times H}$$

$$h^* = \frac{H}{2} + \frac{H}{6}$$

$$\boxed{h^* = \frac{2H}{3}}$$

Ex:- Circle



$$\bar{h} = D/2$$

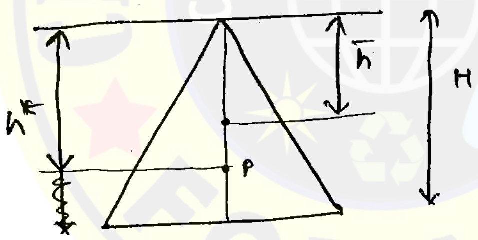
$$A = \frac{\pi D^2}{4}, \quad I = \frac{\pi D^4}{64}$$

$$h^* = \bar{h} + \frac{I}{A\bar{h}}$$

$$= D/2 + \frac{\pi D^4/64}{\frac{D}{2} \cdot \left(\frac{\pi D^2}{4}\right)}$$

$$\boxed{h^* = \frac{5D}{8}}$$

Ex:- Triangle:-



$$\bar{h} = \frac{2}{3}H$$

$$A = \frac{1}{2}BH$$

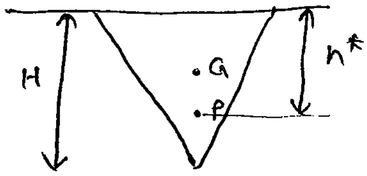
$$I = \frac{BH^3}{36}$$

$$h^* = \bar{h} + \frac{I}{A\bar{h}}$$

$$= \frac{2}{3}H + \frac{BH^3/36}{\frac{BH}{2}\left(\frac{2}{3}H\right)}$$

$$\boxed{\begin{aligned} h^* &= \frac{3}{4}H \\ h^* &= 0.75H \end{aligned}}$$

Ex:- Inverted triangle:-



$$\bar{h} = \frac{1}{3} H$$

$$A = \frac{1}{2} B H$$

$$I = \frac{B H^3}{36}$$

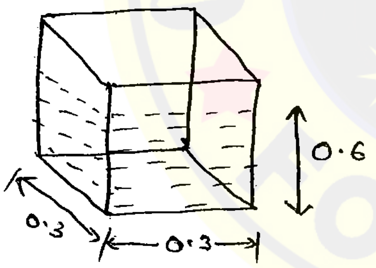
$$h^* = \bar{h} + \frac{I}{A \bar{h}}$$

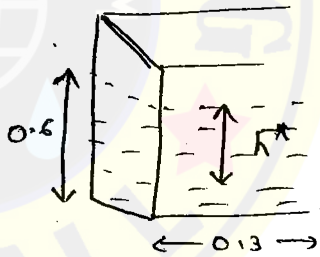
$$= H/3 + \frac{BH^3/36}{\frac{BH}{2}\left(\frac{H}{3}\right)}$$

$$\boxed{h^* = \frac{H}{2} = 0.5H}$$

Pg NO:- 34

17.





$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times \frac{H}{2} \times (B \times H)$$

$$= 9810 \times \frac{0.6}{2} \times (0.3 \times 0.6)$$

$$= 529.2 \text{ N}$$

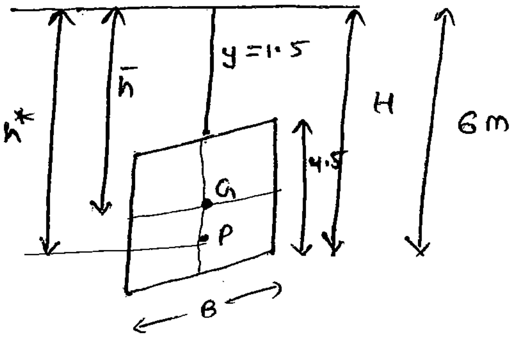
$$h^* = \frac{2}{3} H$$

$$= \frac{2}{3} \times 0.6$$

$h^* = 0.4$ m from top

$= 0.2$ m from bottom.

24.



$$\bar{h} = y + \frac{H}{2}$$

$$= 1.5 + \frac{4.5}{2}$$

$$= 3.75 \text{ m}$$

$$A = B \times H$$

$$= 4.5 B$$

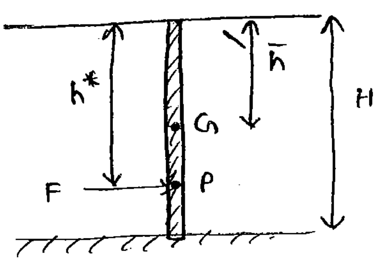
$$I = \frac{BH^3}{12} \Rightarrow \frac{4.5^3 B}{12}$$

$$h^* = \bar{h} + \frac{I}{A\bar{h}}$$

$$= 3.75 + \frac{\frac{B \times 4.5^3}{12}}{4.5 B (3.75)}$$

$$= 4.2 \text{ m}$$

Moment of Hydrostatic force on a rectangular plate:-



Rectangle plate.

$$F = \rho g \frac{H}{2} (BH)$$

$$h^* = \frac{2}{3} H$$

Moment of force 'F' about top $= F \times h^*$

$$= \rho g \frac{H}{2} (BH) \times \frac{2}{3} H$$

$$F = \frac{1}{3} \rho g B H^3 \ (N\text{-}m)$$

23. Force $= \frac{1}{3} \rho g B H^3 (N-m)$

$= \frac{1}{3} \rho \times 1 \times BH^3 \; (kgf \cdot m)$

$= \frac{1}{3} \times 1000 \times 3 \times 3^3$

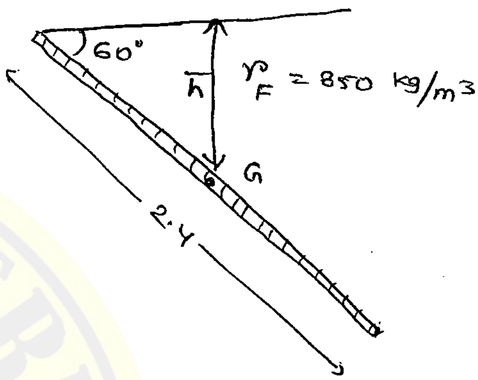
$= 27000 \; kgf-m$

~~Combination of Horizontal and Verticle plates:-~~

P.g NO:- 36.

36.

$F_{\text{normal on inclined plane}} = \rho g A \bar{h}$



$= (0.85 \times 1000)(9.81)\left(\frac{L}{2} \sin\theta\right)$

$(L \times B)$

$= 850 \times 9.81 \times \frac{2.4}{2} \sin 60°$

$\times (2.4 \times 0.75)$

$= 15.59 \; KN$

centre of pressure $h^* = h_{cp} = \bar{h} + \frac{I \sin^2\theta}{A \bar{h}}$

$= \frac{L \sin\theta}{2} + \frac{\frac{BL^3}{12} \times (\sin 60)^2}{\frac{L \sin\theta}{2} \times (\cancel{B} \times L)}$

$= \frac{2.4 \sin 60}{2} + \frac{(2.4)^3 / 12 \, (\sin 60)^2}{\frac{2.4 \sin 60}{2} \times 2.4}$

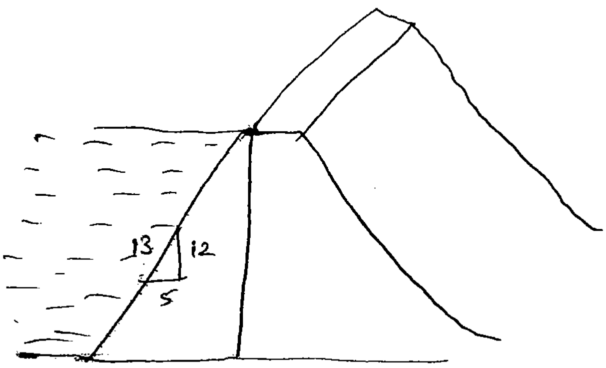
P.g NO:- 39

2. $P = \rho g \bar{h}$

$= 1000 \times 9.81 \times \frac{13}{2}$

$= 63.7 \times 10^3 \; N/m^2$

$= 63.7 \; kN/m^2$

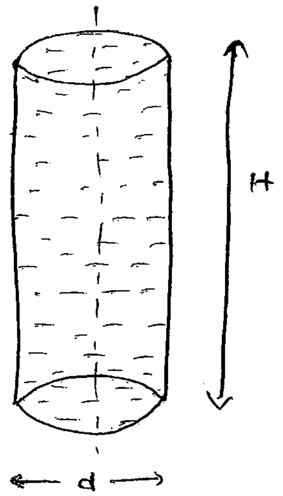


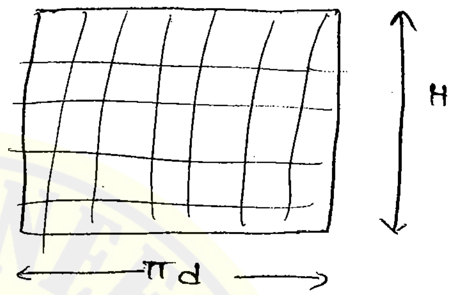
19. $F_{bottom} = F_{verticle\ surface}$

$$(\rho g \bar{h} A)_{bottom} = [\rho g \bar{h} A]_{verticle}$$

$$H \frac{\pi}{4} \cdot d^2 = \frac{H}{2} \pi d \cdot H$$

$$\boxed{H = \frac{d}{2}}$$

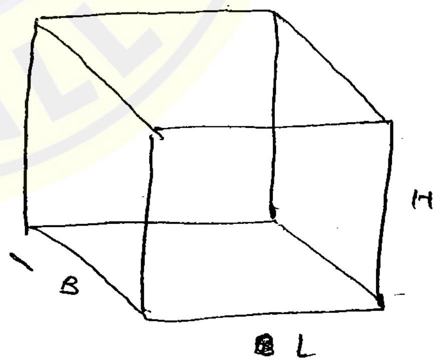




Ex:-) Consider a rectangular water tank of inner dimensions L×B×H is filled with a liquid fully. Determine relation b/w height H, and ~~length~~ width 'B'. If hydrostatic force on floor equal to Hydrostatic force on Large verticle wall.

A) $F_{bottom} = F_{Large\ wall}$

$$\rho g A_{bottom} \bar{h}_{bottom} = \rho g \bar{h}_{large\ wall} A_{large\ wall}$$



$$H \cdot (B \times L) = \frac{H}{2} (L \times H)$$

$$B = \frac{H}{2}$$

$$\frac{H}{B} = 2$$

Ex:-2) For the above question data. Determine H and L if hydrostatic force on bottom equal to that on small verticle wall.

A) $F_{bottom} = F_{small\ wall}$

$$\rho g \bar{h}_{bottom} A_{bottom} = \rho g \bar{h}_{\substack{small \\ wall}} A_{\substack{small \\ wall}}$$

$$H \times (L \times B) = \left(\frac{H}{2}\right) \times (B \times H)$$

$$L = \frac{H}{2}$$

$$\boxed{\frac{H}{L} = 2}$$

Ex:-3) A rectangular tank whose dimensions L : B : H = 2 : 1 : 2. Determine the ratio of hydrostatic force on bottom to that on large wall.

A) $F_{bottom} = F_{large\ wall}$

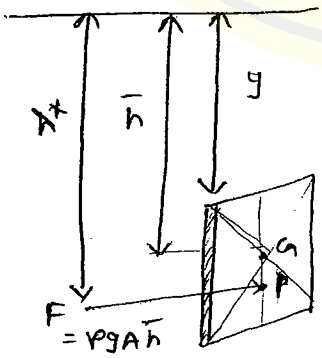
$$\cancel{\rho g}\, \bar{h}_{bottom} A_{bottom} = \cancel{\rho g}\, \bar{h}_{\substack{large \\ wall}} \cdot A_{\substack{large \\ wall}}$$

$$H\,(L \times B) = \frac{H}{2}\,(L \cdot H)$$

$$= \frac{2B}{H}$$

P.g. No:-36.

38.



If $\bar{h}$ is not ~~no~~ changed then force is also not change.

Note:- Hydrostatic force on any plate surface is function of sp. wt. of the fluid, wetted surface area and its verticle depth from free surface to its centroid. It is independent upon surface orientation (angular position)

Pressure diagram of a static fluid on the verticle surface.

By hydrostatic law pressure varies linearly with its depth.

$$P = \rho g h$$
$$= \gamma h$$
$$= w h$$
$$dp = w \cdot dh$$
$$\frac{dp}{dh} = w$$

Strictly speaking

$$\frac{dp}{dh} = -w$$
$$= -\gamma$$
$$= -\rho g$$

$-ve$ indicates that pressure decreases when heigh increases

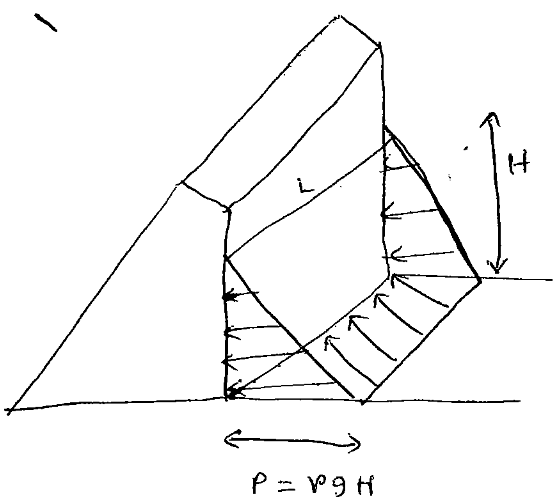
Ex:- At M.S.L $(+0.000)$ $P_{atm} = 101$ kpa

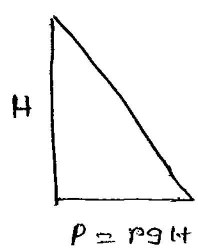
At Hyd M.S.L $(+380.00)$ $P_{atm} = 99.5$ kpa

If depth increases pressure increases

$$\frac{dp}{dh} = -(-\gamma)$$
$$= \gamma$$

$$P \propto h$$





Area of pressure diagram indicates $= \frac{1}{2} \times$ Base $\times$ height (30)

$$= \frac{1}{2} \times P \times H$$

$$= \frac{1}{2} \times \rho g H \times H$$

$$F = \rho g \frac{H}{2} \cdot H$$

Area of pressure diagram indicates or represents

→ Hydrostatic force on wetted surface per unit Area

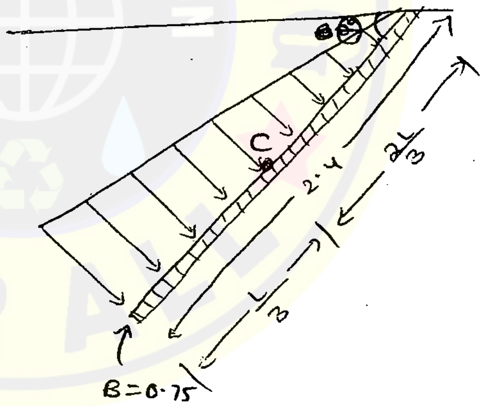
$$\frac{F}{L} = \frac{1}{2} \rho g H \cdot H$$

$$= \frac{w \cdot H^2}{2}$$

$$\frac{F}{L} = \frac{\gamma H^2}{2}$$

Ex:-) Determine the centre of pressure of an inclined plate shown in fig.

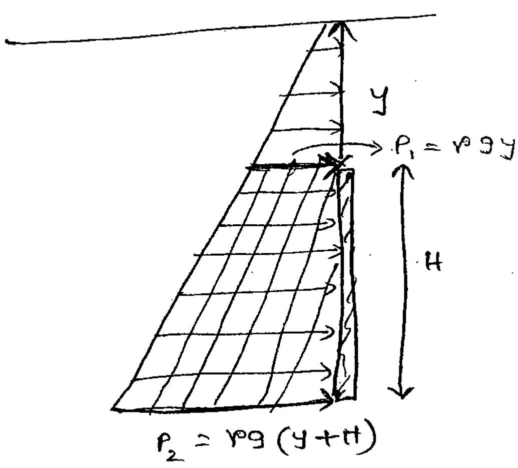
A) $\sin\theta = \dfrac{h^*}{\frac{2L}{3}}$



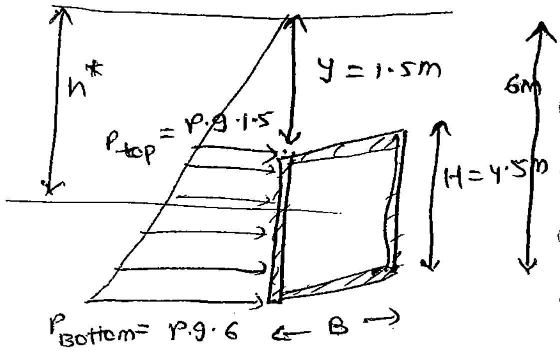
$$h^* = \frac{2 \times 2 \cdot 4}{3} \sin 60^\circ$$

$$h^* = 1 \cdot 39\ m$$

Ex:-2)



33. $h^* = \bar{h} + \dfrac{I \sin^2\theta}{A\bar{h}}$



$A = B \times H$

$= B \times 4.5$

$I = \dfrac{BH^3}{12} = \dfrac{B \times 4.5^3}{12}$

$\bar{h} = y + \dfrac{H}{2}$

$= 1.5 + \dfrac{4.5}{2}$

$= 3.75 m$

$h^* = 3.75 + \dfrac{\frac{4.5^3 B}{12} \sin^2(90)}{4.5 B \times 3.75}$

$= 3.75 + 0.45$

$= 4.2 \ m$

(or) $h^* = y + \bar{y}$

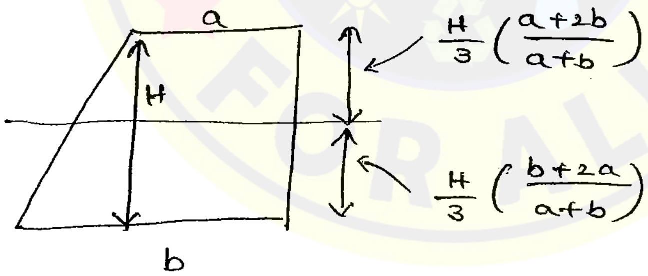
$= 1.5 + \dfrac{H}{3}\left[\dfrac{P_{Top} + 2P_{Bottom}}{P_{Top} + P_{Bottom}}\right]$

$= 1.5 + \dfrac{4.5}{3}\left[\dfrac{1.5 + 2 \times 6}{1.5 + 6}\right]$

$= 1.5 + 2.7$

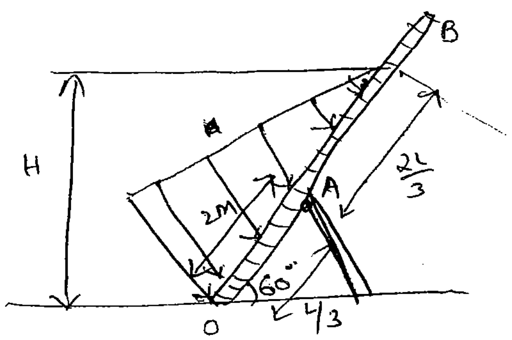
$= 4.2 \ m$

Note:-



Ex:-) Determine at what depth of water causes a rectangular gate tilts.

A)

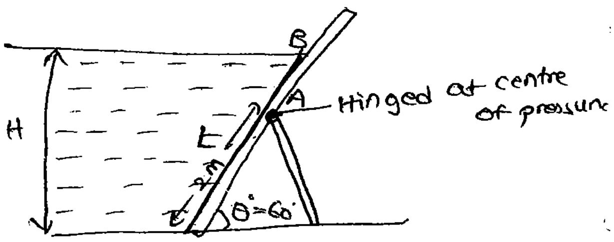


$\theta = 60°$

$H = ?$

$OA = 2m$

$OB = L$



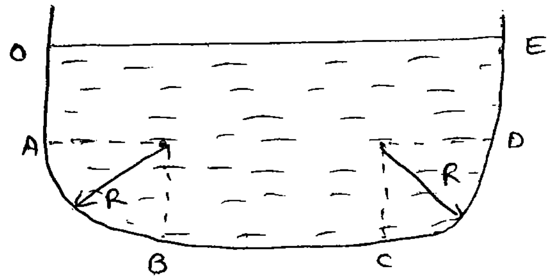
$\dfrac{L}{3} = 2 \Rightarrow L = 6 \ m$

$\sin\theta = H/L$

$H = 5.2m$

Hydrostatic force on curved surface:- (31)

Curved surface is one whose point of rotation is available and it is equal distance from that point.



There are two hydrostatic force components on the curved surface.

1. Horizontal component (F_H)
2. verticle component. (F_V)

Resultant of the hydrostatic force $= \sqrt{F_H^2 + F_V^2}$

1 TMC water $= 10^9 \; ft^3$

$1 \times 1000 \times 10^6 \; ft^3$

$= 10^9 \; ft^3$

1 ft = 12 Inch

$= 12 \times 2.54$ cm

$= 12 \times \frac{2.54}{100}$ m

$1 \; ft^3 = (0.3028)^3 \; m^3$

$= 0.028 \; m^3$

$= 0.028 \times 1000$ lit

$1 \; ft^3 = 28$ lit

1 TMC $= 10^9 \; ft^3$

$= 10^9 \times 28$ lit

1 TMC $= 28 \times 10^9$ lit

For 10,000 population,

1 person = 150 lit/day

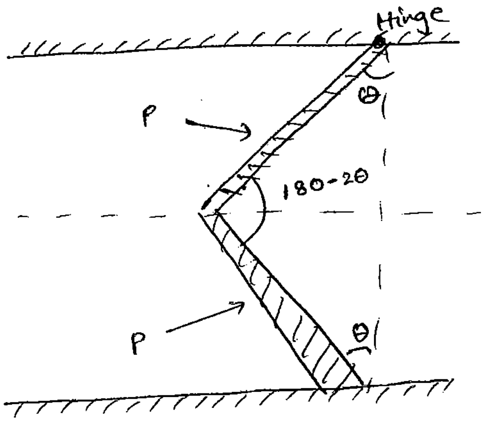
$1 \text{ TMC} = \frac{28 \times 10^9}{10000 \times 150 \times 365}$

1 TMC = 52 years.

cumec $= m^3/sec$

cusec $= ft^3/sec$

* Lock gates:-



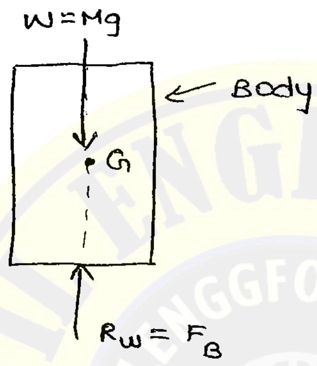
CHAPTER-3

BUOYANCY

When a rigid body touch with a fluid three posibilities based on fluid mass density and rigid body mass density.

1. Fully submerged
2. partly submerged
3. Fully floated

Free body diagram of a floating body :-



Under equilibrium

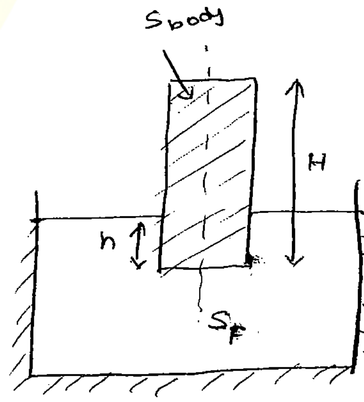
$$\downarrow W_{Body} = F_B \uparrow$$

$$M_{body} \cdot g = \text{weight of the fluid displaced}$$

$$M_{body} \cdot g = W_{fluid\ displaced}$$

$$M_{body} \cdot g = M_{fluid\ displaced} \cdot g$$

$$\rho_{body} \cdot \upsilon_{body} = \rho_{fluid} \cdot \upsilon_{fluid\ displaced}$$



$$\frac{\rho_{body} \cdot \upsilon_{body}}{\rho_{water}} = \frac{\rho_{fluid}}{\rho_{water}} \cdot \upsilon_{fluid\ displaced}$$

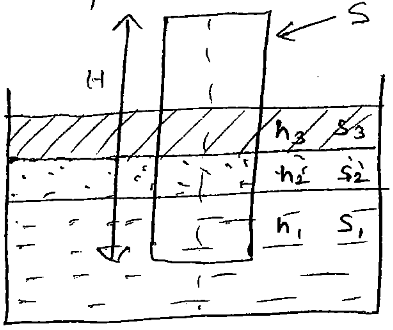
$$S_{body} \cdot \upsilon_{body} = S_{fluid} \cdot \upsilon_{fluid\ displaced}^{\leftarrow wetted}$$

$$S_{body} \cdot \overset{A_{c/s} \cdot H}{\cancel{\upsilon_{body}}} = S_{fluid} \cdot A_{c/s} \cdot h$$

$$\boxed{S_{body} \cdot H = S_{fluid} \cdot h}$$

Area of c/s are same for prisms and cylinders.

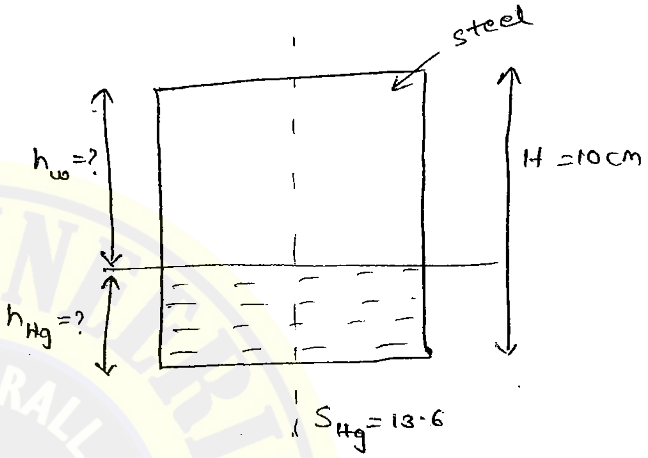
One body Multiple fluid:-



$$S_1 h_1 + S_2 h_2 + S_3 h_3 = S_{body} \cdot H$$

P.g No:- 50

*11. $S_{steel} = 7.6$



$H = h_{Hg} + h_w$

$10 = h_{Hg} + h_w$

$S_{Hg} \cdot h_{Hg} + S_w h_w = S_{steel} \cdot H$

$13.6 (10 - h_w) + 1.0\, h_w = 7.6 \times 10$

$h_w = 4.76$ cm

P.g No:- 53

8. $S_{wood} = 0.75$

$L = 5m$

$B = 2m$

$h = 3m$

$S_{fluid} \cdot V = S_{wood} \cdot V$

$1.0 \cdot V = 0.75 \ (5 \times 2 \times 3)$

$V = 22.5 \ m^3$

13. $\rho_{wood} \cdot V_{\cancel{wood}\ total} = \rho_{fluid} \cdot V$

$800 (3 \times 2 \times 2) = 1000 \times V$

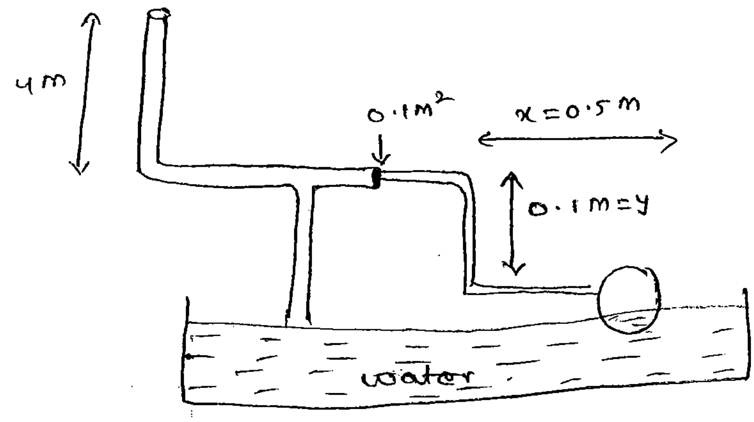
$V = 9.6 m^3$

3. $S_{Body} \cdot H = S_{fluid} \cdot h$

$S_{Body} \cdot 1 = 1 \times 0.81$

$S_{body} = 0.81$

Ex:-) Determine the wetted volume of the plastic ball shown in fig



$F = \rho g \bar{h} A$

$F = \rho g \times 4 \times 0.1$

$= (\rho g \times H \times A_{value}) \cdot y = \rho g v \cdot x$

$= \cancel{\rho g \cdot v \cdot x} \; H \times A_{value} \times y = v \cdot x$

$4 \times 0.1 \times 0.1 = v \times 0.5$

$v = 80m^3$

~~P.g No:- 37~~

Ex:-) A wooden made cylinder of dia 2m and height 4m and relative density of wood is 0.6. The cylinder is kept vertical in a fluid of relative density is 0.8. Take g = 10m/s. Determine the following.

i) Mass of the cylinder in kgs.

ii) Depth of the immersion of cylinder in fluid

iii) volume of the fluid displaced in m^3.

iv) Buoyancy force of the fluid on the cylinder in KN.

v) Gauge pressure at bottom of the immersed cylinder in k-pa

vi) Fraction of the cylinder in air.

vii) How much extra or aditional mass is to be kept on the cylinder such that cylinder top surface coincide with liquid free surface

viii) centre of buoyancy

ix) Different b/w Centre of Gravity (G) and centre of Buoyancy.

A) $S_{wood} = \dfrac{\rho_{wood}}{\rho_{water}}$

$0.6 = \dfrac{\rho_{wood}}{1000}$

$\rho_{wood} = 600 \ kg/m^3$

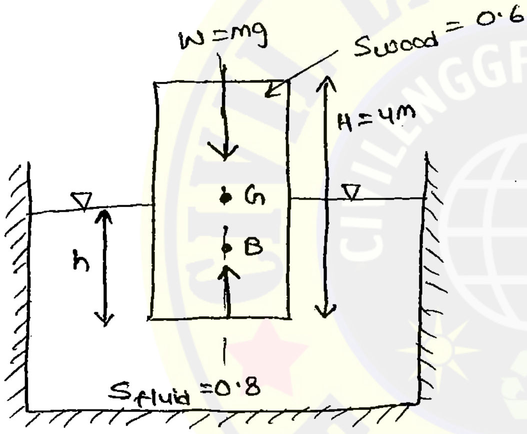
i) $\rho_{wood} = \dfrac{M_{wood}}{V_{wood}}$

$600 = \dfrac{M_{wood}}{\frac{\pi}{4}(2)^2 \times 4}$

$M_{wood} = 7540 \ kg$

$= 7.54$ tonnes

ii) Depth of Immersion (h) :-



Let W_{wood} = self wt. of water

F_B = Buoyancy force.

Buoyancy force :-

It is the reactive forced offered by the fluid immersed or submerged rigid body in N. It is always acts vertically upward (opposite to self wt. of the body). It is passing through the centre of gravity of the fluid volume displaced. Its magnitude always equal to weight of the fluid displaced. It is function of mass density of fluid and not mass density of the rigid body.

F_B = Buoyancy force = weight of the liquid displaced

$$F_B = W_{liquid\ displaced}$$

$$F_B = M_{fluid\ displaced} \cdot g \quad (\text{Newtons})$$

$$= \gamma_{fluid} \times V_{displaced}$$

$$= \rho_{fluid} \times A_{c/s} \times \text{Depth of Immersion}$$

under equilibrium vertically

$$W_{body} \downarrow = F_B \uparrow$$

$$S_{body} \times H = S_{fluid} \times h$$

$$0.6 \times 4 = 0.8 \times h$$

$$h = 3\,m$$

iii) $V_{body} = V_{fluid\ displaced}$

$$= A_{c/s} \times h$$

$$= \frac{\pi D^2}{4} \times h$$

$$= \frac{\pi (2)^2}{4} \times 3$$

$$= 9.42\,m^3$$

iv) Buoyancy force $F_B = W_{wood}$

$$= 7540 \times 10$$

$$= 75400\,N$$

$$= 75.4\,kN$$

(or)

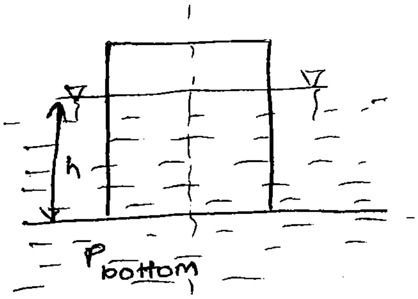
$$F_B = W_{wood}$$

$$\rho_{fluid} \cdot g \cdot V = W_{wood}$$

$$800 \times 10 \times 3\pi$$

$$= 75.4\,kN$$

v)



$$P_{bottom\ cylinder} = \rho_F \cdot g \cdot h$$

$$= 800 \times 10 \times 3$$

$$= 2400\,N/m^2$$

$$= 24\,kN/m^2 \quad \text{(or)}$$

$$P = \frac{F}{A} = \frac{W}{A} = \frac{F_B}{A} = \frac{75.4\,kN}{\frac{\pi (2)^2}{4}} = 24\,kN/m^2$$

vi) Fraction of cylinder in air :-

Fraction with which cylinder above the fluid

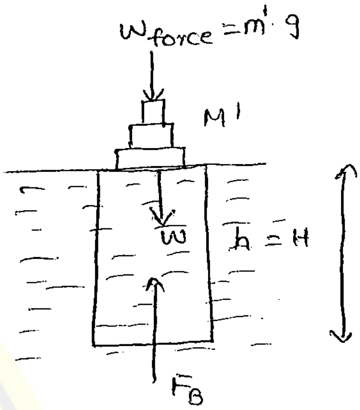
$$= \frac{1m}{4m} = \left(\frac{H-h}{H}\right)$$

$$= 0.25$$

$$= 25\%$$

Fraction of body in fluid $= \frac{3m}{4m} = 0.75 = 75\%$

vii) F.B.D after m' placed on cylinder



$$W_{extra} + W_{wood} = F_B'$$

$$M' \times 10 + 75400 = \rho_{fluid} \times g \cdot V$$

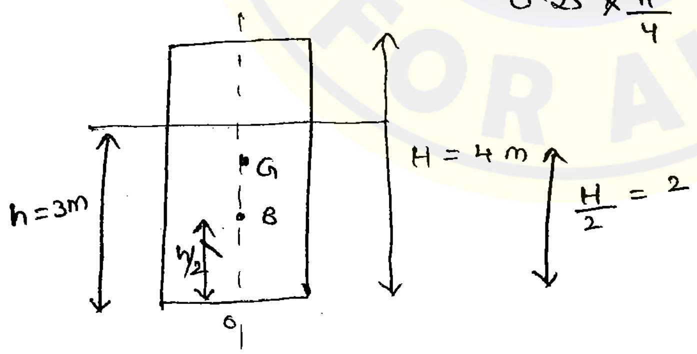
$$= 800 \times 10 \times A_{c/s} \times (h = H)$$

$$M' \times 10 + 75400 = 800 \times 10 \times \frac{\pi}{4}(2)^2 \times 4$$

$$M' = 2513 \text{ kg.}$$

viii) centre of buoyancy (B) :- ~~$= 0.25 \times V \times \rho_{fluid} = m'$~~

~~$0.25 \times \frac{\pi}{4}$~~



$$BG = OG - OB$$

$$= \frac{H}{2} - \frac{h}{2}$$

$$= \frac{4}{2} - \frac{3}{2}$$

$$= 2 - 1.5$$

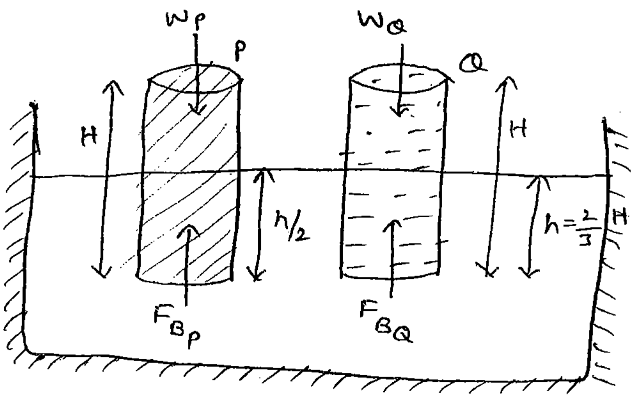
$$= 0.5 m$$

centre of buoyancy from the bottom $= 0.5\,m$.

ix) Distance b/w B & G $= 0.5\,m$.

p.g NO:- 54.

16.



$$S_P \times H_P = S_{fluid} \times h_P$$

$$S_P \times H_P = S_f \times \frac{H_P}{2}$$

$$S_{Fluid} = 2\,S_P \longrightarrow (1)$$

$$S_Q H_Q = S_{fluid} \times h_Q$$

$$S_Q \cdot H_Q = S_{fluid} \times \frac{2}{3} \cdot H_Q$$

$$S_Q = S_{fluid} \times \frac{2}{3}$$

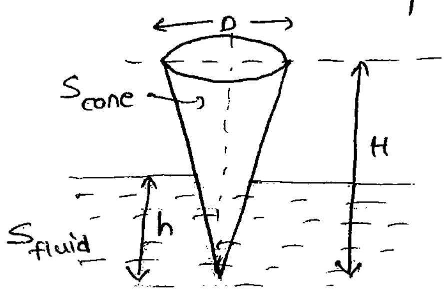
$$S_{fluid} = \frac{2}{3} \cdot S_Q \longrightarrow (2)$$

eq (1) = (2)

$$S_{fluid} = 2 S_P = \frac{2}{3} S_Q$$

$$\frac{S_P}{S_Q} = \frac{3}{4} = 0.75$$

Ex:-) Depth of expression for depth of Immersion of a solid cone in fluid floated with apex downward.



A) $S_{cone} \cdot H = S_{fluid} \cdot h$

$$\frac{D}{H} = \frac{d}{h}$$

$$h = H \cdot \sqrt[3]{\left(\frac{S_{cone}}{S_{fluid}}\right)}$$

$$\frac{R}{H} = \frac{r}{h}$$

$$\frac{R}{r} = \frac{H}{h} \Rightarrow \frac{R^2}{r^2} = \frac{H^2}{h^2}$$

$$W_{cone} \downarrow = F_{B_{fluid}} \uparrow$$

$$M_{cone} \cdot g = M_{fluid} \cdot g$$

$$\frac{\rho_{cone}}{\rho_w} \cdot V_{cone} = \frac{\rho_{fluid}}{\rho_w} \times V_{fluid\ displaced}$$

$$\rho_{cone} \cdot V_{cone} = \rho_{fluid} \cdot V$$

$$S_{cone} \cdot \frac{1}{3} \cdot \pi R^2 \cdot H = S_{fluid} \cdot \frac{1}{3} \pi r^2 \cdot h$$

$$S_{cone} \cdot R^2 H = S_{fluid} \cdot r^2 \cdot h$$

$$S_{cone} \cdot \frac{R^2}{r^2} \cdot H = S_{fluid} \cdot h$$

$$S_{cone} \left(\frac{H^2}{h^2}\right) \cdot H = S_{fluid} \cdot h$$

$$S_{cone} \cdot H^3 = S_{fluid} \cdot h^3$$

$\cancel{S_{cone} \cdot H}$ $$\boxed{h = H \sqrt[3]{\frac{S_{cone}}{S_{fluid}}}}$$

P.9 NO:- 50

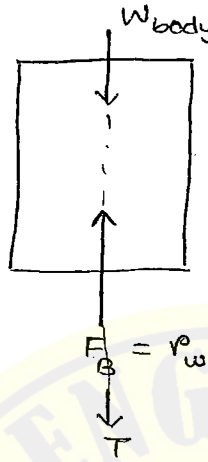
13. $S_{body} \cdot V = S_{fluid} \cdot V_{wetted}$

$$\frac{S_{body}}{S_{fluid}} = \frac{V_{wetted}}{V}$$

$$\frac{3.4}{13.6} = \frac{1}{4} = 0.25 = \frac{V_{wetted}}{V}$$

Metacentric height of floating body in a fluid:- 37

Let a prismatic body of material S_{body}, cross sectional area A, and its height H is kept in a fluid of specific gravit S_{fluid} connected to a tension rope. Determine tension in 'p' in terms of above variables.



$$W_{body} = M_{body} \cdot g = \rho_{body} \cdot \vartheta_{body} \cdot g$$

$$= S_{body} \cdot \rho_{water} \cdot A \cdot H \cdot g.$$

$$F_B = \rho_{water} \cdot \vartheta g = \rho_{water} \cdot A \cdot h \cdot g$$

Under equilibrium vertically

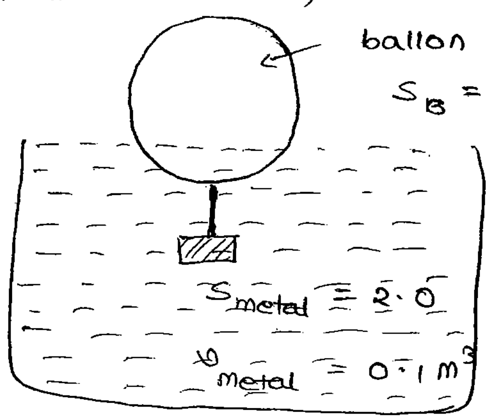
$$\boxed{T = \rho_w A g [h - S_{body} \cdot H]}$$

$$W_{body} + T = F_B$$

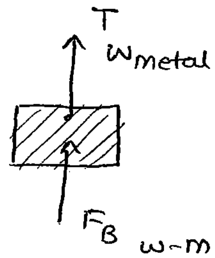
$$S_{body} \cdot \rho_w \cdot A H g + T = \rho_w A h g$$

$$T = \rho_w A g [h - S_{body} \cdot H]$$

Ex:- Two bodies connected by a rope and kept in water show in fig. Determine wetted volume of the plastic button. Find. i) Tension in ballon, ii) volume of Ballon in water



A. F.B.D of metal :-

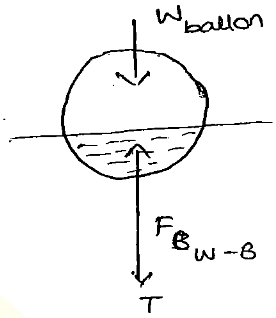


$$W_{metal} = T + F_B$$

$$\rho_{metal} \cdot V \cdot g = T + \rho_{water} \cdot V \cdot g$$

$$2000 \times 0.1 \times 10 = T + 1000 \times 10 \times 0.1$$

$$T = 1000\ N$$



$$W_{ballon} + T = F_{B_{w-B}}$$

$$\rho_{ballon} \cdot V \cdot g + T = \rho_{water} \cdot g \cdot V_{ballon}$$

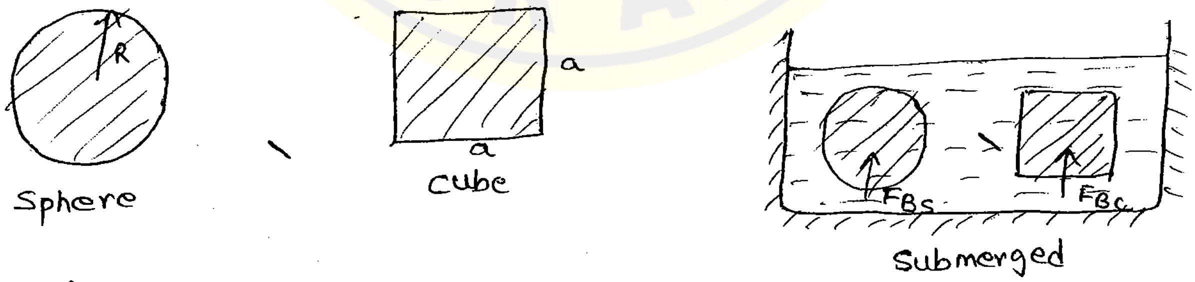
$$100 \times 1 \times 10 + 1000 = 1000 \times 10 \times V_{ballon}$$

$$V_{ballon} = 0.2$$

Ex:-) A solid sphere and a solid cube are having same surface area. This two bodies fully immersed in a fluid. Determine ratio of buoyancy force to that on sphere to that on cube.

a) 1 : 1 b) $\sqrt{6}$: 1 c) 1 : $\sqrt{6}$ d) 1 : 3

A)



$$A_{s_{sphere}} = A_{s_{cube}}$$

$$4\pi R^2 = 6a^2$$

$$\frac{F_{B\,sphere}}{F_{B\,cube}} = \frac{\rho_w \cdot g \cdot V_{sphere}}{\rho_w \cdot g \cdot V_{cube}}$$

$$\frac{V_{sphere}}{V_{cube}} = \frac{\frac{4}{3}\pi R^3}{a^3}$$

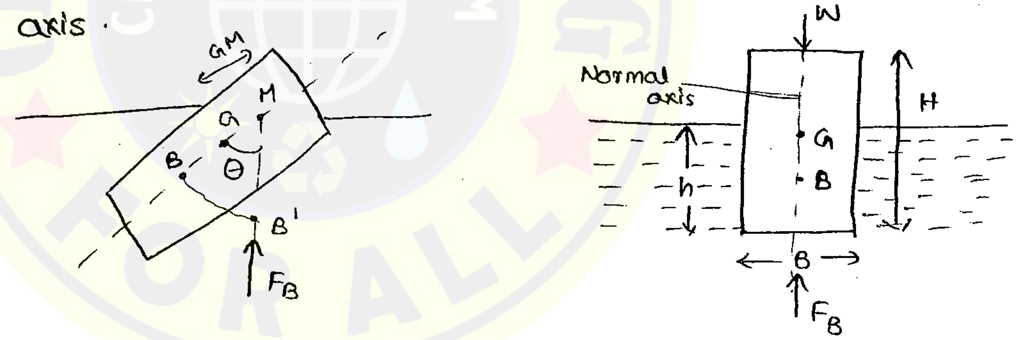
$$= \frac{\frac{4}{3}\pi R^3}{a^2 \cdot a}$$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4\pi R^2}{6}\cdot\sqrt{\frac{4\pi R^2}{6}}} \Rightarrow \frac{1}{\sqrt{\frac{\pi}{6}}}$$

$$= \sqrt{\frac{6}{\pi}}$$

Metacentric height :-

When a floating body is made to disturb by giving small angular displacement shown in fig. It is observed that the floating body oscillates about a point. That point is call Metacentre. It is obtain by intersection of normal axis of the body and action of normal axis cyclinder (or) prism and bouyance forces axis.



It is the distance between Meta centre (M) and centre of gravity of the floating body (GM).

1. Analytical method
2. Experimental method.

$$GM = BM - BG$$

$$= BM - (OG - OB)$$

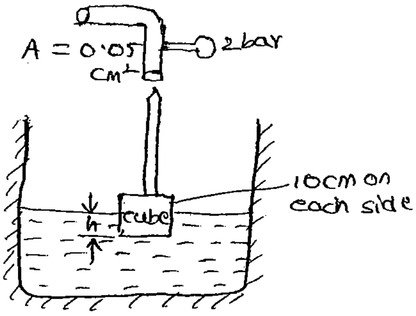
$$GM = \frac{I}{V} - BG$$

$$= \frac{I}{V} - \left(\frac{H}{2} - \frac{h}{2}\right)$$

where

I = moment of Inertia (second moment of Area)

Ex:-) Find depth of immersion?



$$F\downarrow = P \times A = 2\times10^5 \times 0.05\times10^{-4}$$

$$F_B\uparrow = \rho_f \cdot g \cdot V = 1000\times10\times A_{c/s}\times h$$

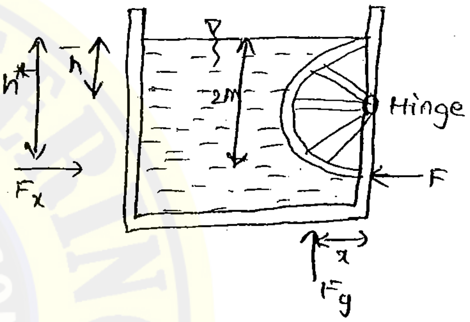
$$= 1000\times10\times0.1\times0.1\times h$$

Equating

$$h = 0.01m$$

$$h = 1\ cm$$

Ex:-) The force 'F' required at equilibrium on the semi cylindrical gate shown below is



A. $F_x = \rho g \bar{h} A$

$$= 1000\times9.81\times\frac{2}{2}\times2\times1$$

$$F_x = 19620\ N$$

$$h^* = \bar{h} + \frac{I_G}{A\bar{h}}$$

$$= 1 + \frac{\frac{bd^3}{12}}{b\times h\times\bar{h}}$$

$$= 1 + \frac{1\times2^3}{12\times2\times1}$$

$$= 1 + \frac{1}{3}$$

$$h^* = 1.33\ m$$

$$F_y = \rho g V$$

$$= 1000\times9.81\times\frac{\pi R^2}{2}\times1$$

$$= 1000\times9.81\times\frac{\pi(1)^2}{2}$$

$$= 15409\ N$$

$$\bar{x} = \frac{4R}{3\pi} = \frac{4\times1}{3\pi}$$

$$= 0.42m$$

Taking moments about hinge

$$F_x \times (h^* - \bar{h}) - F_y(\bar{x}) - F \times 1 = 0$$

$$19.62 \times (1.33 - 1) - (15.41 \times 0.42) - F = 0$$

$$F = 0 \text{ KN}$$

Significance of meta centric height of a floating body (not submerged):-

1. Stable equilibrium:-

If a floating body made disturbed after certain oscillations it is able to return its initial position. For which GM value positive (M above G) (or) (G below the M).

2. Neutral equilibrium:-

If a floating body made disturbed by giving θ for which θ never taken place instead body moves linearly. For which GM = 0 (M and G coincides).

3. Unstable equilibrium:-

When a floating body made to disturb and body orientation changed and never returns its initial position for which G.M. value negative (M is below G (or) G is above M).

Ex:- A piece weighing 3 kg(f) in air was found to weight 2.5 kg when submerged in water. specific gravity of piece metal is.

A.
$$S_{body} = \left(\frac{3}{3-2.5}\right) \times 1.0$$

$$= 6.0$$

$$S_{body} = \left(\frac{W_{air}}{W_{air} - W_{water}}\right) S_w$$

~~P-g-NO:-~~50

17.

Ex:- Determine specific gravity of body (or) fluid by an experimental method.

A. Weight of body $= 100\,N$, $S_f = 0.8$, $W_{air} = 80$

$$S_{body} = \left(\frac{W_{air}}{W_{air} - W_{fluid}}\right) S_{fluid}$$

$$= \left(\frac{100}{-80+100}\right) \times 0.8$$

$$= \frac{100}{20} \times 0.8$$

$$S_{body} = 4.0$$

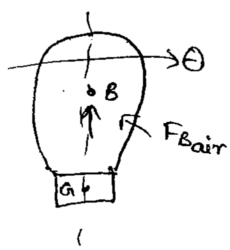
Stability of submerged body in a fluid (ballon) in air :-

For submerged body no meta centre is available.

1. Stable equilibrium of a submerged body:-

A submerged body is said to be stable if it returns its initial position for which B is above G (or) G is below B.

2. Neutral equilibrium of a submerged body:-

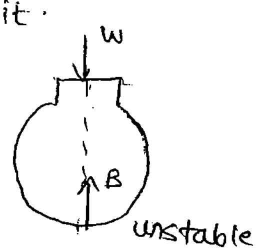


When a body not executing θ, B and G coincide is said to neutral equilibrium.

3. Unstable submerged body :-

If a submerged body never returns its initial stage after certain oscillations. It is remains at disturbed body is said for which B is below G or G is above B.

Ex:-) A metal block is thrown into a deep lake. As it sinks deeper in water, the buoyant force acting on it.

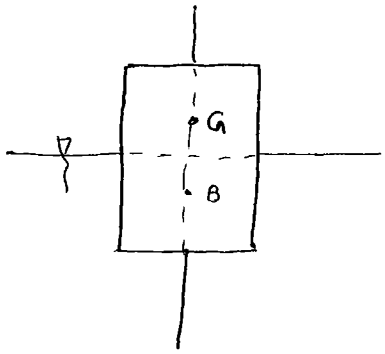


A. $F_B = \rho g V$

Buoyant force is same of submerged body

It is independent of depth for submerged body.

2. Stable equilibrium:-



BM > BG the M is above G

GM > BG

For floating GM is ≯ than BG

Ex:-) A solid weight 45KN in air and when immersed in a liquid of unit weights 10 and 20 KN. What is specific grav of solid?

A. $W_{air} = 45$ kN, $W_w = 20$ kN

$$S_{body} = \left(\frac{W_{air}}{W_{air} - W_f}\right) S_{fluid}$$

$$V_B = \left(\frac{W_{air}}{W_{air} - W_f}\right) V_f$$

$$\frac{V_B}{V_w} = \left(\frac{W_{air}}{W_{air} - W_f}\right) \frac{V_f}{V_w}$$

$$= \left(\frac{45}{45-20}\right) \times 10 \text{ kN/m}^3$$

$$V_B = 1.8 \text{ kN/m}^3$$

$S_{body} =$

24. $\tan\theta = \frac{h}{L}$

$$\frac{a_x}{g} = \frac{h}{L}$$

$$\frac{1.5}{9.81} = \frac{h}{6}$$

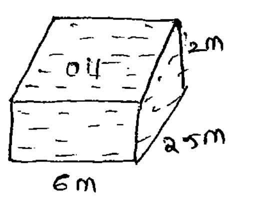
$$h = 0.9 \text{ m}$$

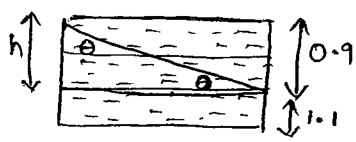
Volume = area x width

$= (\frac{1}{2} \times 0.9 \times 6) \times 2.5$

$= 6.88 \text{ m}^3$

$= 6880$ lit





$$\boxed{\therefore \tan\theta = \frac{a_x}{g}}$$

25. $P_{bottom} = \rho(g+a) \cdot h$ upward

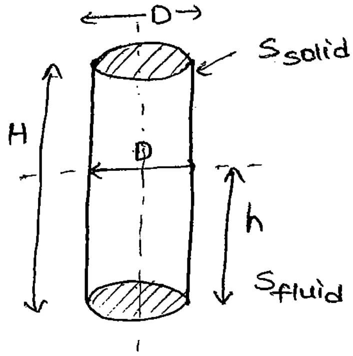
$g = -a$ i.e., tan

flow more downward

$a = g = 9.81 \text{ m/s}^2 \downarrow$

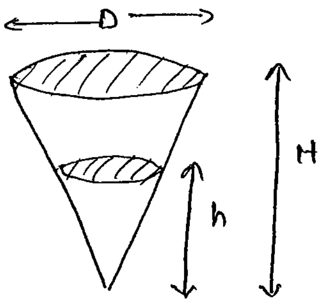
Note:-

1. Regular cylinder

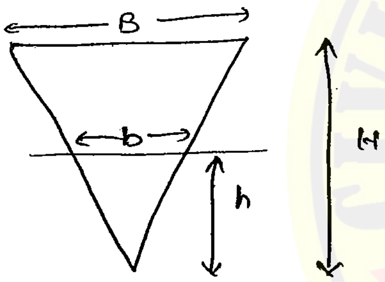


$$S_{solid} \cdot H = S_{fluid} \cdot h$$

This formula is used for cylinders or prisms



$$S_{cone} \cdot H^3 = S_{fluid} \cdot h^3$$



$$S_{pyramid} \cdot H^2 = S_{fluid} \cdot h^2$$

proof:-

$$W_{solid} = W_{fluid\ displaced}$$

$$\frac{\rho_{solid}}{\rho_{water}} \cdot V_{solid} \cdot g = \frac{\rho_{fluid}}{\rho_{water}} \cdot V \cdot g$$

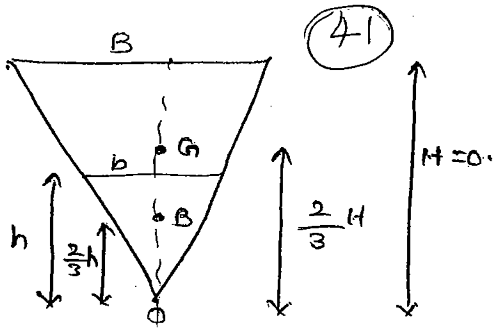
$$S_{solid} \cdot A_{c/s} \cdot l = S_{fluid} \cdot A_{c/s} \cdot l$$

$$S_{solid} \times \frac{1}{2} BH = S_{fluid} \times \frac{1}{2} \times b \times h$$

Similar Δ^{le} $\frac{B}{H} = \frac{b}{h}$

$$S_{solid} \times \frac{H \cdot b}{h} \cdot H = S_{fluid} \times b \times h$$

$$S_{solid} \times H^2 = S_{fluid} \times h^2.$$



2. $S_{solid} \cdot H^2 = S_{fluid} \cdot h^2$

$0.64\,(0.6)^2 = 1.0\,h^2$

$h = 0.48\,m$

$BG = OG - OB$

$= \frac{2}{3}H - \frac{2}{3}h$

$= \frac{2}{3}(0.6 - 0.48)$

$= 0.08\,m$

$= 80\,mm$

Sensitivity:-

$$K = S = \frac{1}{\sin\theta} = \frac{\rho \cdot g \cdot L}{P_{gas}}$$

$P_{gas} = \rho \cdot g \cdot H$

$\cancel{P_o\, g\, L} = \rho\, g\, L \sin\theta$

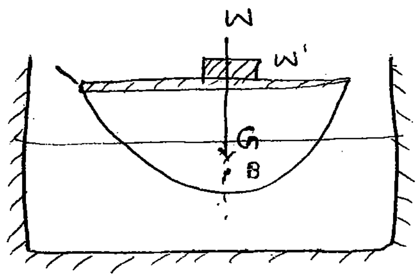
Hydrostatic paradotex:-

1 kg of water = 1 lit of water

1000 kg of water = 1 m^3 of water

1 gm of water = 1 cm^3 of water

Experimental method to determine Metacentric height:-



$$\tan\theta = \frac{x'}{L}$$

where, L = Length of pendulum

x' = displacement of mass

$$\boxed{GM = \frac{W' x}{W \tan\theta}}$$

Time period, $\boxed{T = 2\pi\sqrt{\frac{K^2}{g \cdot GM}}}$ sec

K = radius of Gyration.

P.g No:- 52

5. $W = 10{,}000$ Tons

$w' = 50$ Tons

$x = 6$ m

$\theta = 3^\circ$

$$GM = \frac{50 \times 6}{10000 \times \tan 3^\circ}$$

$= 0.57$ m

Ex:-) A sea board Radius of gyration is found to be 4.5m & metacentric height 1.2 m. Determine the time period of the ship during its rolling oscillations.

A) $$T = 2\pi\sqrt{\frac{(4.5)^2}{9.81 \times 1.2}} = \sqrt{\left(\frac{m^2}{\frac{m}{s^2} \times m}\right)}$$

$= 8.24$ sec.

Note:-

Significance of Meta centric height with reference to stability and comfort.

1. More the meta centric height, more the stabity.
2. More the meta centric height, less the time period more dis-comfort $T \propto \frac{1}{\sqrt{GM}}$

Ex:- For picnic ships or entertainment boats G.M is moderate (not much value) for comfort.

Ex:- For war ships more metacentric height required for stability not comfort purpose.

UNIT - 4

FLUID KINEMATICS

Fluid kinematics is a branch of Fluid mechanics which deals a fluid motion like displacement, velocity, acceleration, mass flow rate, volumetric flow rate, angular displacement, angular velocity and angular acceleration.

Fluid kinematics is used for flow analysis through a pipe, open channel flow, flow through pumps and turbines. There are two approaches to study kinematics.

1. Lagrangerian approach
2. Eulerian approach.

In Lagrangerian approach the observer moves along with the fluid to study fluid characteristics its a system analysis approach.

Eulerian approach the observer is stationary (at origin) and observing flow characteristics of the moving fluid. It is a control volume analysis.

Note:-

In generall Eulerian approach is adopted.

Classification of fluid flows:-

1. Steady and Unsteady flow.
2. Uniform and Non uniform flow
3. compressible and Incompressible fluid flow
4. Laminar and Turbulent flow
5. One, two, three dimensional flow
6. Rotational and Irrotational flow.

Steady flow:-

Any flow properties (or) any fluid property does not change with change of time (remains same with lapse of time) is said to be steady flow.

Ex:-) $\frac{\partial(T)}{\partial t} = 0$, $\frac{\partial(P)}{\partial t} = 0$, $\frac{\partial(\mu)}{\partial t} = 0$, $\frac{\partial(v)}{\partial t} = 0$

Note:-

Unless otherwise stated every physical flow treated as steady flow only.

Unsteady flow:-

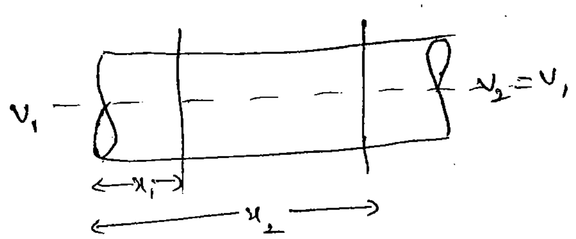
System variable does change with respect to time is called unsteady flow.

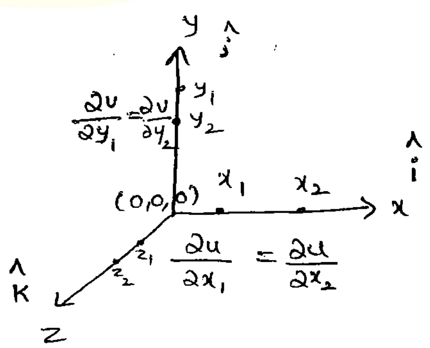
Ex:-) $\frac{\partial(T)}{\partial t} \neq 0$, $\frac{\partial(P)}{\partial t} \neq 0$, $\frac{\partial(V)}{\partial t} \neq 0$

Uniform and Non-uniform flow:-

<u>Only velocity</u> (no other parameter) change with respect to location (space coordinate does not change) is said to be uniform flow.

Ex:-) $\frac{\partial(V)}{\partial s \to x,y,z} = 0$, $\frac{\partial u}{\partial x} = 0$, $\frac{\partial v}{\partial y} = 0$, $\frac{\partial w}{\partial z} = 0$



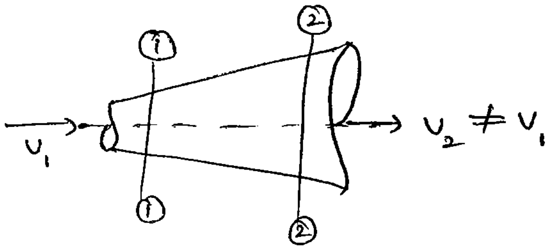


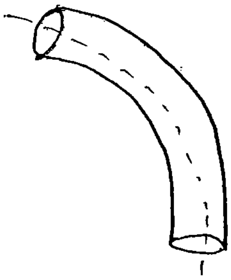
Non uniform flow:-

<u>Only velocity</u> changes with respect to space coordinate

Ex:-) $\frac{\partial V}{\partial s} \neq 0$, $\frac{\partial u}{\partial x} \neq 0$, $\frac{\partial v}{\partial y} \neq 0$, $\frac{\partial w}{\partial z} \neq 0$

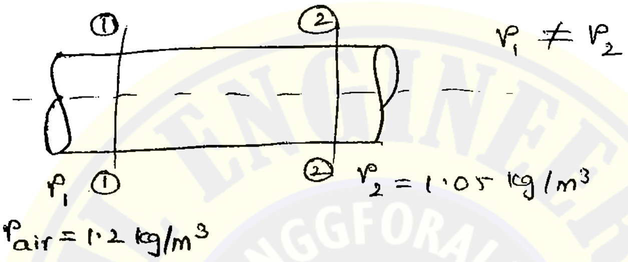
Ex:-) Flow through a tappered pipe (Nozzle) and flow through a bend pipe





Compressible and Incompressible fluid flow:-

A fluid mass density changes from location to location even time to time (Mass density does not constant).



$$PV = mRT$$

$$P = \frac{m}{V} RT$$

$$P = \rho RT$$

$$\rho \downarrow = \frac{P}{RT} \uparrow$$

Incompressible fluid flow:-

Mass density of the fluid remains constant throughout the control volume is said to be Incompressible fluid flow.

$$\rho_1 = \rho_2$$

Eg:-) Water flow, Oil flow.

Laminar flow:-

Laminar flow is a stream line flow (or) parallel flow (or) viscous flow (or) rotational flow (or) sandwich flow and the Reynolds number always through the pipe flow less than 2000

$$R_e \leq 2000$$
(or)

$$\text{Reynold's number} = \frac{\text{Inertial flow}}{\text{viscous flow}} = \frac{\rho v D}{\mu}$$

Turbulent flow:-

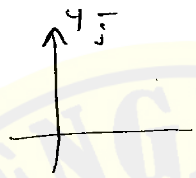
It is a disorder flow, zigzag flow, the velocity fluctuations more and Reynolds number more than 4000. (In b/w 2000 to 4000 range is called Transitional flow)

One dimensional flow:-

It is a flow in which changes consider in one particular direction and neglected other directions (normal, (or) transverse directions).

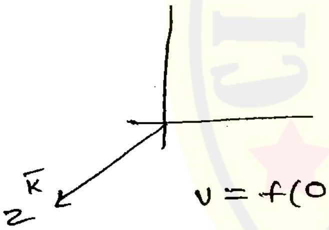
$v = f(x,0,0)$

$\bar{v} = (u)\bar{i} + (0)\bar{j} + (0)\bar{k}$



$v = f(0,y,0)$

$\bar{v} = (0)\bar{i} + (v)\bar{j} + (0)\bar{k}$



$v = f(0,0,z)$

$\bar{v} = (0)\bar{i} + (0)\bar{j} + (w)\bar{k}$

$v = f(x,0,z)$

$v = f(x,y,z)$

$v = f(x,y,z,t)$

$\downarrow$

2, D, unsteady

Two dimensional and un-steady flow:-

$v = f(x,y,0,t)$

$v = f(0,y,z,t)$

$v = f(x,0,z,t)$

Ex:- $\bar{v} = \underset{\left(\frac{1}{m\text{-}sec}\right)}{\underset{\downarrow}{(ax^2)}}\bar{i} + \underset{\left(\frac{1}{sec^2}\right)}{\underset{\downarrow}{(by\cdot t)}}\bar{j}$

Three dimensional and un-steady flow:-

$v = f(x,y,z,t)$

$\bar{v} = \underset{\left(\frac{1}{sec}\right)}{\underset{\downarrow}{(ax)}}\bar{i} + \underset{\left(\frac{1}{m\text{-}sec}\right)}{\underset{\downarrow}{(bxy)}}\bar{j} + \underset{\left(\frac{1}{m\text{-}sec^2}\right)}{\underset{\downarrow}{(cz^2\cdot t)}}\bar{k}$

Rotational and Irrotational flow :-

A flow is said to be rotational if a fluid is executing two motions (one motion rotates its own axis).

Ex:- water motion in centrifugal pump impeller.

Water flow through reaction turbines (Francise and kepler turbi:

viscous fluid flow in a pipe.

A fluid executing only one type of motion in space is called Irrotational flow.

Ex:- Ideal fluids

Flow pattern :-

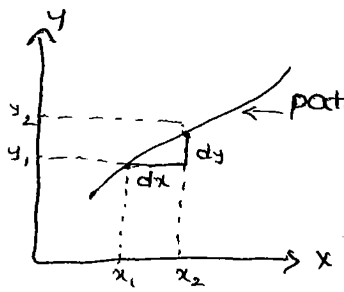
Any fluid flow is described by the following flow pattern.

Types of Lines:-

1. path line
2. stream line
3. streak line
4. Time line
5. stream tube
6. Flow Net.

path line:-

A single fluid particle places the path is called path line.



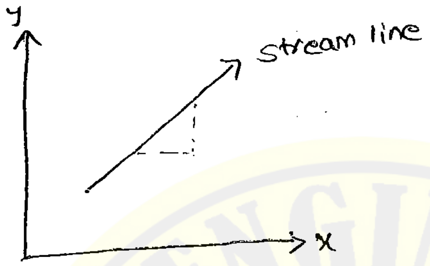
$$v = \frac{dy}{dt}$$

$$\int_0^x dx = \int_0^t u \cdot dt$$

$$u = \frac{dx}{dt}$$

Stream line:-

It is an imaginary line by an observer. It is a tangent line at given point in space. No two stream lines intersect each other and stream lines are always parallel to each other. For flow analysis stream line flow only considered (not pathline not streak line) b/w two stream lines flow rate (discharge takes place). It represents velocity of the flow



slope of a stream line $= \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\boxed{u \cdot dy - v \cdot dx = 0}$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v}}$$

} Two dimensional

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$ — Three dimensional

P. g. No:- 67

18. $\bar{V} = u\bar{i} + v\bar{j}$

$= (-x)\bar{i} + (2y)\bar{j}$

At $P(x, y) = (1, 1)$ find stream line equation

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{-x} = \frac{dy}{2y}$$

$$\int \frac{dx}{x} = -\frac{1}{2}\int\left(\frac{dy}{y}\right)$$

$$\log_e x = -\frac{1}{2}\log_e y + c$$

$$\log_e x = -\log_e y^{\frac{1}{2}} + c.$$

$$\log_e x + \log_e \sqrt{y} = c \qquad \log(a)+\log(b) = \log(ab)$$

$$\log_e x\sqrt{y} = \log_e c$$

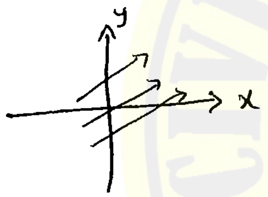
$$x\sqrt{y} = c$$

At $(1,1)$ $\quad 1\sqrt{1} = c$

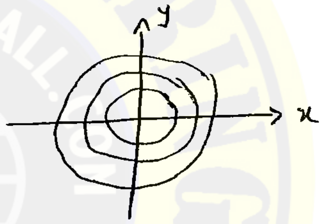
$$1 = c$$

Ex:- Determine stream line equation for the given velocity field $\bar{v} = (x)\bar{i} + (-y)\bar{j}$. Find given equation and also identify stream line patterns.

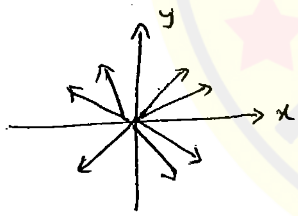
a)



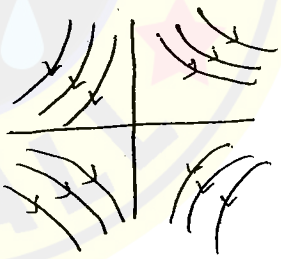
b)



c)



d)



(✓)

A) stream line equation

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\log_e x + \log_e y = c$$

$$\log_e xy = \log_e c$$

$$\boxed{xy = c} \longrightarrow \text{Rectangular Hyperbola.}$$

$$x \propto \frac{1}{y}$$

Ex:-) From above problem $\bar{V} = (y)\bar{i} + (-x)\bar{j}$

A) stream line equation

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{y} = -\frac{dy}{x}$$

$$\int x\,dx = -\int y\,dy$$

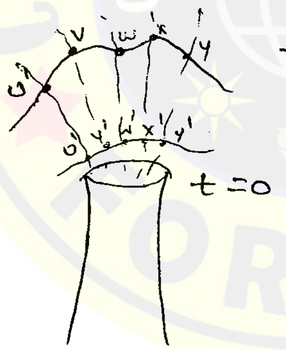
$$\frac{x^2}{2} = -\frac{y^2}{2} + c$$

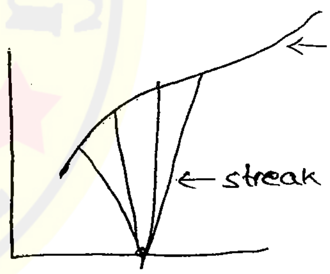
$$\boxed{x^2+y^2=2c} \longrightarrow \text{circle.}$$

(B)

Streak line :-

It is a line joining fluid particles at given instant of time.

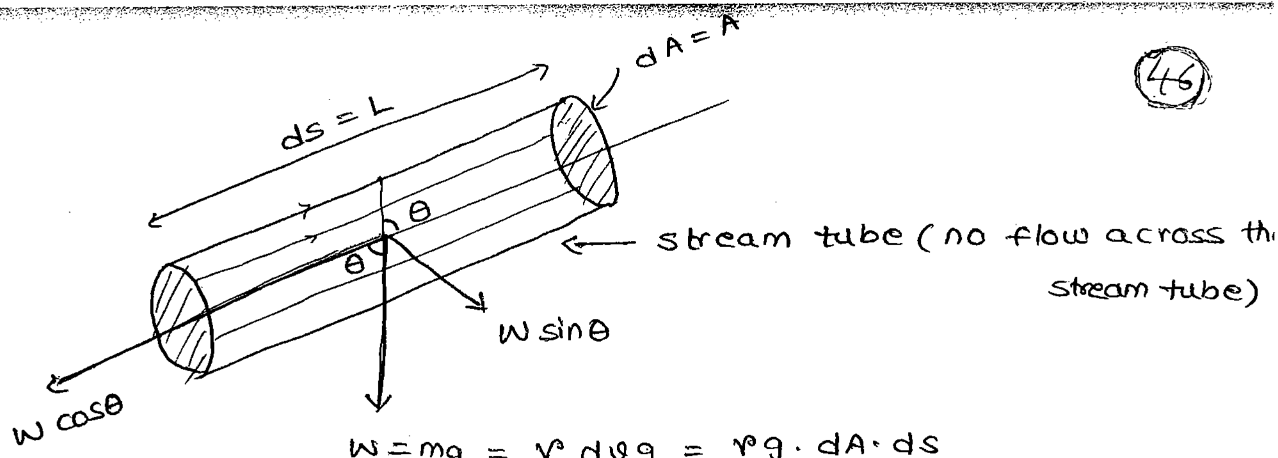




→ In steady flow, path line, stream line, streak line are identical and equal.

Stream tube :-

Stream tube is a combination of stream lines and it is specified region (control volume) in which certain fluid matter is enveloped by 'N' number of streamlines across which no fluid flow takes place.



$$W = mg = \rho\, d\forall\, g = \rho g \cdot dA \cdot ds$$
$$= \rho g \cdot \frac{\pi D^2}{4} \cdot L \cdot$$

Velocity and Acceleration vector:-

$$\bar{V} = (u)\bar{i} + (v)\bar{j} + (w)\bar{k}$$

$$V = |V| = \sqrt{u^2 + v^2 + w^2}$$

Acceleration vector:-

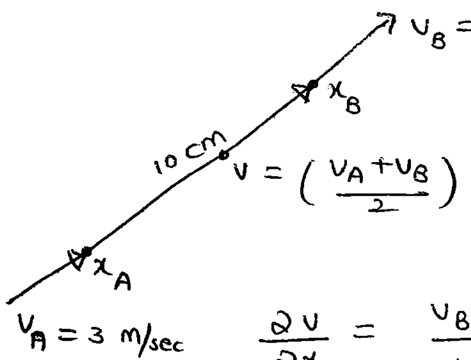
$$\bar{a} = (a_x)\bar{i} + (a_y)\bar{j} + (a_z)\bar{k}$$

a_x = acceleration along x-axis $= a_{local} + a_{convective}$

$$a_x = \left[\frac{\partial u}{\partial t}\right] + \left[u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z}\right]$$

$$a_y = \left[\frac{\partial v}{\partial t}\right] + \left[u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z}\right]$$

$$a_z = \left[\frac{\partial w}{\partial t}\right] + \left[u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z}\right]$$



$$\frac{\partial v}{\partial x} = \frac{V_B - V_A}{x_B - x_A}$$

Ex:-

$$a_{convective} = 3.5 \times \frac{1\ m/sec}{0.1\ m} \qquad V = \left(\frac{3+4}{2}\right) = 3.5$$

$$= 35\ m/sec$$

14. $a_{convective} = \left(\frac{2.5+3}{2}\right) \times \left(\frac{3-2.5}{0.1}\right)$

$$\boxed{a_{con} = \left(\frac{V_a+V_b}{2}\right) \times \left(\frac{V_b-V_a}{x_b-x_a}\right)}$$

$= 2.75 \times 5$

$= 13.75 \ m/s^2$

20. $\bar{V} = (2x)\bar{i} + (y)\bar{j}$

$u = 2x \ (m/sec)$

$v = y \ (m/sec)$

$a_{(x,y)} = a_{(1,1)} = ?$

$$\boxed{a_x = \left[\frac{\partial u}{\partial t}\right] + \left(u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z}\right)}$$

$= (0) + \left(2x \cdot \frac{\partial(2x)}{\partial x} + y \cdot \frac{\partial u}{\partial y} + 0\right)$

$= 2x \cdot (2) + y(0) + 0$

$a_x = 4x$

$= 4(1) = 4 \ m/sec^2$

$$\boxed{a_y = \left(\frac{\partial v}{\partial t}\right) + \left(u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z}\right)}$$

$a_y = (0) + \left(2x \cdot \frac{\partial(y)}{\partial x} + y \cdot \frac{\partial y}{\partial y} + 0\right)$

$a_y = 0 + (0 + y + 0)$

$a_y = y = 1 \ m/sec^2$

$\bar{a} = (a_x)\bar{i} + (a_y)\bar{j}$

$a = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{4^2 + 1^2}$

$\boxed{a = \sqrt{17}}$

3. $V = 4x^2t\,\bar{i} - 5y^2\bar{j} + 6zt\,\bar{k}$

$u = 4x^2t$

$v = -5y^2$

$w = 6zt$

∴ if time is not given take 1 sec

$$a_x = \left(\frac{\partial u}{\partial t}\right) + \left(u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}\right)$$

$$= \overset{(4x^2\times 1)}{\cancel{(8x)}} + \left(4x^2t\cdot\frac{\partial(4x^2t)}{\partial x} + (-5y^2)(0) + 6zt(0)\right)$$

$$= 4x^2 + (4x^2t\cdot 8xt)$$

$$a_x = 4x^2 + (32x^3t^2)$$

(2, 3, 2)

∴ $t = 1$

$$= 4(2)^2 + (32(2)^3(1)^2)$$

$$= \underbrace{16}_{\text{local acceleration}} + \underbrace{\overset{256}{\cancel{128}}}_{\text{convective acceleration}}$$

$\boxed{a_x = 16}$ (he is asking only local acceleration)

$$a_y = \left(\frac{\partial v}{\partial t}\right)$$

$$= \frac{\partial(-5y^2)}{\partial t}$$

$\boxed{a_y = 0}$

$$a_z = \left(\frac{\partial z}{\partial t}\right)$$

$$= \left(\frac{\partial(6zt)}{\partial t}\right)$$

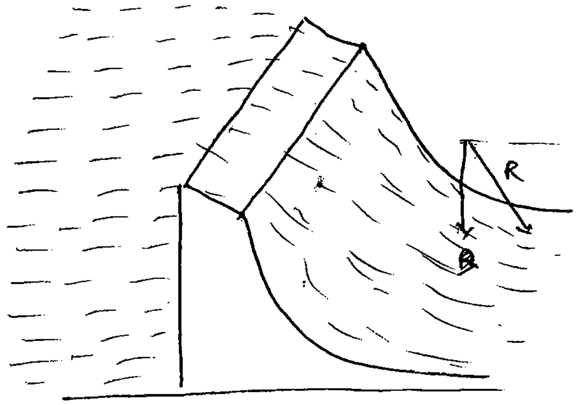
$$a_z = 6z$$

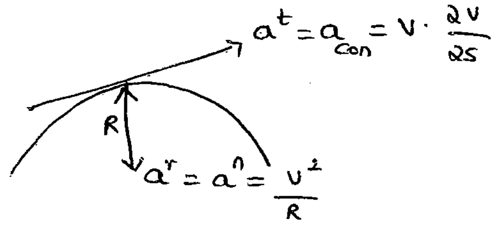
$$= 6(2)$$

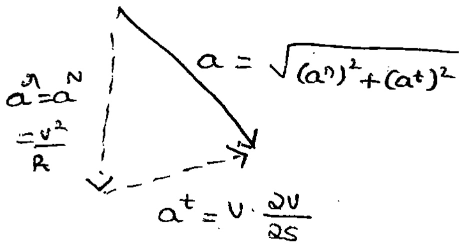
$\boxed{a_z = 12}$

$$a = \sqrt{16^2 + 0^2 + 12^2} = \sqrt{256 + 144} = \boxed{20\ m/s^2}$$

Motion of a fluid along a curved path :-



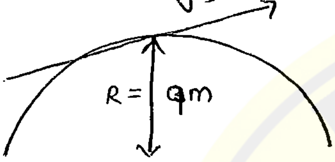




P.g NO:- 68

23.

$V = 3\,m/s \quad \frac{\partial V}{\partial s} = \frac{1}{3}\,(m/sec)/m$



$V = 3\,m/s$

$R = 9\,m$

$\frac{\partial V}{\partial s} = \frac{1}{3}\,(m/sec/m)$

$a^t = a^t = a_{convective} = V \cdot \frac{\partial V}{\partial s}$

$= 3 \times \frac{1}{3} = 1\,m/s^2$

$a^N = a^n = \frac{V^2}{R} = \frac{(3)^2}{9} = 1\,m/s^2$

$a = \sqrt{1^2 + 1^2} = \sqrt{2}\,m/s^2$

24.

$V_r \propto \frac{1}{r^2}$

$a_r \propto\ ?$

$a = \frac{V^2}{r} = \frac{\left(\frac{c}{r^2}\right)^2}{r} = \frac{c^2}{(r^2)^2 \cdot r} = \frac{c^2}{r^5}$

(or)

$a_{conv} = V_r \cdot \frac{\partial (V_r)}{\partial r}$

$= \frac{c}{r^2} \cdot \frac{\partial}{\partial r}\left(\frac{c}{r^2}\right) = \frac{c^2}{r^2} \cdot \frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = \frac{c^2}{r^2}(-2)\,r^{-2} = \frac{-2c^2}{r^5}$

P.g No:- 72

(48)

35. $u = a, \quad v = a$

$$\bar{V} = (u)\bar{i} + (v)\bar{j}$$

$$= (a)\bar{i} + (a)\bar{j}$$

$(x, y) = (2, 6)$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{a} = \frac{dy}{a}$$

$$\int dx = \int dy$$

$$x = y + c \quad \Rightarrow \quad \boxed{y = x + y}$$

$$2 = 6 + c$$

$$\boxed{c = -4}$$

$$\Rightarrow \boxed{y = x + 4}$$

P.g No:- 67

19. $\bar{V} = (u)\bar{i} + (v)\bar{j}$

$$|V| = V = \sqrt{u^2 + v^2}$$

Given $V = 4\sqrt{x^2 + y^2}$

$$= \sqrt{16x^2 + 16y^2}$$

$$= \sqrt{(4x)^2 + (4y)^2}$$

$u = 4x$

$v = 4y$ at $(4, 3)$

$$V = \sqrt{(4\times4)^2 + (4\times3)^2}$$

$$= \sqrt{400}$$

$$= 20 \text{ m/s}$$

$$a_x = \left(\frac{\partial u}{\partial t}\right) + \left[u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y}\right]$$

$$= 0 + (4x \cdot 4 + 0)$$

$$a_x = 16x$$

$= 16(4)$ Given $(4, 3)$

$$= 64$$

$$a_y = \left(\frac{\partial v}{\partial t}\right) + \left(u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y}\right)$$

$$= 0 + (0 + 4y \cdot 4)$$

$$= 16y$$

$$= 16(3)$$

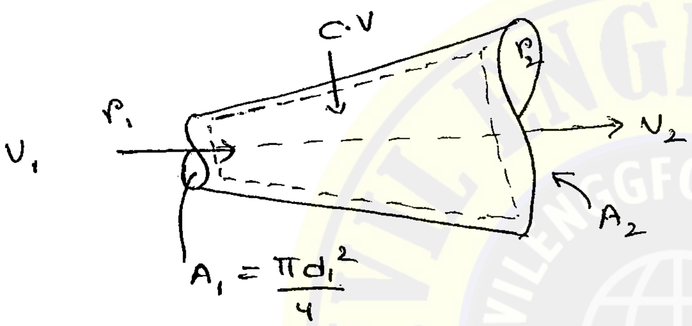
$$= 48$$

$$a = \sqrt{64^2 + 48^2} = 80 \text{ m/s}$$

** Continuity Equation:- (Law of conservation of Mass):-

It states that the total mass of the matter across the control volume remains constant. It means the rate at which fluid flow remains constant at every section of the controlled volume (Eulárian approach). It means rate at which the fluid flow entered into controlled volume must be equal to mass accumulated rate + rate at which mass living.

It means algebraic sum of the mass flow rate across the control volume equal to zero.



Mass = Mass density x volume

$$m = \rho \times \upsilon$$

$$\boxed{\rho = \frac{m}{\upsilon}}$$

$$\frac{m}{time} = \rho \times A \times \frac{L}{time}$$

$$m' = \rho \times (A \times V)$$

$$m' = \rho \times Q$$

(or)

$$Q = \frac{\upsilon}{t} = \upsilon'$$

$$m = \rho \upsilon$$

$$\frac{m}{time} = \rho \cdot \frac{\upsilon}{time}$$

$$m' = \rho \cdot \upsilon'$$

$$m' = \rho \cdot Q$$

$m_1 = \rho_1 A_1 v_1$

$m_2 = \rho_2 A_2 v_2$

Mass = constant

$m = c$

$\rho A v = c$

Differentiate

$d(\rho A v) = 0$

$$\boxed{d\rho A v + \rho\, dA\, v + \rho A\, dv = 0}$$

(or)

Take divide by 'log' on both sides

$\log(\rho A v) = \log c$

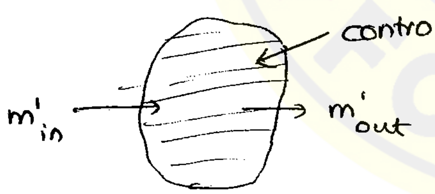
$\log \rho + \log A + \log v = 0$

Differential

$$\boxed{\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0}$$

Differentiate form of continuity equation.

Ex:-



$m = \rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Ex:- $A_1 = 10\, m^2$

$A_2 = 8\, m^2$

$A_{tank} = 100\, m^2$

$v_1 = 10\, m/sec$

$v_2 = 10\, m/sec$.

$\frac{dh}{dt} = ?$

water level in tank (Rate of change of level in tank)

A) $m_{in} = m_{stone} + m_{out}$

$$\rho Q_{in} = \rho Q_{stone} + \rho Q_{out}$$

$$A_1 V_1 = A_{\cancel{stone}\ tank} \frac{dh}{dt} + A_2 V_2$$

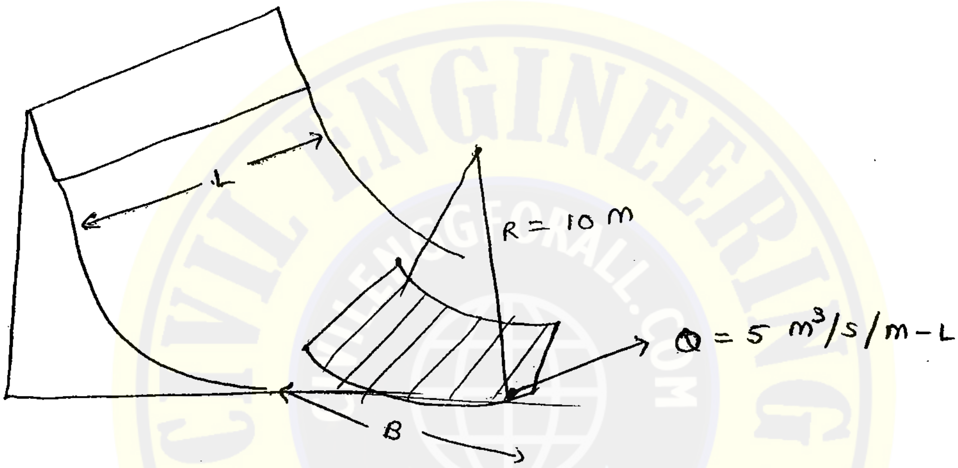
$$10 \times 10 = 100 \times \frac{dh}{dt} + 8 \times 10$$

$$\frac{dh}{dt} = 0.2 \ m/sec$$

$$= 20 \ cm/sec$$

P.g NO:- 67

21.



$$Q = A \cdot V$$

$$Q = (B \cdot L) \cdot V$$

$$5 = (0.5 \times 1) \cdot V$$

$$V = 10 \ m/sec$$

$$a^{n} = a^{N} = a^{centripetal} = \frac{V^2}{R}$$

$$= \frac{10^2}{10}$$

$$= 10 \ m/s^2$$

Conditions for continuity Equation:-

1. For steady, 3-D and incompressible

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

If this satisfies then flow is conserved.

P.g No:- 68.

26. $U = 6xy - 2x^2$

$V = ?$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial(6xy - 2x^2)}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$6y - 4x + \frac{\partial v}{\partial y} = 0$$

$$\cancel{4x = 6y} \quad 4x - 6y = \frac{\partial v}{\partial y}$$

$$\int \partial v = \int (4x - 6y) \cdot \partial y$$

$$v = 4xy - \frac{6y^2}{2} + c$$

$$\boxed{V = 4xy - 3y^2 + c}$$

P.g No:- 69

1. $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(b) $\longrightarrow$ $u = x + y$, $v = x - y$

$$\frac{\partial(x+y)}{\partial x} + \frac{\partial(x-y)}{\partial y} = 0$$

$$1 - 1 = 0$$

$$0 = 0$$

$\therefore$ Hence satisfied.

2. (b) $\longrightarrow$ $$\frac{\partial(2x^2)}{\partial x} + \frac{\partial(2xyz)}{\partial y} + \frac{\partial(-4xz - xz^2 + 5y^2)}{\partial z}$$

$$= 4x + 2xz - 4x - 2xz$$

$$= 0 = 0$$

$\therefore$ Hence satisfied.

3. $U = ax + by$

$V = cx + dy$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$a + d = 0$.

7. (d) $\rightarrow$ $U = x$, $V = -y$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$1 - 1 = 0$

$0 = 0$

$\therefore$ Hence satisfied.

Conditions for continuity equation:-

1. For stead, 3D and compressible

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

2. For unsteady, 3D and compressible.

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial(\rho)}{\partial t} = 0$$

P.g. No:- 68

25. $u = y^2 + 4xy$

$v = ?$

At $y = 0$, $v = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$4y + \frac{\partial v}{\partial y} = 0$$

Integrating on both sides

$$\int \partial v = -\int 4y \cdot \partial y$$

$$v = -\frac{4y^2}{2} + c.$$

$$\boxed{v = -2y^2 + c}$$

$0 = -2(0)^2 + c$

$c = 0$

$$\therefore \boxed{v = -2y^2}$$

Representation of flow in Mathematical functions:-

Any flow can be described by two mathematical forms

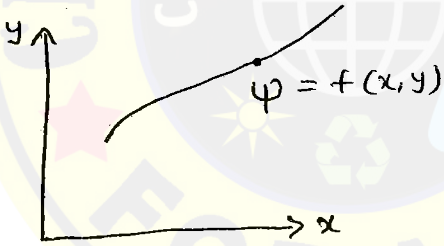
1. stream function (ψ)
2. velocity potential function (ϕ)

Stream function (ψ):-

It is a mathematical representation of fluid flow in scalar form, valid for 2-dimensional flow only. It must satisfy continuity equation. It also satisfies Laplace equation

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0} \longrightarrow \text{Laplace equation.}$$

Stream function used for estimating discharge (volumetric flow rate) for unit width across the stream lines. It depends represents velocity components about normal plains.



$$\frac{\partial \psi}{\partial x} = -v$$

$$\frac{\partial \psi}{\partial y} = +u$$

P.g No:- 70

12. $\psi = 3x^2 - y^3$

$$u = \frac{\partial \psi}{\partial y} = 0 - 3y^2$$

$$u = -3y^2$$

$$v = -\frac{\partial \psi}{\partial x} = -6x - 0$$

$$= -6x$$

$$\bar{v} = (u)\bar{i} + (v)\bar{j}$$

Given (2, 1)

$$= (-3y^2)\bar{i} + (-6x)\bar{j}$$

$$= -3\bar{i} - 12\bar{j}$$

$$|v| = \sqrt{9+144} = \sqrt{153} = 12.37$$

P.g NO:- 68

28. $\psi = 2xy$ $(2,-2) \rightarrow$ Given

$u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

$u = 2x$ $v = -2y$

$\bar{V} = (2x)\bar{i} + (-2y)\bar{j}$

$= (4)\bar{i} + (4)\bar{j}$

$|V| = \sqrt{4^2+4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ m/s.

P.g NO:- 70

3. $KE = \frac{1}{2} m v^2$

where $w = mg$

$m = \frac{w}{g}$

$KE = \frac{1}{2} \cdot \frac{w}{g} v^2$

$\frac{KE}{w} = \frac{v^2}{2g}$.

6. $\psi = ax^2y - 2y^3$ is to be stream function the value of $a = ?$

$\frac{\partial \psi}{\partial x} = 2axy - 0 = 2axy$

$\frac{\partial \psi^2}{\partial y x^2} = 2ay$

$\frac{\partial \psi}{\partial y} = ax^2 - 6y^2 \Rightarrow \frac{\partial \psi^2}{\partial y^2} = 0 - 12y = -12y$

$\frac{\partial \psi^2}{\partial x^2} + \frac{\partial \psi^2}{\partial y^2} = 0 \Rightarrow 2ay - 12y = 0$

$2ay = 12y$

$a = 6$

P.g. NO:- 67

22. $\psi = x^2 - y^2$

For stream function, $\frac{\partial \psi^2}{\partial x^2} + \frac{\partial \psi^2}{\partial y^2} = 0$

$\frac{\partial \psi}{\partial x} = -v \Rightarrow \frac{\partial (x^2-y^2)}{\partial x} = 2x - 0 \Rightarrow v = -2x$

$\frac{\partial \psi}{\partial y} = u \Rightarrow \frac{\partial (x^2-y^2)}{\partial y} = 0 - 2y \Rightarrow u = -2y$

⑤

check for continuity equation :-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial(-2y)}{\partial x} + \frac{\partial(-2x)}{\partial y} = 0$$

$$= 0$$

Total acceleration / vector :-

$$\bar{a} = a_x \bar{i} + a_y \bar{j}$$

$$a_x = \frac{\partial u}{\partial t} + \left[u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y}\right]$$

$$= 0 + (-2y(0) + (-2x)(-2))$$

$$= 4x$$

$$a_y = +4y$$

$$\bar{a} = 4x\bar{i} + 4y\bar{j}$$

Stream function:-

Stream function is used to estimate discharge

$$Q = A \cdot V \; (m^3/sec)$$

$$= d\psi \times \text{width of flow}$$

$$= |d\psi|$$

$$= (\psi_2 - \psi_1) \Rightarrow \frac{Q \; (m^3/sec)}{\text{unit width } (m)} = q$$

$$\Rightarrow |d\psi|$$

$$= |\psi_2 - \psi_1|$$

P.g NO:- 68.

29. $\phi = 3xy \quad \psi = \frac{3}{2}(y^2 - x^2), \quad (x_1, y_1) = (1, 3), \quad (x_2, y_2) = (3, 3)$

$$q = |\psi_2 - \psi_1| = \left|\frac{3}{2}(3^2 - 3^2) - \frac{3}{2}(3^2 - 1^2)\right|$$

$$= \left|0 - \frac{3}{2}(8)\right|$$

$$= |-12|$$

$$= 12 \text{ units}$$

$$= 12 \; m^3/sec/m$$

8. Given $\psi = x^2 - y^2$

$$u = -2y = -2\times1 = -2$$

$$v = -2x = -2\times1 = -2$$

$$V = \sqrt{u^2+v^2} = \sqrt{(-2)^2+(-2)^2}$$

$$V = 2\sqrt{2}$$

potential function (or) velocity potential function:-

It is a mathematical function used to represent fluid flow in 3-d quadrant. It is also a scalar function.

1. It represents the direction of the flow.
2. Flow of the fluid takes place based on the difference of total energy.
3. It must satisfy laplace transform equation $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial \phi^2}{\partial y^2} + \frac{\partial \phi^2}{\partial z^2} = 0$
4. It need not be satisfied continuity equation.
5. It is only valid for Irrotational flow (non viscous flow).

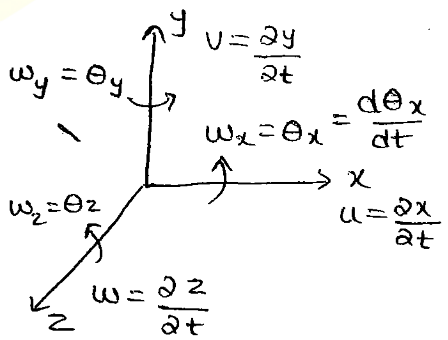
$$\frac{\partial \phi}{\partial x} = -u, \quad \frac{\partial \phi}{\partial y} = -v, \quad \frac{\partial \phi}{\partial z} = -w$$

'-ve' indicates that fluid flow always higher potential (higher total energy) to lower potential (Lower total energy).

Rotational and Irrotational flow conditions for rectangular coordination:-

$$\bar{V} = (u)\bar{i} + (v)\bar{j} + (w)\bar{k}$$

$\bar{\omega}$ = Angular speed vector



Rotational matrix:-

$$\omega = \frac{1}{2}\begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix}$$

$$\omega_x = \frac{1}{2}\begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix} \rightarrow 0 \quad = \frac{1}{2}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ v & w \end{vmatrix}$$

53

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{-1}{2}\begin{bmatrix} \partial/\partial x & \partial/\partial z \\ u & w \end{bmatrix}$$

$$= -\frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \quad \text{rad/sec}$$

Note:-

1. If $\omega \neq 0$, then the given velocity field is Rotational
2. If $\omega = 0$, then flow is Irrotational.

Velocity (Ω):-

Two times of angular speed

$$\Omega = 2\times\omega$$

$$\Omega_z = 2\times\omega_z$$

$$\Omega_z = 2\times\frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad \text{if flow is irrotational}$$

$$\Omega_z = 0$$

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

velocity vector for irrotational flow is "curlq"

$$\omega_y = -\frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right]$$

P.g No:- 68

27.35. $\cancel{\omega_z = 0}$ $V = (ax+by)i + (cx+dy)j$

condition for irrotational, $\omega_z = 0$

$u = ax+by \qquad v = cx+dy$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$$

$$= \frac{1}{2}\left[\frac{\partial(cx+dy)}{\partial x} - \frac{\partial(ax+by)}{\partial y}\right]$$

$$= \frac{1}{2}[c-b] = 0$$

$$b = c$$

For continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$a + d = 0$$

$$a = -d.$$

P.g No:- 70

7. $V = 6x^3\bar{i} - 8x^2y\bar{j}$, $u = 6x^3$, $v = -8x^2y$

$$\Omega_z = 2\omega_z$$

$$= 2 \times \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= \frac{\partial(8x^2y)}{\partial x} - \frac{\partial(6x^3)}{\partial y}$$

$$= -16xy - 0$$

$$= -16xy$$

Shear strain rate:-

$$\text{shear strain} = \frac{\theta_1 + \theta_2}{2}$$

$$= \frac{1}{2}\left[\frac{\partial v}{\partial x} - \left(-\frac{\partial u}{\partial y}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

Flow net:-

Flow net is graphical representation of grid (or) square matrix (or) perpendicular lines made by stream lines and equi-potential lines.

P.g No:- 72

24. d.

Note:-

1. It is valid for 2D, irrotational flow and also steady.
2. This analysis ignores gravity forces of fluid only pressure energies and velocity energies.

3. It can not be applied for viscous fluids

4. It cannot estimate loss.

5. Only it gives the intensity of flow discharge.

6. Slope of stream line x slope of velocity potential line = -1

$$m_1 \cdot m_2 = -1$$

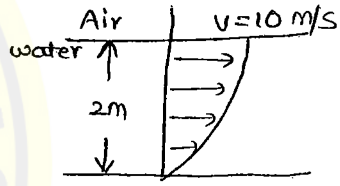
Stream function:-

Satisfy continuity equation and Laplace equation.

potential function:-

Valid for only irrotational flow.

Ex:-) consider incompressible flow through a two dimensional open channel. At a certain section A-A the velocity profile is parabolic. Neglecting air resistance at the free surface. Find the volume flow rate per unit width of the channel.



A. Mean velocity for parabola $= \frac{2}{3}(0+10)$

$$V_{mean} = 6.66 \text{ m/sec}$$

volume of flow rate (Q) = A V

$$= 2 \times 1 \times 6.66$$

$$Q = 13.33 \text{ m}^3/\text{sec}$$

Ex:-) The velocity along the centreline of a nozzle of length 'L' is given by $v = 2t\left(1-\frac{x}{2L}\right)^2$, where v = velocity in m/sec; x = distance from ∅ inlet to ~~outlet~~ nozzle. At t = 3 sec, x = 0.5 and L = 0.8.

A. a. The Local acceleration in m/s^2 is

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial t}\left[2t\left(1-\frac{x}{2L}\right)^2\right) = 2\cdot 1\left(1-\frac{x}{2L}\right)^2$$

$$a = 2\left(1-\frac{0.5}{2\times 0.8}\right)^2 = 0.94 \text{ m/s}^2$$

b. the convective acceleration in m/sec^2 is

$$a_{convective} = v \times \frac{\partial v}{\partial s} = 2t\left(1-\frac{x}{2L}\right)^2 \times \frac{\partial}{\partial x}\left(2t\left(1-\frac{x}{2L}\right)^2\right)$$

$$= 2\times 3\left(1-\frac{0.5}{2\times 0.8}\right)^2 \times 2t \times 2\left(1-\frac{x}{2L}\right)\times\left(0-\frac{1}{2L}\right)$$

$$= 6\,(1-0.31)^2 \times 4\cancel{t}\times 3\times 2\left(1-\frac{0.5}{2\times 0.8}\right)\times\left(0-\frac{1}{2\times 0.8}\right)$$

$$a_{conv} = -14.72\ m/s^2$$

Ex:-) The velocity vector corresponding to a 2-D flow field $V = 3x\,\vec{i} + 4xy\,\vec{j}$. The magnitude of rotation at the point (2,2) in rad/sec is.

A) The magnitude of rotation, $\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$$= \frac{1}{2}\left[4y - 0\right]$$

$$= 2\times 2$$

$$= 4 \text{ units (rad/sec)}$$

UNIT - 5

FLUID DYNAMICS

Fluid dynamics is a branch of fluid mechanics whic deals steady motion of the incompressible, non viscous, irrotational fluid flow under influence of different forces, different energi and power causing motion.

Fluid dynamics is basis for conservation of mass, energy and momentum. It covers the flow analysis through pipes flow open channel flow (external flow), flow through pumps and water turbines.

It is also applied to fluid flow quantities of measuring devices.

Different forces in fluid flow:-

1. self weight of fluid in the given control volume, $w = mg$
2. pressure force (static or dynamic)
3. viscous force, Frictional, surface tension force, compressibility force, Force due to turbulency.

Different energies in fluid motion:-

1. potential energy (P.E):-

This energy due to elevation (or) position of the fluid in control volume.

$$P.E = mgz$$

$$= wz \quad (kg \cdot \frac{m}{s^2} \cdot m) \text{ (or) } (N-m) \text{ (or) Joule.}$$

2. Kinetic energy (K.E):-

$$K.E = \frac{1}{2}mv^2$$

3. Pressure energy (P_x):-

$$P_x = \frac{\text{Energy}}{\text{volume}}$$

pressure energy = Pressure x volume

$$= p \times v \quad \left(\frac{N}{m^2} \times m^3\right) \text{ (or) Joule (or) } N-m$$

Ex:-) A water jet with a velocity 10 m/sec ejecting from nozzle horizontally in verticle plane. It is turned by 90° in verticle plane. Determine max. possible height, water can reach.

A. $K.E = \frac{1}{2}mv^2$

$$\frac{1}{2}mv^2 = mgz$$

$$\frac{1}{2}v^2 = gz$$

$$v^2 = 2gz$$

$$v = \sqrt{2gz}$$

$$z = \frac{v^2}{2g}$$

$$= \frac{10^2}{2\times 9.81}$$

$$= 5m.$$

Assumptions in fluid flow:-

1. One dimensional flow
2. Steady flow
3. Flow along stream line
4. Fluid is incompressible
5. Fluid is non viscous
6. velocity of the fluid flow at given section is uniform
7. Fluid is homogeneous
8. Only pressure potential and kinetic energy considered
9. Frictional forces, surface tension, Bulk modulus properties ignored
10. Flow is irrotational.

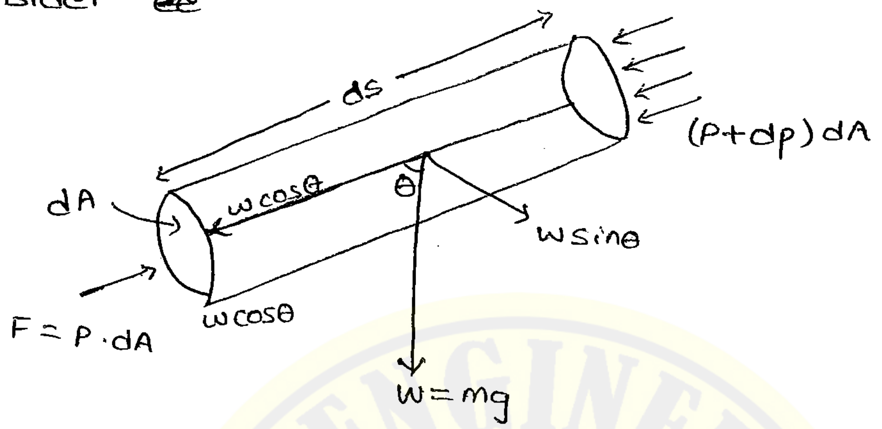
Different fluid flow motion equations:-

* 1. Equation of continuity ($m' = \rho Q = \rho AV$)
* 2. Newtons second Law of motion ($F = ma$)
* 3. Eulers motion equation.
4. Reynolds motion equation
5. Navier stokes motion equation.

Eulers motion equation:-

When a fluid subjected to motion taken place due to momentum transfer. Eulers equation is based on momentum concept and deals gravity force and pressure force under motion.

Consider ee



$$\Sigma F = ma$$

$$P\,dA - w\sin\theta\cos\theta - (P+dp)\,dA = m\left[\frac{\partial v}{\partial t} + v\cdot\frac{\partial v}{\partial s}\right]$$

$$\boxed{\frac{dp}{\rho} + g\cdot dz + v\cdot dv = 0}$$

Bernoulli's equation:-

Dimensional formula for pressure gradient

$$\frac{dp}{dx} = \frac{N/m^2}{m} = \frac{N}{m^3} = \frac{kg\text{-}m/sec^2}{m^3}$$

$$= \frac{kg}{m^2\text{-}sec^2} = M^1L^{-2}T^{-2}$$

The Bernoullis theorem states that the total energy of the moving fluid is remains constant at every section or point in the control volume or on the streamline.

Total energy of the moving fluid = constant.

It is obtained by integrating the Eulers Equation motion

i) $\int\frac{dp}{\rho} + g\int dz + v\int dv = \int 0$.

For Incompressible fluid ρ = constant.

ii) $\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant} \; (\text{Joule/Kg})$

Divide by 'g' on both sides

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \frac{\text{constant}}{g} \; \frac{(N-m)}{m} = m$$

$$m = \frac{N/m^2}{N/m^3} \quad \frac{\left(\frac{m}{s}\right)^2}{\frac{m}{s^2}} \quad m$$

iii) $p + \rho g z + \frac{v^2 \rho}{2} = \text{constant}$

$$\left[\frac{\text{energy}}{\text{unit volume}}\right] = \frac{J}{m^3} = \frac{N-m}{m^3} = \frac{N}{m^2}$$

$$P_{static} = \rho g z$$

$$P_{dynamic} = \frac{\rho v^2}{2} = \frac{kg/m^3 \times \left(\frac{m}{s^2}\right)^2}{m}$$

$$= \frac{kg}{m^3} \cdot \frac{m^2}{s^2} \Rightarrow \frac{N-m}{m^3} = \frac{J}{m^3} = \frac{N}{m^2}$$

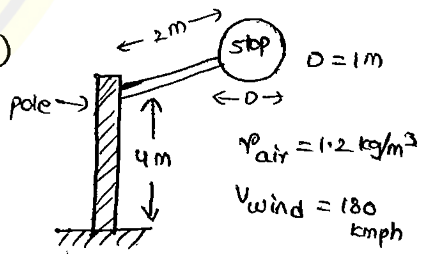
Ex:-) Find

a. Dynamic pressure on stop board (kN/m^2)

b. Dynamic force on stop board (kN)

c. Max. BM (kN-m)

d. Torque



A. a) $P_{dynamic} = \frac{\rho v^2}{2} = \frac{1.2 \times \left(180 \times \frac{5}{18}\right)^2}{2} = 1500 \; N/m^2$

b) Dynamic force $= P_D \times A$

$= 1500 \times \frac{\pi}{4} \times 1^2$

$= 1178$ N

c) Max B.M occur at point 'o' $= F \times x$

$= 1.178 \times 4$

$= 4.712$ kN-m

d) Torque $(T) = M \times r$

$= 1178 \times 2 = 2356$ N-m

suitable dia of pole $(d) = \frac{\pi}{16} \times \tau_{rod} \times d^3 = \sqrt{M^2+T^2}$

Bernoullis Equation :-

$$\left(\frac{P}{\rho g} + z\right) + \frac{v^2}{2g} = const$$

piezometric head + velocity head = Total head

P.g. NO:- 887.

3. $KE = \frac{1}{2} m u^2$

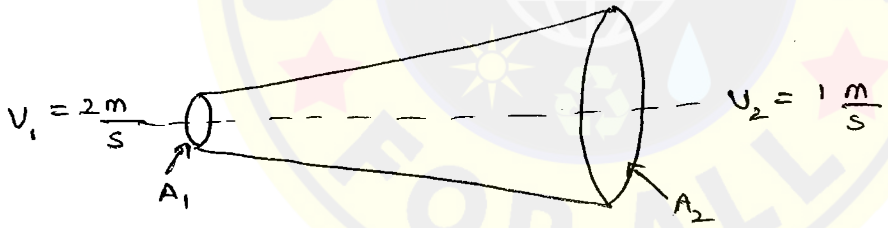
where $w = mg$

$m = \frac{w}{g}$

$KE = \frac{1}{2} \frac{W}{g} . u^2$

$\frac{KE}{W} = \frac{u^2}{2g}$.

5.



$Q = A_1 V_1 = A_2 V_2$

$= 1(2) = 2(1)$ assume

$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}.$$

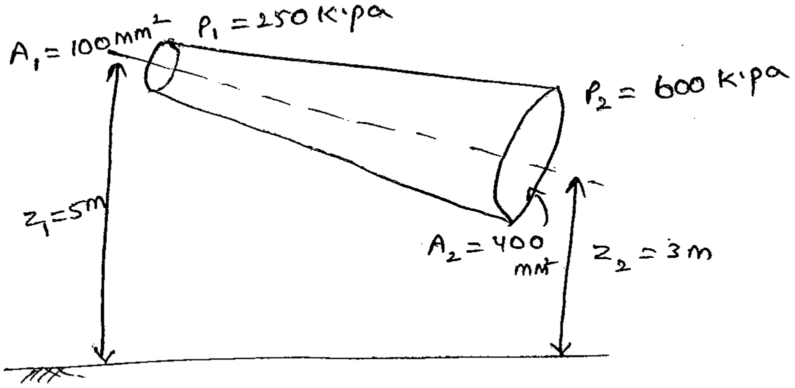
$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{v_1^2}{2g} - \frac{v_2^2}{2g}$$

$$= \frac{(2)^2}{2g} - \frac{(1)^2}{2g}$$

$$= \frac{1.5}{g}.$$

Applications of Bernoullis Energy Equation:-

1. Total energy of fluid at given points and identify the direction of the flow.



$Q = 1$ LPS
$= 1 \times 10^{-3}$ m^3/sec.

a. Determine water flow velocities at section-1 & 2

b. Mass flow rate in kg/sec.

c. Total energy head of water at section-1

d. Total energy at section-2

e. State the direction of the water flow 1-2 (or) 2-1.

f. Hydraulic losses in the duct flow.

g. power loss in watts and in Horse power.

a. A) $Q = A_1 V_1 = A_2 V_2$

$$1 \times 10^{-3} = (100 \times 10^{-6})(V_1) = (400 \times 10^{-6})(V_2)$$
$$(m^2) \quad (m/sec)$$

$V_1 = 10$ m/s

$V_2 = 2.5$ m/s

b. A) Mass flow rate ($\dot{m}$)

$$m = \rho Q$$
$$= 1000 (1 \times 10^{-3}) \quad \left(\frac{Kg}{m^3}\right) \cdot \left(\frac{m^3}{sec}\right)$$
$$= 1 \text{ Kg/sec}.$$

c. A) Total energy head at section - 1

$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g}$$

$$\Rightarrow \frac{250000}{1000 \times 10} + 5 + \frac{(10)^2}{2 \times 10}$$

$$\Rightarrow 25 + 5 + 5$$

$$\Rightarrow 35\ m \left(\frac{\text{Joule}}{\text{Newton}}\right) \text{ (or) } \left(\frac{\text{Energy}}{\text{unit weight}}\right)$$

d. A) Total energy head at section - 2

$$\frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$$

$$\Rightarrow \frac{600000}{10000} + 3 + \frac{(2.5)^2}{2 \times 10}$$

$$\Rightarrow 60 + 3 + 0.31$$

$$\Rightarrow 63.31\ m \left(\frac{\text{Joule}}{\text{Newton}}\right)$$

e. A) $E_1 = H_1 = 35\ m$

$E_2 = H_2 = 63.31$

$E_2 > E_1$. Hence water fluid from 2 to 1 (bottom to top.

f. A) Losses = H_{loss}

Flow from 2 to 1

$$H_2 - H_{Loss} = H_1$$

$$H_{Loss} = H_2 - H_1$$

$$= 63.31 - 35$$

$$= 28.31\ m$$

9. A) P_{Loss} = Power loss $= m g H_{Loss}$

$$= \rho g Q H_{Loss}$$

$$\left(\frac{kg}{m^3}\right) \cdot \left(\frac{m}{sec^2}\right) \cdot \left(\frac{m^3}{sec}\right) (m)$$

$$= 1000 \times 10 \times 1 \times 10^{-3} \times 28.31$$

$$= 283.1 \text{ watt}$$

$$= \frac{283.1}{736} \text{ H.P}$$

$$= 0.38 \text{ H.P.}$$

$$1 \text{ Hp} = \frac{75 \text{ kgf} - 1m}{sec.}$$

1 Metric H.P = 736 watt

$$= \frac{75 \times 9.81 \text{ N-m}}{sec}$$

$$= 736 \frac{\text{Joule}}{sec}$$

$$= 736 \text{ watt}$$

Convert dynamic pressure to static pressure:-

P.g No:- 86.

33. $H_A = \frac{P_A}{\rho g} + Z_A + \frac{V_A^2}{2g}$

$H_B = \frac{P_B}{\rho g} + Z_B + \frac{V_B^2}{2g}$

a. If $H_A > H_B$ then flow from Ⓐ to Ⓑ

$H_A - H_{Loss} = H_B$

$H_A = H_{Loss} + H_B$

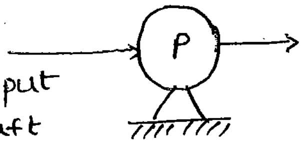
b. If $H_B > H_A$ then flow direction from B to A

$H_B - H_{Loss} = H_A$

$H_B = H_A + H_{Loss}$

Bernoulli's Energy Equation applied to pumps & Turbines.

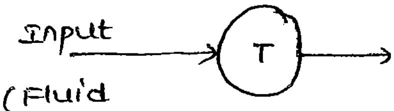
pump



input (shaft power) $= \frac{2\pi n t}{60}$

output (Fluid power) $= mgH$ $= \rho g Q H$

water turbine



Input (Fluid power) $= mgH$ $= \rho g Q H$

output (shaft power) $= \frac{2\pi n T}{60}$

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\rho g Q H}{P_{shaft}}$$

$$\eta_{turbine} = \frac{P_{shaft}}{\rho g Q H}$$

P.g NO:- 86

36.
$$\eta_{Turbine} = \frac{P_{shaft}}{P_{fluid}} = \frac{P_{shaft}}{\rho g Q H}$$

$$0.9 = \frac{P_{shaft}}{1000 \times 9.81 \times 1.2 \times 120} \quad \left(\frac{kg - m}{sec} \times \frac{m^3}{s} \times m\right) = \frac{N-m}{sec} = \frac{Joul}{sec} = watt$$

$$P_{shaft} = 1271 \cdot KW$$

37.
$$\eta = \frac{P_{shaft}}{\rho g Q H}$$

1000 lit = 1m³

$$Q = 50 \text{ lit/sec} \Rightarrow \frac{1}{20} \text{ m}^3/\text{sec}$$

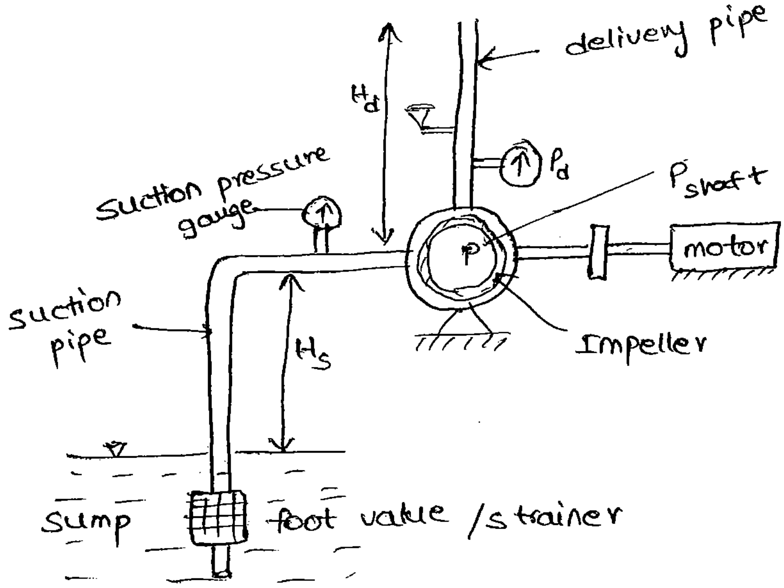
$$\eta = 100\%$$

$$1 = \frac{7.5 \text{ KW}}{1000 \times 9.81 \times \frac{1}{20} \times H}$$

$$H = \frac{7.5 \times 20 \times 1000}{9810}$$

$$= 15.3 \text{ m}$$

Fluid pump Output :-



Apply Bernoullis equation between 1 – 2

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 - h_{loss} + H_{\cancel{applied}\ added} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2}$$

energy added to the fluid

$$\eta_{pump} = \frac{\rho g Q H_{added}}{P_{shaft}}$$

$$Q = A_s V_s = A_d \cdot V_d$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 - h_{loss} + H_{added} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

velocity is same, datum head is not given

$$\frac{P_1}{\rho g} - h_{loss} + H_{added} = \frac{P_2}{\rho g}$$

P.g No:- 86.

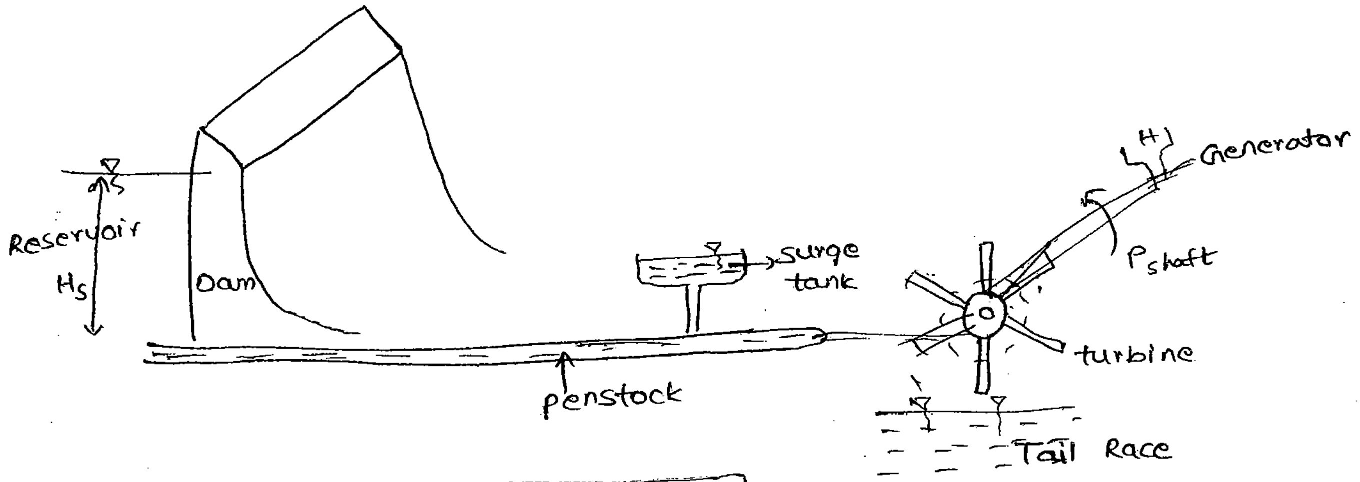
34. $$\frac{P_A}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_{loss} + H_{added} = \frac{P_B}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_A}{9810} - 3.0 + 10 = \frac{120}{9810}$$

$$P_A = -9810 \times 7 + 120$$ divide by 1000 for k·pa.

$$P_A = 51.5 \text{ k·pa}.$$

Hydraulic power plant (Turbine):-



$$\boxed{\eta = \frac{P_{shaft}}{\rho g Q H_{\cancel{external}\ extracted}}}$$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 - h_{loss} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_{Extracted}}$$

$$\frac{0}{\rho g} + H_{supply} + \frac{0^2}{2g} - h_{loss} = \frac{0}{\rho g} + \frac{0^2}{2g} + 0 + H_{extracted}$$

$$\boxed{H_{supply} - h_{loss\ pipe} - h_{loss\ D/s} = H_{extract}}$$

P.g NO:- 86

35. $Q = 3.0\ m^3/sec$ $\quad P_{shaft} = 1000\ kW$

$H = 50\ m$

$H_L = 5m$

$$\eta = \frac{P_{shaft}}{\rho g Q H_{extracted}}$$

$$1 = \frac{1000 \times 1000}{1000 \times 9.81 \times 3 \times H_{extracted}}$$

$$H_{extract} = \frac{1000}{29.43}$$

$$= 33.97$$

$$\cong 34$$

$$H_{supply} - h_{loss\ pipe} - h_{loss\ D/s} = H_{ex}$$

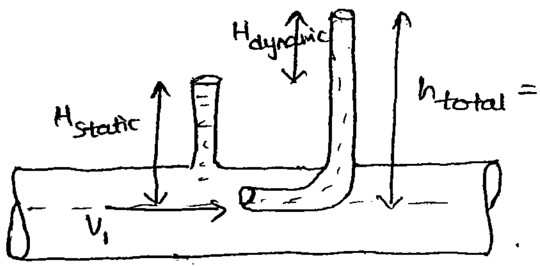
$$50 - 5 - h_{loss\ D/s} = 34$$

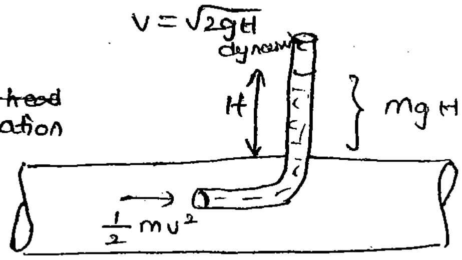
$$h_{loss\ D/s} = 11.5\ m$$

Velocity of fluid flow in a pipe - measurement:-

pitot tube:-

Pitot tube is a measuring device used for fluid flow velocity in a large pipe. It is a glass tube of L-shape which receives the large K.E.





$$V = C_v \sqrt{2g H_{Dynamic}}$$

$$= C_v \sqrt{2g\,(h_{stag} - h_{static})}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

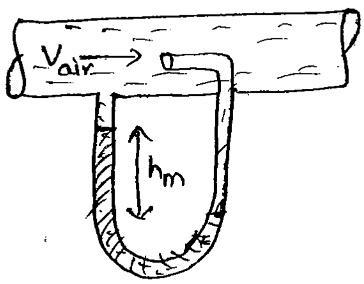
$$h_{static} + 0 + \frac{V_1^2}{2g} = h_{stag} + 0 + \frac{0^2}{2g} \qquad \therefore V_2 = 0$$

$$h_{static} + \frac{V_1^2}{2g} = h_{stagnation}$$

$$\frac{V_1^2}{2g} = h_{stag} - h_{static}$$

$$V_1 = \sqrt{2g\,(h_{stag} - h_{static})}$$

Differential pitot tube:-



$$V_{air} = C_v \sqrt{2g h_d}$$

$$= C_v \sqrt{2g\, h_m \left(\frac{S_m}{S_{air}} - 1\right)}$$

$$V_{air} = C_v \sqrt{2g\, H_m \left(\frac{S_m}{S_{air}} - 1\right)}$$

18. $\rho_{air} = 1.2\ kg/m^3$

$c_v = 1.0$

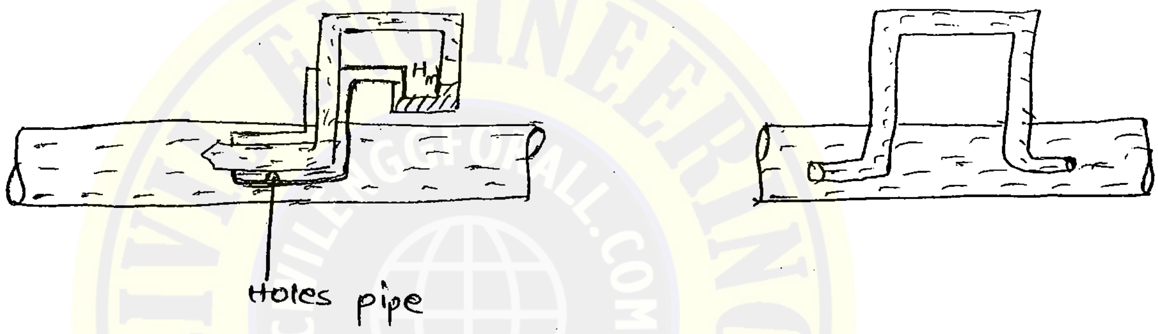
$h_m = h_{water} = 12\ mm = 0.012\ m$

$$V_{air} = c_v \sqrt{2 g h_m \left(\frac{\rho_{water}}{\rho_{air}} - 1 \right)}$$

$$= 1.0 \sqrt{2 \times 9.81 \times 0.012 \left(\frac{1000}{1.2} - 1 \right)}$$

$$V_{air} = 14$$

pitot static tube:-



It measures dynamic pressure

P.g No:- 90

2. $V = c_v \sqrt{2 g H}$

$3 = c_v \sqrt{2 \times 9.81 \times 0.475}$

$c_v = 0.983$

P.g No:- 86.

39. $d = 3\ m$

$h_{stag} = 0.3\ m$

$h_{static} = 0.24\ m$

$$V = c_v \sqrt{2g(h_{stag} - h_{static})}$$

$$= 1.0 \sqrt{2 \times 9.81 (0.06)} = \sqrt{1.177}$$

$V = 1.085$

Other flow velocity measuring devices:-

i) current meter (open channel water flow velocity)

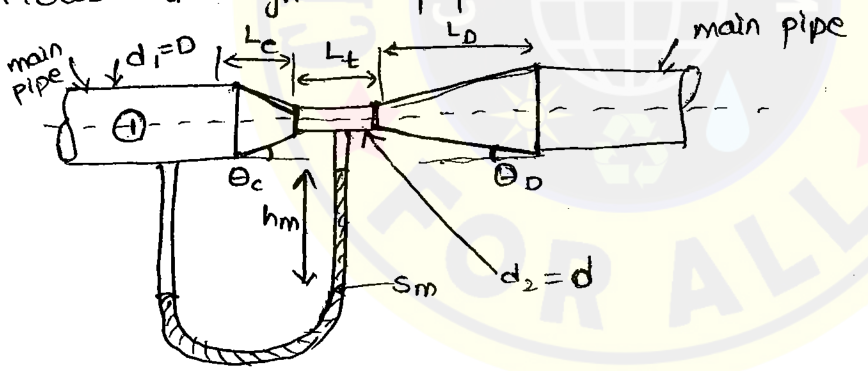
2) Anemometer (velocity of the gases and velocity of aeroplane)

Measurement of volumetric flow rate:- (or) (Discharge or mass flow rate).

1. Venturimeter
2. Orifice plate meter
3. Nozzle meter
4. Bend meter
5. Rota meter
6. Turbine meter

Venturimeter:-

It is one of the primary flow rate measuring instrument. It measures flow rate of compressible or Incompressible fluid flows through a pipe line



$$\frac{d_2}{d_1} = \frac{d}{D} = \frac{1}{4} \text{ to } \frac{1}{2}$$

$d = 25\%$ to 50%

1. * It measures flow rate (volumetric or mass flow or discharge).
2. It is only suitable for pipe flow rate measurement
3. Can be used for incompressible and compressible fluid flow rates.
4. It works on Bernoullis principle.
5. Ratio of throat dia to main pipe dia $= \frac{1}{4}$ to $\frac{1}{2}$ (20% to 50%)

6. $L_D = 4$ to 5 times L_c. (This arrangement to avoid flow separation otherwise cavitation takes place &, more length means pressure recovery). (62)

7. $\theta_c = 25^\circ$ (or) 20°, $\theta_D = 5^\circ$ (or) 4°. ($\theta_D = \theta_c/5$ to $\theta_c/4$)

8. Differential manometer only essential to measure pressure difference between main pipe and throat.

9. The reading of H_m is independent upon the main pipe orientation.

10. coefficient of discharge (C_d) value for venturimeter is 0.97 to 0.99.

$$Q_{fluid} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g(\Delta h)_{fluid}}$$

$$\Delta h_{fluid} = h_m \left(\frac{\gamma_m}{\gamma_F} - 1 \right)$$

P.g No:-94 Note:-

57. Error in measurement manometric head

$$(\Delta h)_{error} = (1 - C_d^2) \cdot h_m$$

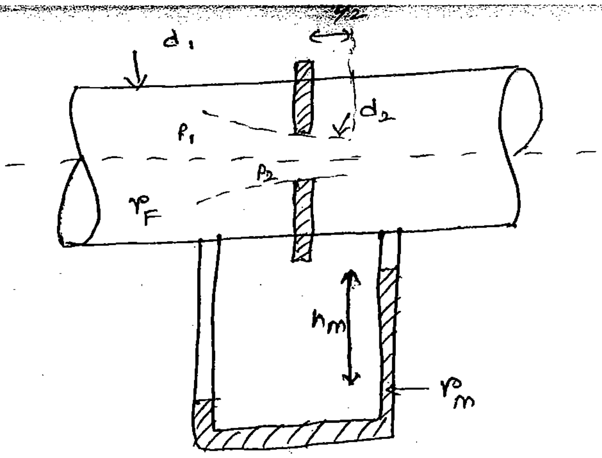
P.g No. 92

17. $(\Delta h)_{error} = (1 - 0.95^2) \times 2.8$

$= 0.273$ m.

Orifice meter:-

Orifice plate meter is one of the flow rate discharge measuring device. It consists a angular circular plate inserted accros the pipe shown in the fig.



The minimum pressure occurs at $\frac{d_2}{2}$

P.g NO:-95

61. The minimum pressure occurs at a distance of $\frac{d_2}{2}$

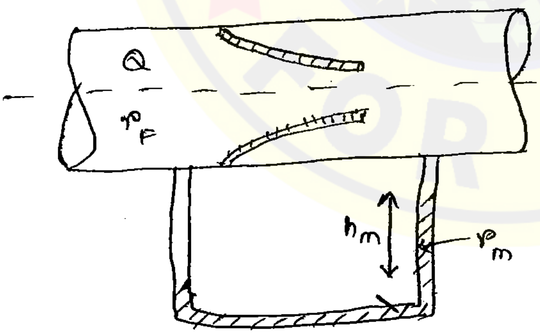
$$d_1 = 100 \text{ mm} \qquad d_2 = 40 \text{mm}$$

$$\Rightarrow \frac{d_2}{2} = \frac{40}{2} = 20 \text{ mm}$$

Nozzle meter:-

It is used for flow rate measurement of pipe flow. Across the pipe a nozzle is inserted to gauge the differential pressure due to velocity change.

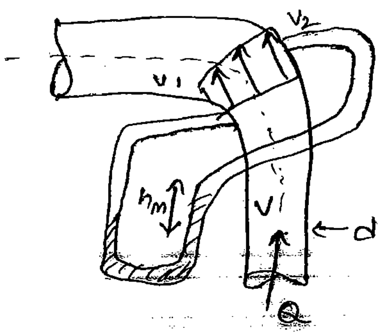
C_d of nozzle $= 0.8$



Note:-

Nozzle meter gives estimation of discharge fluid (flow rate)

Bend meter or Elbow meter:-



$$Q = C_d \cdot A V$$

$$= C_d \times \frac{\pi d^2}{4} \sqrt{2 g h_m \left(\frac{\gamma_m}{\gamma_F} - 1 \right)}$$

bend metre

C_d varies 0.86 to 0.88

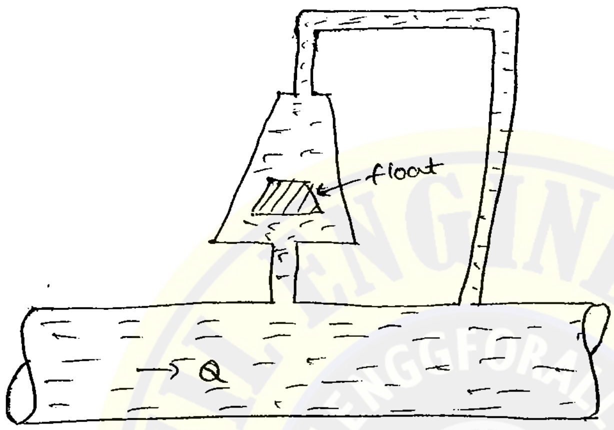
Note:-

$C_{d_{venturimeter}} > C_{d_{bend}} > C_{d_{nozzle}} > C_{d_{\cancel{orifice}\ rota}} > C_{d_{orifice}}$

$0.98 > 0.85 > 0.8 > 0.65 > 0.6$

Rotameter:-

It is a direct discharge & measuring device. It is connected to the pressure raised to the fluid pipe. It works on Archamedics (buouncy).

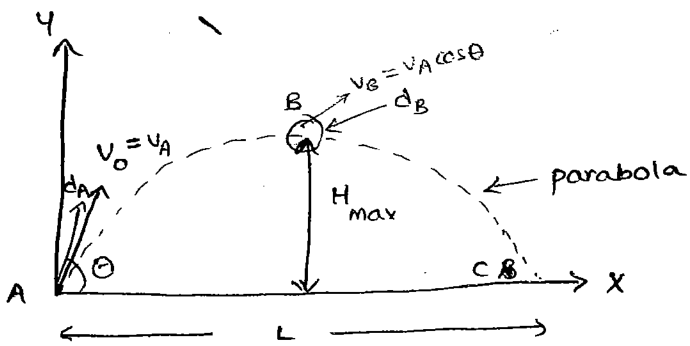


Note:-

Other devices for measuring velocity of flow (Anemometer) gases or wind flow velocity.

Current meter for measurement of flow of water channel velocity

Analysis of free liquid jet:-



$$Q = A_A V_A = A_B \cdot V_B$$

$$\frac{\pi}{4} d_A^2 \cdot V_A = \frac{\pi}{4} d_B^2 \cdot V_A \cdot \cos\theta$$

$$d_A^2 = d_B^2 \cos\theta$$

$$d_B = \frac{d_A}{\sqrt{\cos\theta}}$$

Time of Ascent (t_{A-B}) :-

$$V_{B_y} = V_{A_y} + a_y t_{A-B}$$

$$0 = V_A \sin\theta - g\, t_{A-B}$$

$$t_{A-B} = \frac{V_A \sin\theta}{g} \quad \left(\frac{m/sec}{m/sec^2} = sec\right)$$

Find jet reaches at maximum height :-

$$\text{Velocity} = \frac{\text{displacement}}{\text{Time}}$$

$$V_A \cos\theta = \frac{x}{t_{A-B}}$$

$$x = V_A \cos\theta \cdot x \frac{V_A \sin\theta}{g}$$

$$x = \frac{V_A^2 \sin\theta \cos\theta}{g} \, (m)$$

P.g No:- 89

23. $$x = \frac{V_A^2 \sin\theta \cos\theta}{g}$$

$$= \frac{20^2 \sin 45 \cos 45^\circ}{9.81}$$

$$= \frac{200}{9.81} = 20.38 \text{ m}$$

Expression for maximum height reach by jet :-

$$V_{B_y}^2 - V_{A_y}^2 = 2 a_y S_y$$

$$0^2 - (V_A \sin\theta)^2 = 2(g)(H_{max})$$

$$\boxed{H_{max} = \frac{V_A^2 \sin^2\theta}{2g}}$$

41. $H_{max} = \frac{V_A^2 \sin^2\theta}{2g}$

$= \frac{18^2 \sin(60)^2}{2 \times 9.81}$

$= 12.39\ m$

Total time of flight (t_{A-B-C}) :-

Total time of flight (t_{A-B-C}) = Time of ascent + Time of descent

$= t_{A-B} + t_{B-C}$ $\quad \therefore t_{A-B} = t_{B-C}$

$= \frac{2 V_0 \sin\theta}{g}$

→ Air resistance offers 10% loss

$0.9 \times \frac{1}{2} m (V_0 \sin\theta)^2 = m g H_{max}$

$0.9 \times \frac{V_0^2 \sin^2\theta}{2g} = H_{max}$

43. $H_{max} = 0.9 \times \frac{V_0^2 \sin^2\theta}{2g}$

$= \frac{0.9 \times (20)^2 \sin^2\theta}{2 \times 9.81}$ $\quad \therefore \theta = 90°$

$= \frac{0.9 \times 20^2}{2 \times 9.81}$

$= 18.35\ m$

Ex:- Bernoullis equation written in conventional form which represents total energy per unit of certain quantity. Identify this quantity

$\frac{P}{\rho g} + \frac{v^2}{2g} + z = const$

$\frac{P}{\rho} + \frac{v^2}{2} + zg = const$

$P + \rho g z + \frac{\rho v^2}{2} = const$

a) Energy per unit volume

✓ b) Energy per unit mass

c) Energy per unit weight

d) Energy per unit sp. weight

Ex:-) $\frac{v^2}{2g} + \frac{P}{w} + z = const$, each terms speaks

→ options same above question

Ans:- C. (Energy per unit weight)

Note:-

Bernoulli's equation not valid for Rotational flow. It is valid & Irrotational flow.

Kinetic energy correction factor (α):-

1. Kinetic energy correction factor refers Bernoulli's Energy Equation.

2. It is a multiple factor with kinetic energy head.

$$\frac{P}{\rho g} + z + \alpha \frac{v^2}{2g} = constant \quad \left(\frac{energy}{unit\ wt.} = \frac{J}{N} = \frac{N-m}{N} = m \right)$$

v = avg. velocity or Mean velocity.

3. It is taken care of Non-uniformity of velocity distribution, at given section. While deriving bernoullis equation it was assumed that velocity of fluid flow is uniform (rectangular) at given section ~~as~~ which is not correct for real fluid flow. $\alpha = 1.0$ (uniform (or) Rectangular velocity distribution)

4. 'α' is defined as the ratio of kinetic energy of the fluid at given section based on actual velocity to that ~~vel~~ K.E based on average velocity.

$$\alpha = \frac{\frac{1}{2} m_{actual} \cdot V^2_{actual}}{\frac{1}{2} m_{avg} \cdot v^2_{avg}} = \frac{\rho \cdot Q_{actual} \cdot V^2_{actual}}{\rho\ Q_{avg} \cdot V^2_{avg}}$$

$$= \frac{\cancel{m_{actual}}\ A_{actual} \cdot V_{actual} \cdot V^2_{actual}}{A_{avg} \cdot V_{avg} \cdot V^2_{avg}}$$

$$\alpha = \frac{A_{actual} \cdot V^3_{actual}}{A_{avg} \cdot V^3_{avg}}$$

$$\alpha = \frac{1}{A}\int_A \left(\frac{V_{actual}}{V_{avg}}\right)^3 dA$$

$$= \frac{1}{A}\int \left(\frac{v}{V}\right)^3 \cdot dA$$

Note :-

1. $\alpha = 1$ for uniform velocity distribution (rectangle)
2. $\alpha = 2$ for laminar flow in a pipe
3. $\alpha = 1.01$ to 1.2 for turbulent flowing pipe.

P.g NO:- ~~83~~ 91

16. $d = 15$ cm $= 150$ mm $= 0.15$ m

$Q = 70 \times 10^{-3}$ m^3/s $= 0.07$ m^3/sec

$P = 2$ cm of Hg (vacuum)

$\alpha = 1.1$, $\rho_{oil} = 750$ kg/m^3

$z = 12$ cm $= 0.12$ m

Total energy head = ?

$S_{oil} = \frac{\rho_{oil}}{\rho_{water}}$

i) $Q = AV$

$$0.07 = \frac{\pi (0.15)^2}{4} \cdot V$$

$$V = \frac{0.28}{0.07} = 4 \text{ m/sec}$$

ii) $P = \rho_{hg} \cdot g\,(-h_{Hg})$ (N/m^2)

$P = 13600 \cdot 9.81\,(-0.02)$

$= -2668.32$

iii) $\frac{P}{\rho_{oil} g} + \alpha\frac{V^2}{2g} + z = \cancel{const}$ Total energy head

$$\Rightarrow \frac{-2668}{\cancel{9810}\ 750 \times 9.81} + 1.1\,\frac{(4)^2}{2 \times 9.81} + 0.12$$

$\Rightarrow$ ~~$-0.271 + 1.017$~~ $\Rightarrow -0.3626 + 1.017$

$\Rightarrow$ ~~0.746~~ $\Rightarrow$ 0.636 m

Vortex motion:-

vortex motion is one type of liquid matter rotational motion. certain quantity of fluid made to rotate is named as vortex motion.

Two types of vortex motion:-

1. Free vortex motion
2. Forced vortex motion.

Free vortex motion:-

certain mass of the fluid rotated without efforts (without force or without Torque or without power).

Ex:- A whirl pool

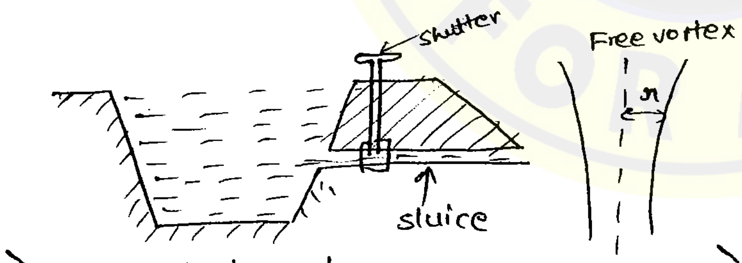
Flow at bend of the pipe.

water rushes towards man hole of the wash basin

water motion in centrifugal pump casing (not in impellar)

water motion in turbine casin (not across the runner).

pressure intensity in free vortex motion:-



open channel flow

$$\boxed{\frac{dp}{dr} = \frac{\rho v^2}{r}}$$

$$\boxed{v \propto \frac{1}{r}}$$

vr = constant

$v_1 r_1 = v_2 r_2$

$$\boxed{r \to 0, \quad v \to \infty}$$

P.g No:- 89

26. $v_1 r_1 = v_2 r_2$

$7.2 (0.12) = v_2 (0.24)$

$$v_2 = \frac{7.2(0.12)}{0.24}$$

$= 3.6$

UNIT - 6

IMPULSE MOMENTUM EQUATION

In fluid mechanics moving fluids possess momentum (mass x velocity). This momentum of the fluid exchange to th contact surface causes force on that surface.

$$\text{Mass} \times \text{velocity} = \text{Momentum}$$

$$m \times v = M$$

$$\text{Impulse } (I) = \text{force} \times \text{Time}$$

$$= N \times s$$

$$= \frac{kg-m}{sec^2} \times sec$$

$$= \frac{kg \cdot m}{s^2}$$

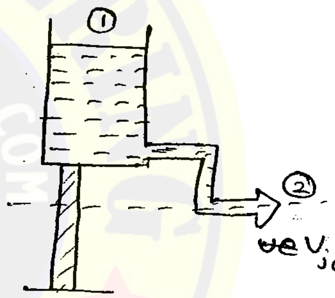
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{0}{\rho g} + \frac{0^2}{2g} + H = 0 + \frac{V_{jet}^2}{2g} + 0$$

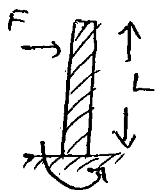
$$H = \frac{V_{jet}^2}{2g}$$

$$V_{jet} = \sqrt{2gH}$$

$$P_{shaft} = \frac{2\pi n T}{60}$$



ve $V_{jet} = \sqrt{2gH}$



Impulse momentum equation is used for estimation of force produced by the moving fluid on the contact surfa It is based on "Newtons second Law of motion".

$$F = m \times a$$

$$N = kg \times \frac{m}{sec^2}$$

a = Rate of change of velocity.

$$F = m \cdot \frac{\partial v}{\partial t}$$

$$F \cdot \partial t = m \cdot \partial v$$

Change in impulse = change in momentum

$$F \cdot t = mv$$

$$F(t_2 - t_1) = m(v_2 - v_1)$$

$$F(t_2 - 0) = m(v - u)$$

$$Ft = m(\Delta v)$$

$$F = \frac{m \cdot \Delta v}{t}$$

$$F = m' \cdot \Delta v$$

$$F_x = m(V_{fx} - V_{ix})$$

$$F_y = m(V_{fy} - V_{iy})$$

$$F_{result} = \sqrt{F_x^2 + F_y^2}$$

$$\Sigma F = m \cdot a$$

$$\Sigma F = m \cdot \Delta V$$

$$\Sigma F = \rho Q \cdot \Delta V$$

$$\Sigma F = \rho Q (V_2 - V_1)$$

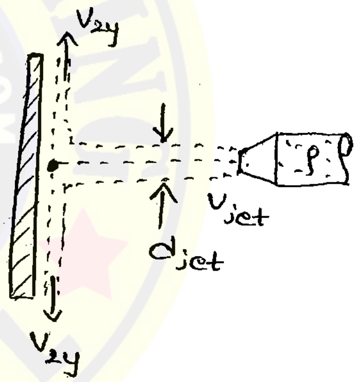
Application of Impulse momentum equation:-

1. Force exerted by a liquid jet

a. Impact of jet

a. Fixed plate normally :-

$$A_{jet} = \frac{\pi}{4} d_{jet}^2 \quad (\text{only in } m^2)$$



$$R_{N_{plate}} = F_x = m^{\cdot}(V_{2x} - V_{1x})$$

$$F_x = m^{\cdot}(0 - V_j)$$

$$F_x = -m^{\cdot} V_j$$

-ve sign indicates loss of momentum

$$F_x = m^{\cdot} V_j = \rho Q V_j = \rho \cdot A \cdot V \cdot V.$$

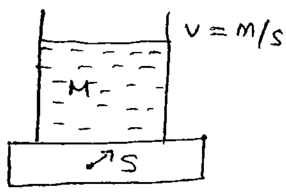
$$F_x = \rho A V^2$$

b. Impact of jet on moving plate :-

$$\boxed{S = Mg + \rho Q V} \longrightarrow$$

$$F_x = \rho A V_{01}^2$$

$$F_x = \rho A (v - u)^2$$



M = Mass of tub + water

P.g No:- 104

13. $U = \frac{1}{3} V_j$; $F = K \rho A V^2$

$$F = \rho A (V - u)^2 = \rho A (V - \tfrac{1}{3} V)^2$$

$$F = \rho A \left(\frac{3V - V}{3}\right)^2 = \rho A \left(\frac{2V}{3}\right)^2$$

$$F = \frac{4}{9} \rho A V^2 \qquad K = \frac{4}{9}$$

$$F_j = \rho A (\bar{V} + \bar{u})^2$$

$$= \rho A (V + v)^2$$

P.g NO:- 103.

1. $A_j = 1200\ cm^2 = 0.12\ m^2$, $v = 5\ m/sec$, $\gamma = 10\ kN/m^3$

$\gamma = \rho g$

$\rho = \frac{\gamma}{g} = \frac{10\times10^3}{9.81} = 1000\ kg/m^3$

$F = \rho A v^2$

$= 1000 \times 0.12 \times 5^2$

$= 3000\ N$

$F = 3\ kN$

5. $V_j = 30\ m/s$, $u = 10\ m/s$, $\eta = ?$

$\eta = 40\%$.

5. $V_j = 100\ m/s$, $u = 50\ m/s$, $d_j = 0.1\ m$

$F = \rho A (v-u)^2$

$= 1000 \times \frac{\pi (0.1)^2}{4} \times (100-50)^2$

$F = 19634.9\ N$

$= 19.635\ kN$

P.g NO:- 102

3. $A_j = 0.015\ m^2$, $v = 15\ m/s$, $u = 5\ m/s$

$F = \rho A (v-u)^2$

$= 1000 \times 0.015 \times (15-5)^2$

$= 1500\ N$

2. $A_j = 0.03\ m^2$, $F = 1\ kN$

$F = \rho A v^2$

$v = 5.77\ m/s^2$

1. $G_0 = 0.8$ $\rho = 0.8 \times 1000 = 800\ kg/m^3$

$F = 800 \times 0.02 \times 10^2 = 1600\ N$

P.g NO:- 101

11. $F = \rho A (v-u)^2$

$= 1000 \times 0.01 \times (20-10)^2$

$= 1000\ N$

12. power $= F \times u$ = Force x moving plate velocity

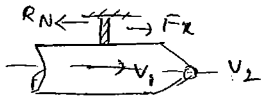
$= 1000 \times 10 = 10{,}000 \, W = 10 \, KW$

13. $F = \rho A (v+u)^2$

$= 1000 \times 0.01 \, (20+10)^2$

$= 9000 \, N$

3. $V_1 V_2$ are not same



P.g NO:- 102

14. $F_{moving\ plate} = \rho A (v-u)^2$

$= \rho A (v-u)(v-u)$

$= \rho Q (v-u)$

$= \rho \, (A V_r) \, V_r$

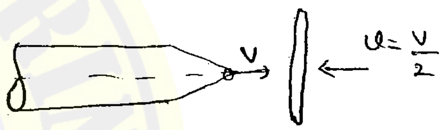
$Q = A V_r$

$= A (v+u)$

$= A \left(v + \frac{v}{2}\right)$

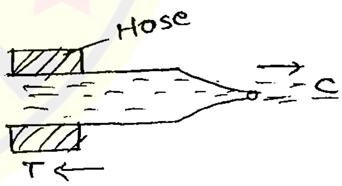
$Q = \frac{3}{2} A V$

$= 1.5 Q$



15. Nozzle force = compression

Hose = Tension



Impact of jet on inclined fixed plate:-

$F_N = \rho A v^2 \sin\theta$

$F_x = F_N \sin\theta = \rho A v^2 \sin^2\theta$

$F_y = F_N \cos\theta = \rho A v^2 \cos\theta \cdot \sin\theta$

P.g NO:- 103

6. $F_N = \rho A v^2 \sin\theta$

$=$

14. Given $\theta = 30°$, $F_N = 600\,N$, $F_x = F_N \sin\theta$

$$F_x = 600 \times \sin 30°$$
$$= 300\ N.$$

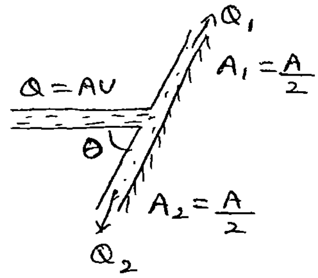
Discharges in plates:-

$$Q = Q_1 + Q_2$$

$$Q_1 = \frac{Q}{2}(1+\cos\theta)$$

$$Q_2 = \frac{Q}{2}(1-\cos\theta)$$

$$\boxed{\frac{Q_1}{Q_2} = \frac{1+\cos\theta}{1-\cos\theta}}$$



P.g. NO:- 102

16. $\frac{Q_1}{Q_2} = \frac{1+\cos 45°}{1-\cos 45°} = 5.83$

Impact of jet on Fixed curved plate:-

$$F_x = \rho Q(V_f - V_i)$$
$$= \rho AV(-V\cos\theta - V)$$
$$F_x = -\rho AV^2(1+\cos\theta)$$

P.g NO:- 103

2. Given $A_j = 2000\ mm^2 = 2\times10^{-3}\ m^2$, $V = 10\ m/s$, $\theta = 120°$

$$F = 1000 \times 2\times10^{-3} \times 10^2 (1+\cos 120°)$$
$$F = 100\ N$$

Jet force on unsymmetrical fixed curved plate:-

$$F_x = -\rho AV(V\cos\beta + V\cos\alpha)$$
$$F_x = -\rho AV^2(\cos\beta + \cos\alpha)$$
$$F_y = \rho AV^2(\sin\beta - \sin\alpha)$$

Ex:- 2)

$$F_x = R_N = \rho Q(V_f - V_i)$$
$$= \rho Q(-V - V)$$
$$= \rho Q - 2V$$
$$R_N = F_x = -2\rho QV$$

$$F_{clamp-1} = -\rho Q V$$

$$F_{clamp-2} = \rho Q V$$

$$F_{result} = \sqrt{2}\,\rho Q V$$

$$\Sigma F_y = m^{\circ}(V_{fy} - V_{iy})$$
$$= \rho Q\,(0 - V)$$
$$= -\rho Q V$$

$$\Sigma F_x = m^{\circ}(V_{fx} - V_{ix})$$
$$= \rho Q\,(0 - V)$$
$$= -\rho Q V$$

Ex:-3) $\cancel{\Sigma F_x =}$ $Q = A_p \cdot V_p = A_2 \cdot V_2$

$$= 1\times10^{-4} \times 1\times10^{-2}$$
$$= 1\times10^{-6} \times V_2$$
$$V_2 = 1\,m/sec$$

$$F = m^{\circ}(V_2 - V_1)$$
$$= \rho Q\,(V_2 - V_1)$$
$$= 1000 \times 1\times10^{-4}\,(1\times10^{-2})\,(1 - 0.01)$$
$$F = 9.9\times10^{-4}\ N$$

$$A_{piston} + V_{piston} = Q_{air} + A_{neg} \cdot V_{Blood}$$

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$0 + \frac{V_N^2}{2g} = H + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{V_N^2 - 2gH}$$

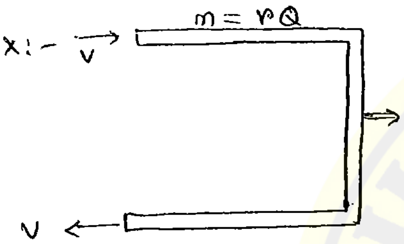
$$V_2 = \sqrt{10^2 - 2\times10\times4} = 4.47\,m/sec$$

$$\Sigma F_y = m^{\circ}(V_{fy} - V_{iy})$$
$$-mg = \rho Q\,(0 - V_2)$$
$$mg = \rho Q V_2$$
$$m = \frac{1000\times0.1\times10\times4.47}{9.81} = 455.66\ kg$$
$$W = 4470\ N.$$

Control volume:-

1. In fluid science control volume refers to a region in spac upon which attention is focus.
2. control volume is an imaginary and arbitary selected spac in which a fixed quantity of matter dealt.
3. Across the control volume the matter transfers. and also energy can cross the boundary.
4. control volume stationary as well as movable.
5. For the movable control volume relative velocity is consider
6. Once control volume moves it is indication of work done.

Ex:-



$F = 2 \cdot m \cdot v$

$= 2 \times \rho Q v$

proof:- $F = m(v_f - v_s)$

$= m'(-v - v_s)$

$= -2mv$

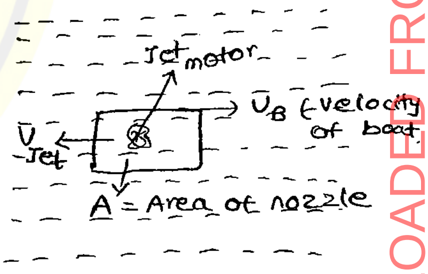
Boat analysis:-

1. Boat in a river (Relative motion):-

$F_{boat} = m'(\Delta v)$

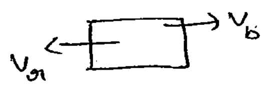
$= \rho Q (V_r - V_B)$

$F_{boat} = \rho A V_r (V_r - V_B)$



P.g No:- 103

4. $V_{boat} = 36$ kmph $\Rightarrow 36 \times \frac{5}{18} = 10$ m/sec



$V_{Jet} = 30$ m/sec

$V_{relative}$ velocity $= 30 - (-10) = 40$ m/sec

$A = 20{,}000 \text{ mm}^2$

$F_{boat} = \rho A V_r (V_r - V_B)$

$= 1000 \times 20000 \times 10^{-6} \times 40 (40 - 10)$

$= 24000$ N

$= 24$ kN

5. $\eta_{boat\ propulsion} = \frac{O/P}{I/P} = \frac{\text{workdone/sec}}{\text{K.E of Jet/sec}}$

$$= \frac{F_{boat} \cdot U_B}{\frac{1}{2} m' V_r^2}$$

$$= \frac{2 \cdot U_B}{V_r + U_B}$$

$$= \frac{2 \times 10}{40 + 10} = \frac{20}{50} = 0.4 = 40\%$$

Efficiency (η) :-

$$\eta = \frac{\text{output}}{\text{input}}$$

$$= \frac{\text{workdone by the jet on plate}}{\text{Kinetic energy of the jet}}$$

$$= \frac{\text{power}}{\frac{1}{2} m U^2}$$

$$= \frac{F \times \frac{\text{displacement}}{\text{time}}}{\frac{1}{2} m' U^2}$$

UNIT - 7

FLOW THROUGH PIPES

1. pipe is a close conduit through which fluid under pressure convey or transported.
2. Due to friction, viscosity, bend in the pipe, pipe enlargmen and contractions. Loss of pressure energy takes place.
3. While designing pipe (size of the pipe) is to be decided, and also power required to pump the fluid can be estimated.
4. Type of the flow in the pipe is decided by the Reynolds number range. If Reynolds no. less than 2000 then that flow is called Laminar flow.
5. Reynolds no. more than 4000 then flow is called Turbulen
6. In between 2000 to 4000 that flow is called Transistiona flow.

$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$= \frac{\text{Mass} \times \text{Acceleration}}{\tau \times \text{Area}}$$

$$= \frac{\rho L^2 V^2}{\mu \cdot VL}$$

$$= \frac{\rho L V}{\mu}$$

'L' is replaced by 'D'

$$\boxed{R_e = \frac{\rho D V}{\mu} = \frac{VD}{\upsilon}}$$

For non circular pipes, $\boxed{R_e = \frac{V D_H}{\upsilon}}$ $D_H = \frac{4A}{P}$

7. Reynolds no is a bench mark to decide the flow patterns in the pipe as it covers physical system dimension (D), flow characteristic (V) and fluid property (υ)

Ex:- prove that water flow through pipes always turbulent flow?

A. $\uparrow R_e = \dfrac{vd}{\nu\downarrow}$

$$R_e = \frac{vd}{1\times10^{-6}}$$

$$R_e = vd\times10^{6} > 4000$$

Due to low kinematic viscosity of water for any size of the pipe and any value of velocity the Reynolds no. always greater than 4000.

Reynolds number for non circular pipe:-

$$R_{e\,\text{circular pipe}} = \frac{VD_H}{\nu}$$

$$D_H = \text{Hydraulic diameter} = \frac{4A}{P}$$

$$= \frac{4\times \text{c/s Area of duct}}{\text{Perimeter}}$$

$$D_{H\,\text{circular}} = \frac{4\times\frac{\pi D^2}{4}}{\pi D} = D$$

$$D_{H\,\cancel{\text{rectangle}}\ \text{square}} = \frac{4\times D^2}{4D} = D$$

$$D_{H\,\text{rectangle}} = \frac{4\times B\times D}{2(B+D)} = \frac{2BD}{B+D}$$

Based on the type of flow the losses can be estimated. For laminar flow Hygeins formula used. For turbulant flow Darcy's formula is used.

There are two types of losses in pipe flow.

1. Major losses (frictional loss)
2. Minor losses (Loss other than friction)

Ex:- Bend in the pipe, enlargement of the pipe, contraction of the pipe, pipe cuplings and valves ~~etc~~, entrance loss and exit loss.

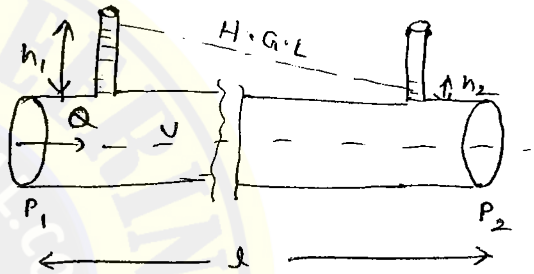
The above losses causing pressure drop. pressure drop means pumping work increase. pump is a device which supply energy to the fluid. The pipe analysis means.

1. Find the size.
2. Find the power required to maintain the flow.
3. No. of pump stations.
4. pipe Network (series and parallel connections).

Expression for loss of pressure head due to friction (Darcy head loss) :-

$$h_f = h_1 - h_2$$

$$= \frac{P_1 - P_2}{\rho g}$$



$$\boxed{h_f = \frac{f l v^2}{2gd}}$$

$$= \frac{fl}{d}\left(\frac{v^2}{2g}\right)$$

Replace 'v' in terms of 'Q' and 'd'

$$Q = A v$$

$$= \frac{\pi d^2}{4} \cdot v$$

$$v = \frac{4Q}{\pi d^2}$$

$$h_f = \frac{fl}{d}\left(\frac{[4Q/\pi d^2]^2}{2g}\right)$$

$$= \frac{fl}{d} \cdot \frac{Q^2}{2g} \cdot \frac{16}{\pi^2 d^4}$$

$$= \frac{8 f l Q^2}{g \pi^2 d^5}$$

$$\boxed{h_f = \frac{f l Q^2}{12.1 d^5}}$$

For laminar flow, friction factor

$$f = \frac{64}{Re}$$

For same discharge

$$h_f \propto \frac{1}{d^5}$$

$$\frac{h_{f_1}}{h_{f_2}} = \left(\frac{d_2}{d_1}\right)^5$$

Note:-

For same discharge diameter of the pipe is doubled then head loss reduced by $\frac{1}{32} h_{f_1}$

P.g No:- 131.

1. Given $d = 0.3\ m$, $V = 0.5\ m/sec$, $h_f = 0.4m$, $l = 39.25\ m$, $f = ?$

$$h_f = \frac{flv^2}{2gd}$$

$$0.4 = \frac{f(39.25)(0.5)^2}{2 \times 9.81 \times 0.3}$$

$$f = 0.24$$

$$f = 4f'$$

$$f' \text{(coefficient of friction)} = \frac{f}{4} = \frac{0.24}{4} = 0.06$$

3. Given $l = 10\ m$, $d = 0.12\ m$ $h_f = 0.84m$ $f' = 0.1$

$$f = 4f'$$
$$f = 0.4$$

$$h_f = \frac{flv^2}{2gd}$$

$$0.84 = \frac{0.4 \times 10 \times v^2}{2 \times 9.81 \times 0.12}$$

$$v = 0.7\ m/s$$

9. Given $\nu = 2 \times 10^{-5}\ m^2/sec$, $V = 1.2\ m/sec$ $d = 60\ mm = 0.06\ m$

$$R_e = \frac{Vd}{\nu}$$

$$= \frac{1.2 \times 0.06}{2 \times 10^{-5}}$$

$$R_e = \frac{7200}{2} = 3600$$

72

15. $\rho = 850\ kg/m^2$ $\quad \mu = \frac{1.86}{10} \left(\frac{N-s}{m^2}\right)$ $\quad d = 0.15\ m$ $\quad v = 0.05\ (m/sec)$

$= 0.186$

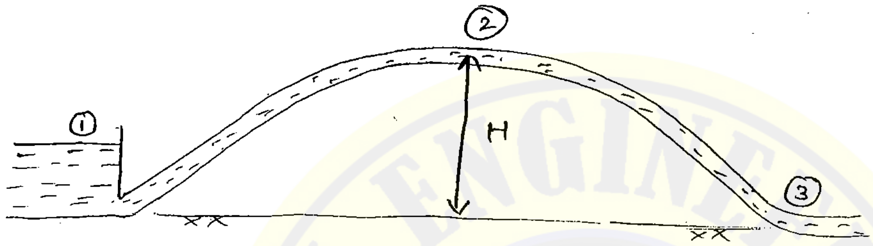
$$R_e = \frac{\rho v d}{\mu}$$

$$= \frac{850 \times 0.05 \times 0.15}{0.186}$$

$$R_e = 34 < 2000$$

$\therefore$ Laminar flow

Siphon pipe:-



Bernoullis equation between 1 and 2

$$\left(\frac{P_1}{\rho g} + z_1 + \frac{h^2}{2g}\right) - h_f = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$$

P.g NO:- 133

26. Given $\quad H = 4m$

$$\frac{0}{2\rho g} + 0 + \frac{0^2}{2g} - 2 = \frac{P_2\ (N/m^2)}{1000 \times 9.81} + 4 + 0.5$$

$$P_2 = 63.65 \times 10^{-3}\ N/m^2$$

$$= -63.65\ kpa$$

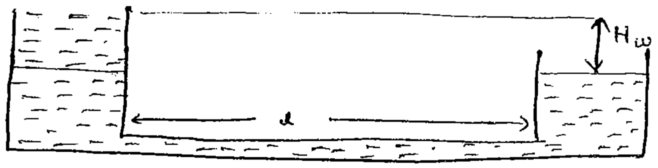
P.g NO:- 127

34. $$\frac{P_2}{\rho g} = -h_f - H - \frac{v_2^2}{2g}$$

$$= -2 - 4 - 0.5$$

$$= -6.5m \text{ of water.}$$

Ageing of a pipe:-



$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} - h_f = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g}$$

$$\frac{0}{\rho g} + H + \frac{0^2}{2g} - h_f = \frac{0}{\rho g} + 0 + 0$$

$$H - h_f = 0$$

$$\therefore H = h_f$$

$$H = \frac{f l Q^2}{12.1 d^5}$$

$$f, Q^2 = \text{constant.}$$

$$\therefore f \propto \frac{1}{Q^2}$$

$$\boxed{f = K \cdot Q^{-2}} \longrightarrow ①$$

$$\frac{f_1}{f_2} = \left(\frac{Q_1}{Q_2}\right)^{-2}$$

This is not valid for aging. This is valid for instant

Differentiate $f = K \cdot Q^{-2}$

$$\frac{df}{dt} = K \cdot -2 \cdot Q^{-3} \cdot \frac{dQ}{dt}$$

$$df = -2K \cdot Q^{-3} \cdot dQ \longrightarrow ②$$

$$\frac{eq ②}{eq ①} = \frac{df}{f} = \frac{-2K \cdot Q^{-3} \cdot dQ}{K \cdot Q^{-2}}$$

$$\boxed{\frac{df}{f} = -2 \cdot \frac{dQ}{Q}}$$

P.g NO:- 132.

11. $\frac{df}{f} = -2 \cdot \frac{dQ}{Q}$

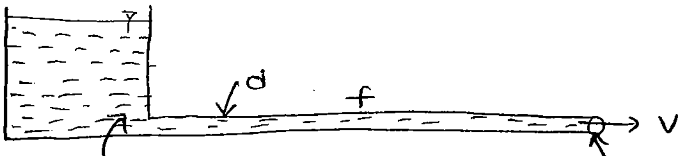
$= -2(+1\%)$

$= -2\%$

21. $\frac{df}{f} = \cancel{-2(+25\%)}$

$=$

Expression for power transmitted through a pipe, condition for max. power transmission and corresponding max. transmissio efficiency :- (with major loss)



$P_{input} = \rho g Q H$

$P_{outlet\ frm\ pipe} = \rho g Q (H - h_f)$

$$\eta_{pipe} = \frac{O/P}{I/P}$$

$$= \frac{H - h_f}{H}$$

$$\boxed{\eta_{pipe} = 1 - \frac{h_f}{H}}$$

Hydraulic Transmission of power :-

condition for Maximum power transmission by a pipe:

$$P_{output} = \rho \cdot g \cdot Q (H - h_f)$$

$$= \rho \cdot g\, A \cdot V \left(H - \frac{f l V^2}{2gd}\right)$$

$$= \rho g A \left(HV - \frac{f l V^3}{2gd}\right)$$

To maximize power transmission

$$\frac{dP}{dV} = \rho g A \left(H - \frac{3 f l V^2}{2gd}\right)$$

$$0 = \rho g A (H - 3 h_f)$$

$$0 = H - 3h_f$$

$$H = 3h_f$$

$$\boxed{h_f = \frac{H}{3}}$$

$$P_{output\ pipe} = \rho \cdot g \cdot Q \left(H - \frac{H}{3}\right)$$

$$= \rho \cdot g \cdot Q \left(\frac{2}{3} H\right)$$

$$= \frac{2}{3} \cdot \rho g Q H$$

$$\eta_{pipe} = 1 - \frac{h_f}{H}$$

$$= 1 - \frac{\frac{1}{3}H}{H}$$

$$= 1 - \frac{1}{3}$$

$$\boxed{\eta_{pipe} = \frac{2}{3} = 66.6\,\%}$$

P. g NO:- 127

35. Given $w = 10,\quad Q = 1,\quad H = 99$

$$P_{max\ pipe} = \frac{2}{3} \cdot w \cdot Q \cdot H$$

$$= \frac{2}{3}\ \rho g Q H$$

$$= \frac{2}{3}\ \gamma \cdot Q \cdot H$$

$$= \frac{2}{3} \cdot w \cdot Q \cdot H$$

$$= \frac{2}{3} \times 10 \times 1 \times 99$$

$$= 660\ kW.$$

36. $\cancel{P_{max\cdot pipe}} = \cancel{\frac{2}{3} \times 10 \times}$

$$\eta_{turbine} = \frac{P_{shaft}}{P_{water}} = \frac{P_{shaft}}{\rho g Q H} \qquad \therefore\ \eta_{turbine} = 1$$

$= 1$

$$P_{shaft} = \rho \cdot g \cdot Q \cdot H$$

$$= 1000 \times 10 \times 100 \times 75$$

$$= 75000000\ \text{watt}$$

$$= 75000\ KW$$

$$= \frac{75000}{0.75}\ HP$$

$$= 1,00,000\ H.P$$

Note:-

$$1\ \text{metric}\ H.P = 75\ \frac{Kgf-m}{sec}$$

$$= 75 \times 10\ \frac{N-m}{sec}$$

$$= 750\ \frac{N-m}{sec}$$

$$= \cancel{750}\ 750\ W = 0.75\ kW$$

37. $\eta_{pump} = \frac{O/P}{I/P} = \frac{P_{pump\,fluid}}{P_{shaft}} = \frac{\rho g Q H + h_f}{P_{shaft}}$

$\therefore \eta_{pump} = 100\%$ assume

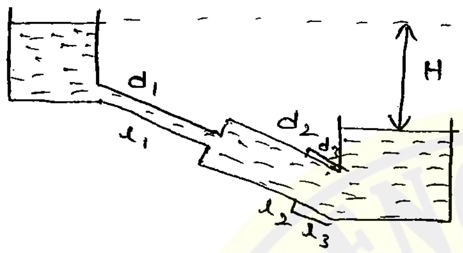
$$P_{shaft} = \rho g Q (H + h_f)$$

$$= 1000 \times 10 \times 0.1 \ (10 + 5)$$

$$= 15000 \ W$$

$$= 15 \ kW$$

pipes in series:-



$Q_1 = Q_2$ (or) $A_1 V_1 = A_2 V_2$

$h_f = h_{f_1} + h_{f_2}$

$$h_{f_e} = \frac{f_1 l_1 Q_1^2}{12.1\, d_1^5} + \frac{f_2 l_2 Q_2^2}{12.1\, d_2^5}$$

$$\frac{f_e l_e Q_e^2}{12.1\, d_e^5} = \frac{f_1 l_1 Q_1^2}{12.1\, d_1^5} + \frac{f_2 l_2 Q_2^2}{12.1\, d_2^5}$$

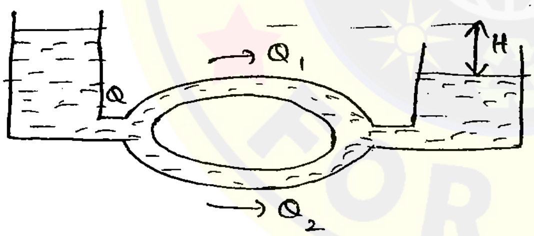
$$Q_e = Q_1 = Q_2$$

$$\frac{f_e l_e}{d_e^5} = \frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5}$$

$$l_e = l_1 = l_2$$

$$\boxed{\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5}}$$

pipes in parallel:-



$$Q = Q_1 + Q_2$$

$$= A_1 V_1 + A_2 V_2$$

$$h_{f_1} = h_{f_2} = h_{f_e}$$

$$\frac{f_1 l_1 Q_1^2}{12.1\, d_1^5} = \frac{f_2 l_2 Q_2^2}{12.1\, d_2^5} = \frac{f_e l_e Q_e^2}{12.1\, d_e^5}$$

Assume $l_1 = l_2 = l_e$

$f_1 = f_2 = f_e$

$$\frac{Q_1^2}{d_1^5} = \frac{Q_2^2}{d_2^5} = \frac{Q_e^2}{d_e^5}$$

$$Q_1 = Q_e \sqrt{\left(\frac{d_1}{d_e}\right)^5}$$

$$Q_2 = Q_e\sqrt{\left(\frac{d_2}{d_e}\right)^5}$$

$$Q_e = Q_1 + Q_2$$

$$Q_e = Q_e\left[\left(\frac{d_1}{d_e}\right)^{5/2} + \left(\frac{d_2}{d_e}\right)^{5/2}\right]$$

$$\boxed{\therefore\ d_e^{5/2} = d_1^{5/2} + d_2^{5/2}}$$

For 'n' pipe same diameter

$$\boxed{d_e = (\cancel{\ }) \, (n)^{2/5} \cdot d}$$

P.g. NO:- 126

18. Given $\nu = 8\ cm^2/sec$, $d = 8\ cm$, $Q = 3200\pi\ cm^3/sec$

$$Re = \frac{Vd}{\nu} \qquad \therefore Q = AV = \frac{\pi d^2}{4} \cdot V$$

$$Re = \frac{\frac{4Q}{\pi d^2} \cdot d}{\nu}$$

$$Re = \frac{4Q}{\pi d \nu}$$

$$= \frac{4 \times 3200\cancel{\pi}}{\cancel{\pi} \times 8 \times 8}$$

$$Re = 200 < 2000$$

$\therefore$ Laminar flow

Note:-

$$\therefore Re \propto \frac{1}{d} \quad (Q = \text{const})$$

$$\frac{Re_1}{Re_2} = \frac{d_2}{d_1}$$

24. $$\frac{h_{fA}}{h_{fB}} = \frac{d_B^5}{d_A^5}$$

$$\therefore d_A = 0.2d_B + d_B = 1.2d_B$$

$$= \frac{\cancel{d_B^5}}{\cancel{0.2d_A}\ \cancel{1.2d_B}} \quad \left(\frac{d_B}{1.2d_B}\right)^5$$

$$= 0.402$$

25. Given $d_A = d_B$, $l_A = l_B$, ~~$d_A = 4 \cdot d_B$~~ $f_A = 4 \cdot f_B$

If parallel $h_{f_A} = h_{f_B}$

$$\frac{f_A l_A Q_A^2}{12.1\, d_A^2} = \frac{f_B l_B \cdot Q_B^2}{12.1\, d_B^2}$$

$$\frac{Q_A}{Q_B} = \sqrt{\left(\frac{f_B}{f_A}\right)}$$

$$= \sqrt{\frac{1}{4}}$$

$$= 0.5$$

28. $d_1 = 10\text{ cm}$ $d_2 = 20\text{ cm}$, $l_1 = l_2$

$l_e = l_1 + l_2$

$l_e = 2l$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5}$$

$$\frac{2\cancel{l}}{d_e^5} = \cancel{l}\left(\frac{1}{10^5} + \frac{1}{20^5}\right)$$

$$\frac{2}{d_e^5} = \frac{20^5 + 10^5}{20^5 \cdot 10^5}$$

$$d_e^5 = \frac{2 \times 20^5 \times 10^5}{20^5 + 10^5}$$

$$d_e^5 = 193939.3$$

$$d_e = (193939.3)^{1/5}$$

$$= 11.41\text{ cm}$$

30. Given $d_e = 30\text{ cm}$

$$d_e = (2)^{2/5} \cdot d$$

$$30 = (2)^{2/5} \cdot d$$

$$d = 22.73\text{ cm}$$

$$\cong 25\text{ cm}.$$

32. $d_1 = 2d, \quad d_2 = d, \quad l_1 = l_2, \quad f_1 = f_2, \quad \frac{Q_1}{Q_2} = ?$

$$h_{f_1} = h_{f_2}$$

$$\frac{f_1 l_1 Q_1^2}{12.1 d_1^5} = \frac{f_2 l_2 Q_2^2}{12.1 d_2^5}$$

$$\frac{Q_1^2}{Q_2^2} = \left(\frac{d_1}{d_2}\right)^5$$

$$\frac{Q_1}{Q_2} = \left(\frac{d_1}{d_2}\right)^{5/2}$$

$$= \left(\frac{2d}{d}\right)^{5/2}$$

$$= (2)^{5/2}$$

$$= 4\sqrt{2}$$

Minor losses in pipe flow:-

1. Any loss other than friction loss (major loss) is known as minor loss.
2. If minor losses less than are equal to 5% then minor losses can be ignored.
3. Minor losses are many.

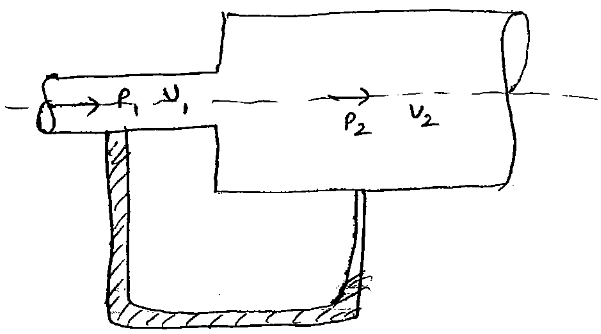
a. Loss due to sudden enlargement of the pipe (sudden expansion)

$$h_{L_{enlargement}} = \boxed{h_{L_{expansion}} = \frac{(V_1 - V_2)^2}{2g}} \qquad \therefore Q = A_1V_1 = A_2V_2$$

$$= \frac{V_1^2}{2g}\left[1 - \frac{V_2}{V_1}\right]^2 \Rightarrow \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

$$= \frac{V_1^2}{2g}\left[1 - \frac{d_1^2}{d_2^2}\right]^2$$

$$h_{L_{enlargment}} = h_{L_{expan}} = \frac{V_1^2}{2g} \cdot K_e$$



P.g NO:- 128

10. $Q = A_1 V_1 = A_2 V_2$

$$\frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} d_2^2 v_2$$

$$(0.1)^2 \times 5 = (0.2)^2 v_2$$

$$v_2 = 1.25 \text{ m/sec}$$

$$h_{L\,expansion} = \frac{(v_1 - v_2)^2}{2g}$$

$$= \frac{(5 - 1.25)^2}{2 \times 9.81}$$

$$= 0.717$$

20. $$h_{L\,exp} = \frac{v_1^2}{2g}\left[1 - \frac{d_1^2}{d_2^2}\right]^2$$

$$= \frac{v_1^2}{2g}\left[1 - \left(\frac{4}{8}\right)^2\right]^2$$

$$= \frac{v_1^2}{2g}\left[1 - \frac{1}{4}\right]^2$$

$$= \frac{v_1^2}{2g}\left[\frac{3}{4}\right]^2$$

$$= \frac{9}{16} \cdot \frac{v_1^2}{2g}$$

P.g NO:- 131.

4. Given $d_1 = 40\,mm$, $d_2 = 60\,mm$, $v_1 = 0.9$ m/s $v_2 = ?$

$$\cancel{\left(\frac{40}{60}\right)^2 \times 0.9 = v_2} \quad \cancel{h_{L\,exp}} =$$

$$\underline{4} \quad h_{L\,exp} = \frac{(v_1 - v_2)^2}{2g} \Rightarrow \frac{(0.9 - 0.4)^2}{2 \times 9.81} = 12.5\ m$$

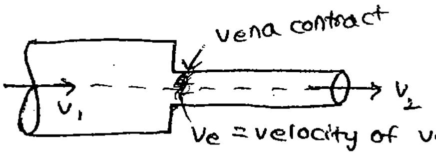
$$Q = A_1 v_1 = A_2 v_2$$

$$= \frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} d_2^2 v_2$$

$$(40)^2 \times 0.9 = (60)^2 \cdot v_2$$

$$v_2 = 0.4 \text{ m/s}$$

b. Loss due to sudden contraction :-



$$\boxed{h_{L_{contract}} = \frac{(V_c - V_2)^2}{2g}}$$

$$Q = A_1V_1 = A_2V_2 = A_cV_c$$

$$\frac{A_c}{A_2} = \frac{V_2}{V_c} \Rightarrow C_c = \frac{V_2}{V_c}$$

$$h_{L_{contract}} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

$$= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$\boxed{h_{L_{contract}} \simeq 0.5\frac{V_2^2}{2g}}$$

P.g. No:- 128

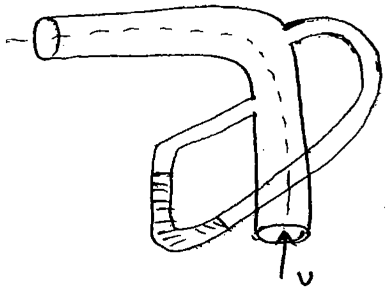
9. Given $d_1 = 5cm$ $d_2 = 2.5cm$, $h_{L_{contract}} = 0.5$, $\sqrt{2g} = 4.43$

$$h_{L_{contract}} = 0.5\frac{V_2^2}{2g}$$

$$\cancel{0}.5 = 0\cancel{.}5\frac{V_2}{\sqrt{2g}}$$

$$V_2 = \sqrt{2g} = 4.43 \text{ m/sec.}$$

c. Loss due to bend in pipe :-



$$\frac{P_1 - P_2}{\rho g} = h_1 - h_2 = h_m\left(\frac{S_m}{S_F} - 1\right)$$

$$h_1 - h_2 = K_{bend} \cdot \frac{V^2}{2g}$$

5. Given $K_{bend} = 12$, $h_1 - h_2 = 0.36$

$$h_1 - h_2 = K_{bend} \cdot \frac{v^2}{2g}$$

$$0.36 = 12 \cdot \frac{v^2}{2 \times 9.81}$$

$$V = 0.767 \ m/s$$

$$Q = A V$$

$$= \frac{\pi}{4}(0.05)^2 \times 0.767 \quad m^3/s$$

$$= 1.5 \ lit/sec.$$

Ex:- A uniform cross sectional pipe carries water at the rate of $360 \ m^3/hr$. The cross sectional area is $1000 \ cm^2$. The bend factor is 2. Take $g = 10 \ m/s^2$. Determine loss of head due to the bend in the pipe.

A. Given $Q = 360 \ m^3/hr = \frac{360}{60 \times 60} \frac{m^3}{s}$

$$A = 100 \ cm^2$$

$$K_{bend} = 2$$

$$g = 10$$

$$Q = A V$$

$$0.1 = 100 \ V$$

$$0.1 = (100 \times 10^{-4}) \ V$$

$$V = 10 \ m/sec.$$

$$(h_1 - h_2) = K_{bend} \cdot \frac{v^2}{2g}$$

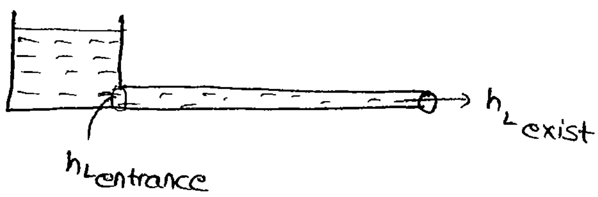
$$= 2 \times \frac{10^2}{2 \times 10}$$

$$(h_1 - h_2) = 10 \ m$$

d. Loss at entrance & exit of a pipe:-

$$h_{L_{entrance}} = 0.5\frac{v^2}{2g}$$

$$h_{L_{exit}} = \frac{v^2}{2g}$$



$$\% \text{ increase in loss} = \frac{\frac{v^2}{2g} - 0.5\frac{v^2}{2g}}{0.5\frac{v^2}{2g}}$$

$$= \frac{0.5\frac{v^2}{2g}}{0.5\frac{v^2}{2g}} = 1 \cancel{\%} = 100\%$$

% increase in loss = 100%.

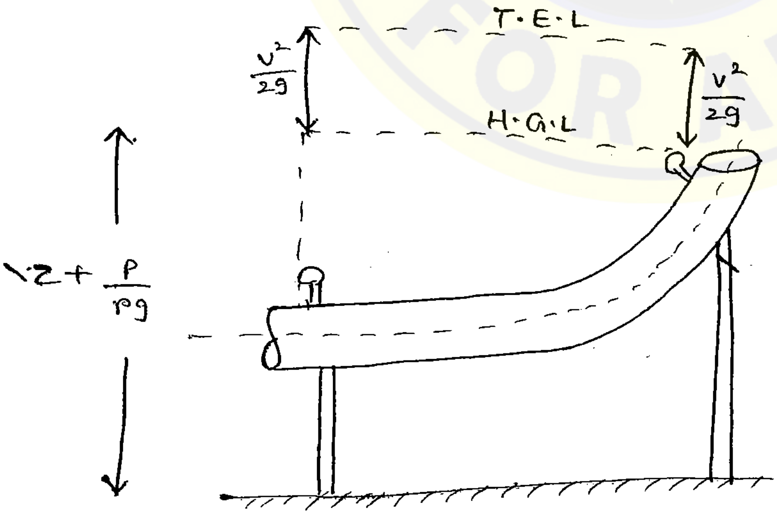
H.G.L and T.E.L in pipe flow:-

H.G.L – Hydraulic gradient line (or) piezometric head line

T.E.L – Total energy Line (head) in a moving fluid $= z + \frac{P}{\rho g} + \frac{v^2}{2g}$

T.E.L = H.G.L + velocity head

$$T.E.L = \left(z + \frac{P}{\rho g}\right) + \frac{v^2}{2g}$$



P.g NO:- 135

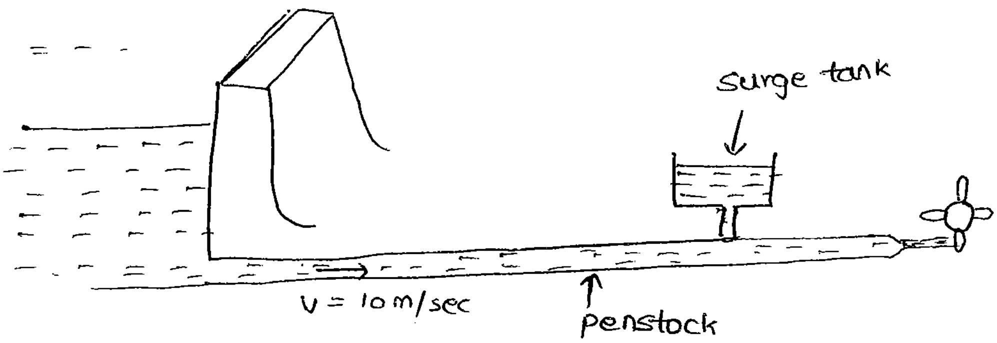
57. H.G.L $= z + \frac{\cancel{v^2}P}{\rho g}$

Given $\frac{v^2}{2g} = 7$, $z = 5$, $\frac{P}{\rho g} = 4$

$= 4 + 5$

$= 9$

Water hammering:-



i. c = velocity of sound in fluid medium $= \sqrt{\frac{K}{\rho}\ \frac{N/m^2}{kg/m^3}}$

$$C_{water} = \sqrt{\frac{2\times10^9}{1000}}$$

$$= 1414\ m/sec$$

ii. $P_{hammer} = \rho c v \ N/m^2$

$$= \frac{kg}{m^3} \times \frac{m}{s} \times \frac{m}{s}$$

$$= \frac{N-m}{m^3}$$

$$= N/m^2$$

$$= 1000 \times 1414 \times 10$$

$$= 14140\ k \cdot pa$$

$$= 14.14 \times 10^6\ M \cdot pa$$

$$= 141.4\ bar$$

1 bar = P_{atm}

$$P_{hammer} = \rho c v$$

$$= \rho \sqrt{\frac{K}{\rho}} \cdot v$$

$$= \sqrt{\rho K} \cdot v$$

$$P \propto \sqrt{\rho}$$

Critical time $= \frac{2L}{c}$

$$\boxed{T_c = \frac{2L}{c}}$$

$$c = \frac{2L}{T_c}$$

Note :-

$T_{actual} > T_c$ (safe or gradually closed) (or) slowly closed.

$$P_{Gradually} = \rho v \cdot \frac{L}{T_{act}}$$

$T_{actual} < T_c$ (gr suddenly closed)

P.g No:- 128

8. Given $L = 3000$ m, $c = 1500$ m/s, $T_{actual} = 4.5$ sec.

$$T_{critical} = \frac{2L}{c} = \frac{2 \times 3000}{1500}$$

$$= 4 \text{ sec.}$$

$\therefore T_{actual} > T_{critical}$

$\therefore$ slowly closure

4. $P_{hammer} = \rho c \Delta V$

$$= \rho c (V_2 - V_1)$$

$$\frac{P_{hammer}}{\rho g} = \frac{\rho c (V_2 - V_1)}{\rho g}$$

$\therefore \frac{P_{hammer}}{\rho g} = H_{hammer}$

$$= \frac{c(V_2 - V_1)}{g} = H_{hammer}$$

3. $c = \frac{2L}{T_c}$

$$= \frac{2L}{0}$$

$c = \infty$ (throughout)

1. $c = \sqrt{\frac{K}{\rho}}$

$$= \sqrt{\frac{2 \times 10^9}{965}}$$

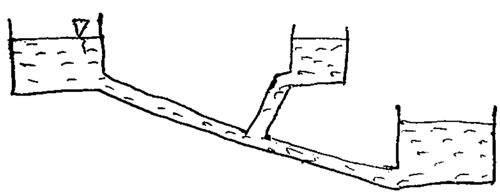
$c = 1440$ m/s

2. $T_{actual} < T_c$

$T_{actual} \leq \frac{2L}{c}$

Branching pipe:-

$$\Sigma h_f = \Sigma H$$



UNIT - 8

LAMINAR FLOW

Introduction:-

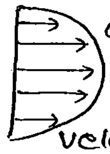
1. Laminar flow is theoretical and study flow. These flow is possible when low density fluid and high viscous fluid with low speed and flowing through narrow passages makes the flow laminar.

$$Re_D = \frac{\rho\downarrow V\downarrow D\downarrow}{\mu\uparrow}$$

2. The following characteristics of the Laminar flow.

   a. Re_D (pipe) $\leq$ 2000 (lower critical $Re = 2000$)
      (upper critical $Re = 2300$ to 2800)

   b. Velocity distribution is not uniform and not linear and mostly parabolic nature.



   c. Laminar flow means viscous fluid flow and essentially incompressible.

   d. Flow is rotational (velocity potential does not exist) i.e., velocity along stream line remains constant.

   e. These is basic for real flows comparison. This flow is also known as stream line flow or parallel flow or sandwich flow. or Hagen's poiselle's flow.

   f. Only two forces are delt in this flow (pressure force and viscous force). Not gravity force, Not inertia (or) friction force, not roughness of the pipe.

   g. Viscous fluid through pipe is analysed with reference to one shear stress distribution across the pipe at a point.

$$C_r = -\left(\frac{dp}{dx}\right)\cdot\frac{r}{2}$$

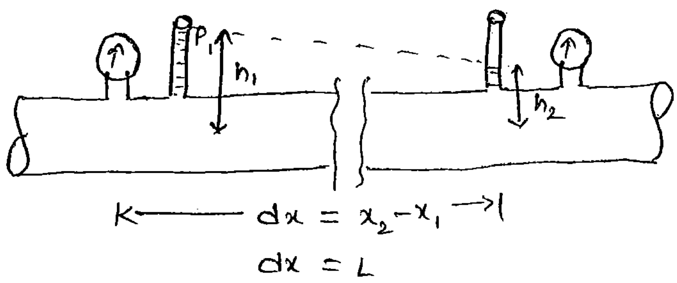
pressure gradient (or) Hydraulic gradient

Dimensional formula for pressure gradient :-

$$\frac{dp}{dx} = \left(\frac{N/m^2}{m}\right) = \frac{32\,\mu v \cdot l}{\rho \cdot g d^2}$$

$$-\frac{dp}{dx} = \frac{-(P_2-P_1)}{L}$$

$$= \frac{P_1-P_2}{L}$$



Loss of pressure head per unit length

$$\frac{h_1-h_2}{L} = \frac{\left(\frac{\Delta P}{\rho g}\right)}{L}$$

$$\frac{dp}{dx} = \frac{N/m^2}{m} = \frac{N}{m^3}$$

$$= \frac{\frac{kg-m}{s^2}}{m^3}$$

$$= \frac{kg - \cancel{m^2}}{m^2 - s^2}$$

$$\boxed{\text{pressure gradient} = M^1 L^{-2} T^{-2}}$$

Drop in pressure is due to viscosity only which is proposed by Hygen's poiselles by balancing external pressure force and internal viscous shear force.

i.e., sum of pressure force

$$\Sigma PA = \Sigma \tau \cdot A_s$$

Hagen's poiselli's equation $= P_1 - P_2 = \dfrac{32\,\mu V \cdot l}{d^2}$

$$Q = AV$$

$$= \frac{\pi d^2}{4} \cdot V$$

$$V = \frac{4Q}{\pi d^2}$$

$$P_1 - P_2 = \frac{32\,\mu \cdot \frac{4Q}{\pi d^2} \cdot l}{d^2}$$

$$P_1 - P_2 = \frac{128 \cdot \mu \cdot Q \cdot l}{\pi d^4}$$

For laminar flow in a pressure pipe is same discharge

$$P_1 - P_2 \propto \frac{1}{d^4} \qquad (Q = \text{const})$$

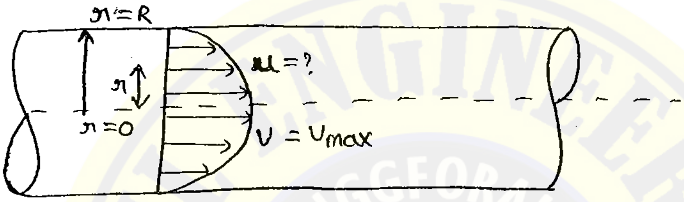
$$\frac{\Delta P_1}{\Delta P_2} = \left(\frac{d_2}{d_1}\right)^4$$

Head loss due to viscosity in a pipe flow

$$h_1 - h_2 = -\Delta h = \frac{P_1 - P_2}{\rho g} = \frac{32\,\mu \cdot V \cdot L}{\rho g \cdot d^2}$$

$$h_1 - h_2 = \frac{128\,\mu \cdot Q \cdot L}{\rho g \cdot \pi d^4}$$

Velocity distribution or variation in laminar pipe flow (parabol



$$u_r = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)$$

If $r = 0$, $u_r = V_{max}$

$$V_{max} = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2)$$

$$\frac{V_r}{V_{max}} = 1 - \frac{r^2}{R^2}$$

$$V_r = V_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

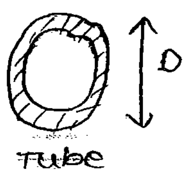
$Q = AV$

= cross sectional area x Mean (or) Average velocity

$= \frac{\pi d^2}{4} \times \frac{\text{Max. velocity (or) centre line velocity}}{2}$

$= \frac{\pi d^2}{4}\left(\frac{V_{max}}{2}\right)$

Note:-



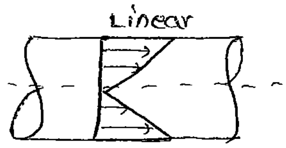
pipe measures inner dia.

Tube measures outer dia and it carries high pressure. (or) full pressure.

Summary of Laminar or viscous flow through a pipe:-

$$\tau_r = -\left(\frac{dP}{dx}\right)\cdot\frac{r}{2}$$

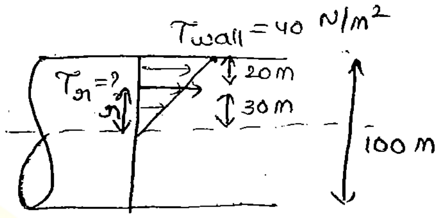
Shear stress variation is Linear



$$\tau_r \propto r$$

$$\tau_{wall} = \boxed{\tau_{max} = \frac{-dP}{dx}\cdot\frac{R}{2} = \frac{-dP}{dx}\cdot\frac{d}{4}}$$

EX:-

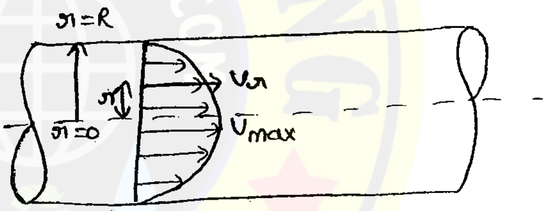


$$\frac{\tau_{wall}}{R} = \frac{\tau_r}{r}$$

$$\frac{40}{50} = \frac{\tau_r}{30}$$

$$\tau_r = 24\ N/m^2$$

Velocity variation of viscous fluid in a pipe:-



$$V_r = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$V_r = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)$$

$$Q = AV$$

$$= A\cdot\frac{V_{max}}{2}$$

$$\boxed{V_{max} = 2V}$$

Pressure drop (or) Head loss due to viscosity (Hagen's poisellies):-

$$P_1 - P_2 = \frac{32\,\mu\cdot V\cdot l}{d^2} = \frac{128\,\mu\cdot Q\cdot l}{\pi d^4}$$

$$h_1 - h_2 = \frac{P_1 - P_2}{\rho g} = \frac{32\,\mu\cdot V\cdot l}{\rho g\cdot d^2} = \frac{128\,\mu\cdot Q\cdot l}{\rho g\,\pi d^4}$$

Relation between Hagen's poiseilles equation and Darcy's eq:- (81)

$$\frac{f L v^2}{2gd} = \frac{32\,\mu \cdot v \cdot L}{\rho \cdot g\, d^2}$$

$$\boxed{f = \frac{64}{R_e}}$$

P.g No:- 144

24. $Q = A\,V$

$$= \frac{\pi D^2}{4} \times \frac{v_{max}}{2}$$

$$= \frac{\pi (0.04)^2}{4} \times \frac{1.5}{2}$$

$$Q = \frac{3\pi}{10000}\ m^3/sec$$

Note :-

$$v_{\text{℄}} = v_{max} = 2V$$

25. $f = \frac{64}{R_e} = \frac{64}{2000} = 0.032$

26. $d = 0.2\ m$

$R = 0.1\ m$

$v_{max} = 1\ m/sec$

$r = 0.05\ m$

$$v_r = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$= 1\left[1 - \left(\frac{0.05}{0.1}\right)^2\right]$$

$$= 0.75\ m/s$$

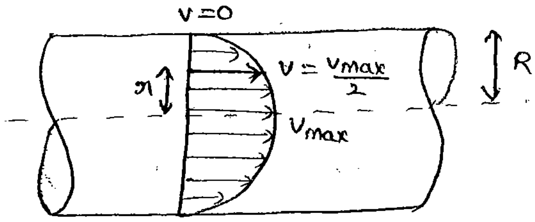
P.g. No:- 148.

4. $\tau_r = \tau_{max} \times \frac{r}{R}$

$$= 28 \times \frac{3}{4}$$

$$= 21.0 \text{ pascal.}$$

7. $$\boxed{\begin{aligned} r &= \frac{R}{\sqrt{2}} \\ &= 0.707R \\ r &= 0.3535\,D \end{aligned}}$$



The radial distance from $\mathbb{C}_L$ of pipe, where avg.(or) mean velocity occur.

$$\bar{r} = \frac{R}{\sqrt{2}}$$

$$= \frac{D}{2\sqrt{2}}$$

$$= 0.707\,R$$

$$= 0.3535\,D$$

proof :-

$$V_r = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)$$

$$V_r = v = \frac{V_{max}}{2} = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)$$

$$\frac{\frac{1}{4\mu}\left(\frac{-dP}{dx}\right)R^2}{2} = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)$$

$$\frac{R^2}{2} = R^2 - r^2$$

$$r^{-2} = \frac{R^2}{2}$$

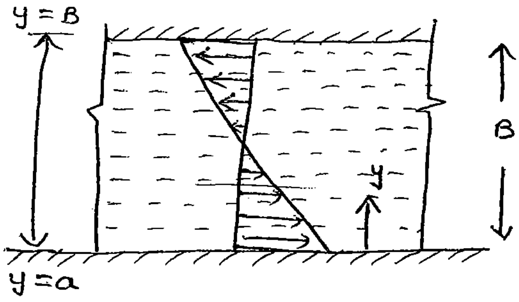
$$r = \frac{R}{\sqrt{2}}$$

$$\boxed{r = 0.71\,R}$$

10. $$R_e = \frac{64}{f}$$

$$f = \frac{64}{R_e} = \frac{64}{0.1} = 640$$

Analysis of Laminar flow or viscous flow in b/w two parall plates :-



$$\tau_y = -\left(\frac{dP}{dx}\right)\left(\frac{B - B2y}{2}\right)$$

$$\tau_{max} = \frac{-dP}{dx} \cdot \frac{B}{2}$$

$$\boxed{V_{max} = 1.5\,V \text{ for parallel plates}}$$

P.g NO:- 145.

1. $V = \frac{2}{3} V_{max}$

$$\frac{V_{max}}{V} = \frac{V_{max}}{\frac{2}{3} V_{max}} = \frac{3}{2}$$

4. $q = \frac{3}{2} \cdot V$

$V = 6$

7. Given $\nu = 0.25$, stokes $= 0.25\ (cm^2/sec)$, $d = 10$ cm, $V = ?$

$$Re = \frac{V d}{\nu}$$

$$2000 = \frac{V \times 10}{0.25}$$

$V = 50\cancel{0}$ cm/sec

$V = 0.5$ m/sec

8. $f = \frac{64}{Re}$

$= \frac{64}{640}$

$= 0.1$

11. For parallel plates (passage) $= 1 \cdot V_{max} = 1.5\,V$

$$= \frac{V_{max}}{V}$$

$$= \frac{V_{max}}{V_{max}/1.5}$$

$= 1.5$

14. Given $\rho = 1000\ kg/m^3$, $\mu = 0.01$ poise, $d = 0.06$

$= 0.01 \times 0.1\ \frac{N\text{-}s}{m^2}$

$$2000 = \frac{\rho V d}{\mu} = \frac{1000 \times V \times 0.06}{0.001}$$

$V = 100/3$ cm/sec

15. $\Delta p = \dfrac{128\,\mu \cdot Q \cdot L}{\pi d^4}$

$Q = \dfrac{\Delta p \cdot \pi \cdot d^4}{128 \cdot \mu \cdot l}$

17. $h_{Loss} = \dfrac{32\,\mu \cdot V \cdot L}{\rho g \cdot D^2}$

$= \dfrac{32\,\mu \cdot V \cdot L}{w \cdot D^2}$

20. $\Delta p = \dfrac{128\,\mu \cdot Q \cdot l}{\pi d^4}$

$\mu \propto \dfrac{1}{Q}$

21. $\Delta p = 100000\ (N/m^2)$, $D = 0.15\,m$ $l = 10\,m$, $\tau_{max} = ?$

$\tau_{max} = \dfrac{-dP}{dx} \cdot \dfrac{R}{2}$

$= \dfrac{100000}{10} \times \dfrac{\left(\frac{0.15}{2}\right)}{2}$

$= 375\ N/m^2$

22. $V_r = V_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$

$\dfrac{V_r}{V_{max}} = \left[1 - \left(\dfrac{D/4}{D/2}\right)^2\right]$

$= \left[1 - \left(\frac{1}{2}\right)^2\right]$

$= 1 - \frac{1}{4}$

$= \frac{3}{4} = 0.75$

23. $f = \dfrac{64}{Re}$

$= \dfrac{64}{1280}$

$= 0.05$

24. $h_{Loss} = \dfrac{128\,\mu \cdot Q \cdot l}{\pi \cdot \rho g \cdot d^4}$

$Re = 100\ (L \cdot F)$

$h_{loss} \propto Q$

$\dfrac{h_{L_1}}{h_{L_2}} = \dfrac{Q_1}{Q_2}$

$h_{L_2} = \dfrac{Q_2}{Q_1} \cdot h_{L_1}$

$= \dfrac{2Q_1}{Q_1} \cdot h_{L_1}$

$= 2 \times h_{L_1}$

$= 2 \times 2$

$= 4\ m$

25. $Re = \dfrac{64}{f}$

$= \dfrac{64}{0.04}$

$= 1600.$

31. $Q = AV$

$= \dfrac{\pi (0.2)^2}{4} \cdot \dfrac{V_{max}}{2}$

$= \dfrac{\pi (0.2)^2}{4} \times \dfrac{1.5}{2}$

$= 0.0235$

32. Given $d = 2.5\,cm = 0.025\,m$, $\Delta p = 0.12\ \frac{kg(f)}{cm^2}$, $V = 2.34\ \frac{m}{sec}$

$l = 100\,m$

power lost = ?

$$\text{power lost} = \rho \cdot g \cdot Q \cdot h_{Loss}$$

$$= \rho \cdot g \cdot Q \cdot \frac{\Delta p}{\rho g}$$

$$= Q\,\Delta p$$

$$= Q\left(\frac{\Delta p}{l} \times L\right)$$

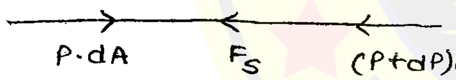
$$= \frac{\pi D^2}{4} \cdot V \left(\frac{\Delta p}{l}\right) \times L$$

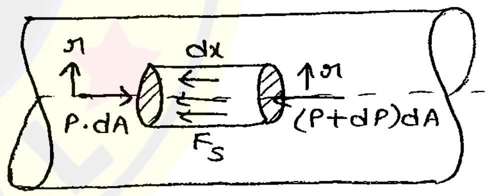
$$= \frac{\pi (0.25)^2}{4} \times 2.34 \times \left(0.12 \times 9.81 \times 10^4\ \frac{N/m^2}{m}\right)(100\,m)$$

$$= 1.84\ H.P \qquad \therefore 1\ H.P = 736\ \text{watts}.$$

Proofs :-

1. $$T_r = \frac{-dP}{dx} \cdot \frac{r}{2}$$





$$\Sigma F = ma$$

$$P \cdot dA - F_s - (P + dP)dA = m \times 0$$

$$-F_s - dP \cdot dA = 0$$

$$F_s = -dP \cdot dA$$

$$T_r \cdot dA_s = -dP \cdot dA$$

$$T_r \cdot 2\pi r \cdot dx = -dp \cdot \pi r^2$$

$$\boxed{T_r = \frac{-dP}{dx} \cdot \frac{r}{2}}$$

2. Velocity distribution :-

Newtons law of viscosity

$$\tau = \mu \cdot \frac{du}{dy}$$

when $y = R - r$

Differentiate

$$dy = 0 - dr$$

$$dy = -dr$$

$$\tau_r = \mu \cdot \frac{du}{-dr}$$

$$\tau_r = -\frac{dP}{dx} \cdot \frac{r}{2} = -\mu \cdot \frac{du}{dr}$$

$$\int_0^{V_r} du = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)\int_R^{r} r \cdot dr$$

$$u_r = \frac{1}{2\mu}\left(\frac{dP}{dx}\right)\left(\frac{r^2}{2}\right)_R^{r}$$

$$U_r = \frac{1}{4\mu}\left(\frac{dP}{dx}\right)(r^2 - R^2)$$

$$\boxed{u_r = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)}$$

prove that for Laminar flow, $P_1 - P_2 = \frac{32 \mu V l}{d^2}$

we have, $V_r = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)(R^2 - r^2)$

$r = 0$, $V_r = V_{max}$

$$V_{max} = \frac{1}{4\mu}\left(\frac{-dP}{dx}\right)R^2$$

$$-\int_{P_1}^{P_2} dP = \frac{4 \mu V_{max}}{R^2}\int_{x_1=0}^{x_2=l} dx$$

$$-(P_2 - P_1) = \frac{4 \mu V_{max}}{\left(\frac{D}{2}\right)^2}[x]_0^l$$

$$P_1 - P_2 = \frac{16 \cdot \mu \cdot V_{max} \cdot l}{D^2}$$

we have, $V_{max} = 2V$

$$P_1 - P_2 = \frac{32 \mu V l}{d^2}$$

→ Hagen's poisellie's eq

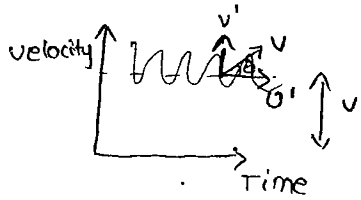
$$h_{loss} = \frac{P_1 - P_2}{\ldots} = \frac{32 \mu V \cdot l}{\ldots} = \frac{128 \mu \cdot Q \cdot l}{\ldots}$$

UNIT - 9

ELEMENTARY TURBULENT FLOW (1-2Q)

1. Turbulent flow is a real fluid flow. Every low viscosit fluid turns to the turbulent flow due to high Reynolds number turbulency.

2. Turbulent flow has the following observations through a pipe line.



3. $u' = v\cos\theta$

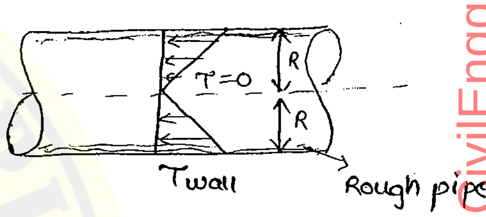
$v' = v\sin\theta$

3. Reynolds shear stress, $\tau_{Reynolds} = -\rho u'v' = \frac{N \cdot m}{m^3} = \frac{N}{m^2}$ (pa)

$$\tau_{wall} = -\left(-\frac{dP}{dx}\right)\frac{R}{2}$$

$$= \frac{dP}{dx}\cdot\frac{D}{4}$$

$$= \frac{\rho g h_{friction}}{L}\cdot\frac{D}{4}$$



4. Friction factor is function of R_e and Roughness Height (K)

In laminar flow, $R_e \leq 2000$

$$\boxed{f = \frac{64}{R_e}}$$

In turbulent flow, $4000 < R_{e_D} < 1\times10^5$

$$\boxed{f = \frac{0.316}{(R_e)^{1/4}}}$$

5. Ageing effect the pipe discharge

$$\frac{df}{f} = -2\cdot\frac{dQ}{Q}$$

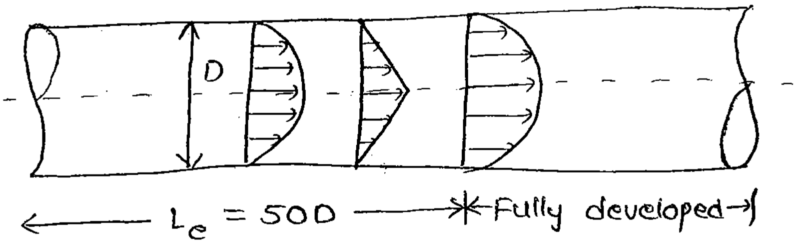
6. In laminar flow, $V_{max} = 2V$ (pipe)

7. In laminar flow between two parallel fixed plates, $V_{max} = 1.5V$

8. In turbulent flow, $V_{max} = V(1 + 1.43\sqrt{f})$

f = friction factor which is functioning of R_e & K.

9. Velocity profile variation in turbulent flow of a pipe



Entrance length of the pipe represents the stabilisation of velocity variation across the length of the pipe. The velocity profile after entrance length is remains same. through out its motion. This type of flow is known as fully developed flow for which acceleration is zero(velocity along the stream line remains constant)

The entrance length, $L_e = 50\,D$ (Dia of pipe).

10. At very high R_e flow through Rough pipe the friction factor is only function of Roughness height of the pipe (K)

11. A turbulent flow decides the friction factor in terms of R_e and 'K' for smooth boundary.

$$\frac{1}{\sqrt{f}} = 2 \log_{10} R_e \sqrt{f} - 0.8$$

12. Friction factor for Rough boundary in turbulent flow

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{D/2}{K} \right) + 1.74$$

Classification of smooth and Rough pipe boundaries:-

The criterion to classify the boundaries ratio of

$$\frac{K}{\delta'} = \frac{\text{Pipe boundary surface roughness height}}{\text{Laminar sub-layer thickness}}$$

$\delta' > K$ (smooth surface)

$\delta' < K$ (Rough surface)

Laminar sub layer thickness in turbulent flow

$$\delta' = \frac{11.68\nu}{v^*}$$

v^* = shear velocity (m/sec)

$$v^* = \sqrt{\frac{\tau_{wall}}{\rho}}$$

$$v^* = v\sqrt{\frac{f}{\nu}}$$

P.g NO:- 161

18. $\tau = \frac{dP}{dx} \cdot \frac{d}{4}$

$$= \frac{\rho g h_f}{L} \cdot \frac{d}{4}$$

$$= \frac{1000 \times 10 \times 10}{100} \times \frac{0.1}{4}$$

$$= 25 \ N/m^2$$

$$= 0.025 \ kN/m^2$$

$$\tau = 0.025 \ k.pa.$$

P.g. NO:- 164.

1. Given $L = 30m$, $d = 75$ mm , $dp = 0.16 \ N/mm^2$

$$\tau = \frac{dP}{dx} \cdot \frac{d}{4}$$

$$= \frac{0.1 \times 10^6 \ (N/m^2)}{30 \ m} \times \frac{0.075 \ m}{4}$$

$$\tau = 100 \ N/m^2$$

10. $f = \frac{0.316}{(10000)^{1/4}} = 0.02$ (turbulent)

$$f_{laminar} = \frac{64}{Re} = \frac{64}{2000} = 0.032$$

$$\frac{V_{max}}{V} = 1 + 1.43\sqrt{f}$$

$$\frac{V}{V_{max}} = \frac{1}{1 + 1.43\sqrt{f}}$$

$$\frac{V}{V_{max}} = \frac{1}{1 + 1.43\sqrt{0.02}} = 0.83$$

Note:- $\boxed{\therefore f_{Laminar\ flow} > f_{turbulent\ flow}}$

P.g No:- 162

6. $\tau_0 = \tau_{wall} = \left(\frac{-dP}{dx}\right)\frac{D}{4}$

$$= \frac{\Delta P}{L} \cdot \frac{D}{4}$$

$$= \frac{\Delta P \cdot D}{4L}$$

8. $U_{max} = V\left[1 + 1.43\sqrt{f}\right]$

$$3.61 = V\left[1 + 1.43\sqrt{0.02}\right]$$

$$V = 3 \text{ m/sec}$$

~~prove that for Laminar flow~~ $\cancel{P_1 - P_2} = \frac{32\mu V L}{d^2}$

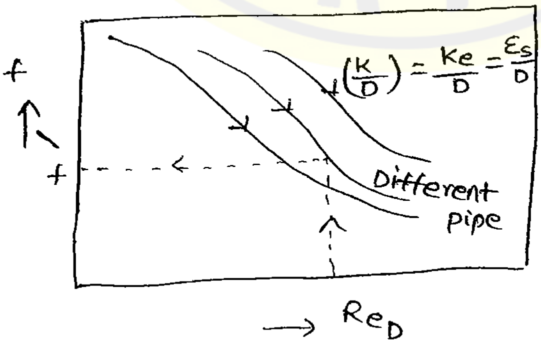
~~we h.~~

Commercial pipes - Friction factor :-

1. While selecting commercial pipes it is necessary to compare performance of different pipes under consideration.
2. For which manufacturers provide experimental result charts. Once such chart is called Moody's chart.

Moody's Diagram:-

1. It is a graphical representation of Reynolds number, friction factor and $\frac{K}{D}$ ratio.

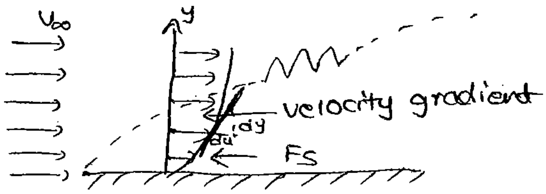


Prandtl mixing length:- (1904)

1. Developed external flow analysis (pressure gradient = zero). When flow takes place over the surface due to inertia force a thin layer is developed is called boundary ~~line~~ layer theory.

2. In boundary layer region velocity gradient always developed due to velocity gradient shear stress developed on the surface.

3. Due to this surface life effected. This is to be prevented.



$$\tau_{Reynolds} = \rho u'v'$$

Assume $u' = v'$

$$\tan\theta = \theta = m = \frac{du}{dy}$$

$$u' = v' = \frac{du}{dy}$$

$$\tau_{prandtl} = \rho \cdot l \cdot \frac{du}{dy} \cdot \frac{du}{dy} \cdot l$$

$$\boxed{\tau_{prandtl} = \rho \cdot l^2 \left(\frac{du}{dy}\right)^2}$$

Note :-

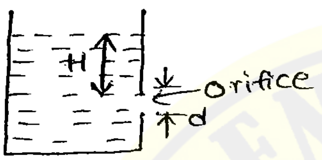
In turbulent flow through pipe the Prandtl mixing length is zero at the rate of pipe surface.

27-11-

UNIT - 10

FLOW THROUGH ORIFICES AND MOUTHPIECES

1. Orifice is a device used for measuring fluid flow rate.
2. Orifice is a small opening at bottom or side of the tank or reservoir through which fluid is permitted through letout.
3. Mouthpiece an extension to the orifice whose function is to create low pressures.
4. Due to low pressure the velocity of the fluid is very high when compared to orifice.



$d << H$

Coefficient of ~~Discharge~~ velocity:-

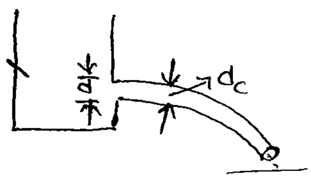
$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{V_a}{V_t} = \frac{V_a}{\sqrt{2gH}}$$

$\sqrt{2gH}$ = spouting velocity.

Coefficient of contraction:-

$$C_c = \frac{\text{Area of jet at vena contracta}}{\text{Area of the orifice}}$$

$C_c = 0.62$ (for small orifice)



$$C_c = \frac{\pi d_c^2/4}{\pi d^2/4} = \left(\frac{d_c}{d}\right)^2$$

Coefficient of discharge (C_d):-

$$C_d = \frac{\text{Actual discharge to orifice}}{\text{Theoritical discharge through orifice}}$$

$$C_d = \frac{Q_{act}}{Q_{the}} = \frac{A_c \cdot V_{actual}}{A \cdot V_{theo}}$$

$$= \left(\frac{A_c}{A}\right)\left(\frac{V_{act}}{V_{the}}\right)$$

$$C_d = \left(\frac{d_c}{d}\right)^2 / \frac{V_{act}}{V_{theo}} \Rightarrow \cancel{C_v} \; C_c \times C_v$$

Standard values :-

1. $C_c = 0.58 - 0.62$

2. $C_v = 0.95 - 0.99$ for sharp edge orifice.

3. $C_d = 0.59 - 0.61 \approx 0.60$

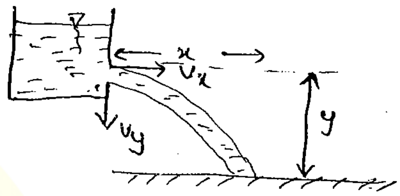
$$\therefore \quad C_v < C_d < C_c$$

Flow through an orifice and mouth piece :-

Actual discharge through an orifice $\boxed{(Q) = C_d A \sqrt{2gH}}$

where $C_d = C_c \times C_v$

Method to find C_v :-



$$V_y = u_y + a_y t$$

$$V_y = 0 + gt$$

$$t = \frac{V_y}{g} \rightarrow (1)$$

$$\text{velocity in horizontal} = \frac{\text{Distance}}{\text{Time of flight}}$$

$$V_x = \frac{x}{t}$$

$$= \frac{x}{\frac{V_y}{g}}$$

$$V_x = \frac{gx}{V_y}$$

$$S_y = u_y t + \frac{1}{2} g t^2$$

$$y = 0t + \frac{1}{2} g t^2$$

$$\frac{2y}{g} = t^2$$

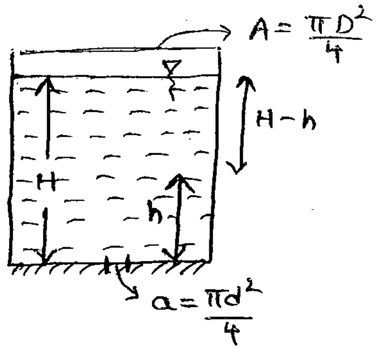
$$C_v = \frac{V_{act}}{V_{the}} = \frac{V_{act}}{\sqrt{2gH}}$$

$$V_{act} = V_x = \frac{x}{t}$$

$$C_v = \frac{x}{t\sqrt{2gH}} = \frac{x}{\sqrt{\frac{2y}{g}} \cdot \sqrt{2gH}} = \frac{x}{2\sqrt{yH}}$$

$$\boxed{C_v = \sqrt{\frac{x^2}{4yH}}}$$

Expression for time consumed to empty the tank of liquid:-



T = time of emptying a tank

$$\boxed{T = \frac{2\,A_{tank}}{C_D \cdot a \cdot \sqrt{2g}} \times \sqrt{H}}$$

proof:-

$$Q = AV$$

$$q = aV$$

$$= a\sqrt{2gh}$$

$$Q = q = \frac{\text{volume}}{\cancel{\text{area}}\ \text{time}}, \quad q = \frac{A \times H}{\text{time}}$$

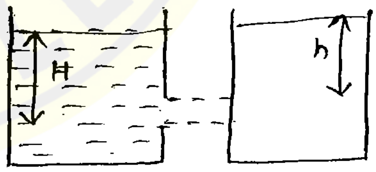
$$\text{Time } (T) = \frac{A \cdot H}{q} = \int_0^T dt = \frac{A\,h}{C_D \cdot a\sqrt{2gH}}$$

$$T = \frac{A}{C_d\, a\sqrt{2g}} \int_0^H \frac{dh}{h}$$

$$= \frac{A}{C_d\, a\sqrt{2g}} \left[\frac{h^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_0^H$$

$$\boxed{T = \frac{2A}{C_d\, a\sqrt{2g}} \sqrt{H}}$$

Expression for discharge through Rectangular orifice:-



$$dq = C_d \cdot dA \cdot V$$

$$dq = C_d \times B \times dh \times \sqrt{2gh}$$

$$dq = C_d \times B \times \sqrt{2g} \cdot \sqrt{h} \cdot dh$$

$$\int_0^Q dq = C_D \times B \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h} \cdot dh$$

$$Q = C_D \times B \times \sqrt{2g} \left(\frac{H^{3/2}}{3/2}\right)_{H_1}^{H_2}$$

$$= \frac{2}{3} C_D \times B \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2}\right)$$

P.9 NO:- 169

10. $Q = C_d \times a \times \sqrt{2gh}$

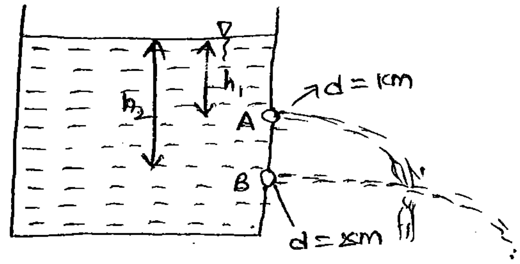
$$= 0.6 \times 0.0003 \times 4.4 \times \sqrt{1}$$

$$= 7.9 \times 10^{-4}\ m^3/sec$$

11. $H = 10\text{cm}, \quad x = 19\text{ cm} \quad y = 10\text{cm}$

$$c_v = \sqrt{\frac{x^2}{4yH}} = \sqrt{\frac{19^2}{4\times10\times10}} = 0.95$$

*12. $Q_A = Q_B$



$$\frac{h_1}{h_2} = ?$$

$$\cancel{C_{d_A}} \cdot A_A \sqrt{2gh_1} = \cancel{C_{d_B}} \cdot A_B \cdot \sqrt{2gh_2}$$

$$d_A^2 \sqrt{h_1} = d_B^2 \sqrt{h_2}$$

$$\sqrt{\frac{h_1}{h_2}} = \frac{d_A^2}{d_B^2}$$

$$\frac{h_1}{h_2} = \left(\frac{d_A}{d_B}\right)^4 = \left(\frac{2}{1}\right)^4 = \frac{16}{1} \Rightarrow 16:1$$

13. $$T = \frac{2A}{C_d\, a \sqrt{2g}} (H)^{1/2}$$

$a \to \pi d^2/4$

$$T \propto \frac{1}{d^2}$$

$$\frac{T_1}{T_2} = \left(\frac{d_2}{d_1}\right)^2$$

$$\frac{T_1}{\cancel{200}} = \left(\frac{\cancel{2d}}{\cancel{2d}}\right)^2 \qquad \frac{200}{T_2} = \left(\frac{2d}{d}\right)^2$$

$$T_2 = 50 \text{ sec}$$

P.g NO:- 170.

1. $$T = \frac{2\, A_{tank}}{C_d \cdot a_{orifice} \times \sqrt{2g}} \times \sqrt{H}$$

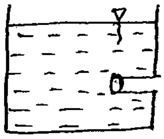
$$= \frac{2\times(1\times1)}{0.9\times(2000\times10^{-6})\times\sqrt{2\times9.81}} \times \sqrt{2}$$

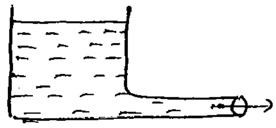
$$= 355 \text{ sec.}$$

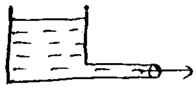
Classification of mouth piece:-

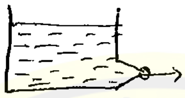
Mouth piece

Internal mouthpiece | External mouthpiece





cylindrical mouth piece 

convergent mouth piece 

convergent - divergent mouth piece 

Note:-

C_c value for mouth piece = 1

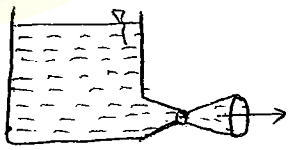
C_v value for mouth piece = 0.85 – 0.86

$\therefore C_d$ mouthpiece = 1 × 0.86 = 0.86

Hence C_d mouthpiece is greater than C_d of orifice. Hence discharge through mouth piece is more.

Flow through a convergent - divergent mouth piece:-

$$\boxed{\frac{Q_1}{Q_2} = \sqrt{1 + \frac{H_{atm} - H_1}{H}}}$$



P.9 No:-170

14. $$\frac{10^2}{5^2} \cdot \frac{5^2}{10^2} = \sqrt{1 + \frac{10.3 - \cancel{(+8)}\,(2.3)}{H}}$$

$15\,H = \cancel{2.3}\ 8$

$H = 0.53$

Note:-

The pressure at throat portion is to be absolute pressure, Given data at the throat 8m at vacuum. Absolute pressure at throat = Atm. press + Gauge p vacuum ⇒ 10.3 + (−8)

⇒ 2.3

UNIT - II

FLOW OVER NOTCHES AND WEIRS

1. Notches and weirs are used measuring volumetric flow rate.
2. The fluid flow takes place in open atmosphere (without pres where as flow through under pressure.
3. Orifice always under submerged condition. It is mostly emptying the tank purpose where as notches and weirs measuring flow rate and discharge flow over excess water in reservoir.
4. The main difference between Notch and weir.

Notch	Weir
1. It is a small plate used in laboratories, canal distributaries ~~whe~~ varies a large obstruction across the flow.	1. It is a large structure found in irrigation tank and across large rivers.
2. It is made from metal plate.	2. It is made from concrete (or) masonry. It is a large structure found in irrigation tank and across large rivers

Classification :-

1. Shape of opening
2. Shape of crest
3. Discharge condition

Expression for discharge through a notch :-

$$dq = C_d \cdot d_a \cdot v$$

$$= C_d \cdot L \times dh \times \sqrt{2gh}$$

$$= C_d \cdot L \cdot \sqrt{2g} \cdot \frac{2}{3} \left[H^{3/2} \right]_{H_2}^{H_1}$$

$$\boxed{d_q = C_d \cdot L \cdot \sqrt{2g} \cdot \frac{2}{3} \left[H_1^{3/2} - H_2^{3/2} \right]}$$

Discharge error measurement :-

$$Q \propto H^{3/2}$$

$$Q = K\, H^{3/2} \longrightarrow (1)$$

Differentiate

$$dQ = K \cdot \frac{3}{2} H^{\frac{3}{2}-1} \cdot dH$$

$$dQ = K \cdot \frac{3}{2} H^{1/2} \cdot dH \longrightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{dQ}{Q} = \frac{3}{2}\left(\frac{dH}{H}\right)$$

$$\boxed{\frac{dQ}{Q} = 1.5\left(\frac{dH}{H}\right)}$$

Flow through V-notch :-

$$Q_{v\text{-notch}} = \frac{8}{15} C_d \sqrt{2g}\, \tan\frac{\theta}{2} \cdot H^{5/2}$$

Discharge error :-

$$Q \propto H^{5/2}$$

$$Q = K \cdot H^{5/2} \longrightarrow (1)$$

Differentiate

$$dQ = K \cdot \frac{5}{2} \cdot H^{5/2 - 1} \cdot dH$$

$$dQ = K \cdot \frac{5}{2} H^{3/2} \cdot dH \longrightarrow (2)$$

$$\frac{(2)}{(1)} \Rightarrow \boxed{\frac{dQ}{Q} = 2.5\left(\frac{dH}{H}\right)}$$

P.g NO:- 177.

3. $$Q = \frac{2}{3} C_d \sqrt{2g} \cdot L \cdot H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times \sqrt{2 \times 9.81} \times 1 \times (0.1)^{3/2}$$

$$= 0.058 \ m^3/sec$$

4. $$V_{theo} = \sqrt{2gH}$$

$$= \sqrt{2 \times 9.81 \times 1.3}$$

$$= 5.0 \ m/s$$

2. $$\frac{dQ}{Q} = \frac{5}{2}\left(\frac{dH}{H}\right)$$

$$= 2.5(4)$$

$$= 10\%$$

5. Given $L = 2m$, $C_d = 0.6$, $H = 0.64\,m$

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

$$= \frac{2}{3} \times 0.6 \times 2 \times \sqrt{2 \times 9.81} \times (0.64)^{3/2}$$

$$Q = 1814 \text{ lit/sec}$$

6. $Q = K H^{2.5}$

8. $\frac{v^2}{2g} = \frac{3.6^2}{2 \times 9.81} = 0.66$

Velocity of approach in notches:-

$$h_a = \frac{v^2}{2g}$$

$$Q = \frac{2}{3} C_d L \sqrt{2g} \left[(H + h_a)^{3/2} - (h_a)^{3/2} \right]$$

11. $h_a = \frac{v^2}{2g}$

$$0.041 = \frac{v^2}{2 \times 9.81}$$

$$v = 0.9 \text{ m/s}$$

12. $2.5\left(\frac{dh}{h}\right)$

$2.5 \times 5\%$

$= 12.5\%$

17. $Q = \frac{8}{15} C_d \sqrt{2g} \tan\theta \cdot H^{5/2}$

$$= \frac{8}{15} \times 0.58 \times \sqrt{2 \times 9.81} \times \tan\frac{90^\circ}{2} \times (0.1)^{5/2}$$

$$= 0 \qquad \times 1000 \times 60 \text{ lit/min}$$

$$= 260 \text{ lit/min}$$

23. $Q_{v\text{-notch}} \propto H^{5/2}$

$$\frac{Q_1}{Q_2} = \left(\frac{H_1}{H_2}\right)^{5/2}$$

$$\frac{Q_2}{Q_1} = \left(\frac{H_2}{H_1}\right)^{5/2} = \left(\frac{0.2}{0.1}\right)^{5/2}$$

$= 5.66$ times increase.

P.g No:- 176.

12. $\frac{Q_2}{Q_1} = \left(\frac{H_2}{H_1}\right)^{5/2} = \left(\frac{0.3}{0.15}\right)^{5/2} = 5.657$

Q. No: = 176

$$\frac{dh}{h} = \frac{0.15}{75} \times 100 = 0.2$$

$$\frac{dQ}{Q} = 2.5 \times 0.2 = 0.5$$

177. $Q = \frac{8}{15} \times C_d \times \sqrt{2g} \tan\frac{\theta}{2} \cdot H^{5/2}$

$$= \frac{8}{15} \times 0.6 \times 4.4 \times \tan 45 \times (0.1)^{5/2}$$

$$= 267 \text{ lit/min.}$$

176:-

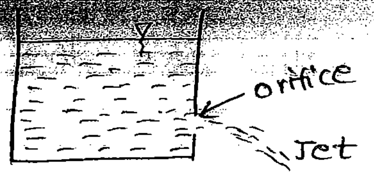
$1. \frac{dQ}{Q} = 1.5 \times \frac{dh}{h} \Rightarrow 1.5 \times 2 \Rightarrow 3\%.$

Notches & weirs:-

1. It is used for measuring the open channel flow rate (where as orifice and mouth piece are used for measuring flow rate a closed fluid system)
2. Notches and weirs are overflow structures (orifice and mouth-piece are through flow devices)
3. Notch is a small size, made from metal. where is bigger size and made from concrete (or) masonry across rivers, canals and large streams.
4. Hydraulic point of view notches and weir same treatment.
5. Notch is limited use (laboratory). weir is in irrigation practise to store and the regulate water levels on upstream of the tank.

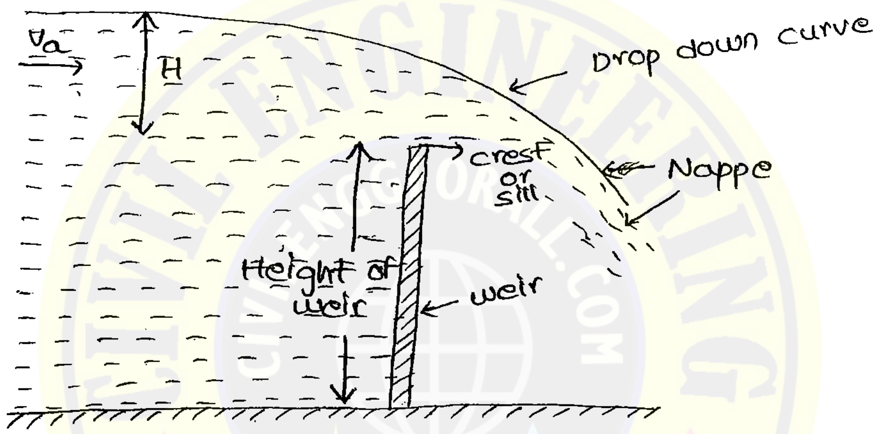
Compare Orifice (or) Mouthpiece with Notch (or) Weir.

Orifice (or) Mouthpiece	Notch (or) weir.
1. Flow is through the device	1. Flow is over the device
2. Head of water (H) is large as compared to size of the orifice (or) Mouthpiece	2. Head of water (H) is less as compared to size of the notch or weir.
3. pressure on upstream is more than downstream (D/s is atmosphere)	3. Pressure is same (both sides atmosphere.
4. water sheet is in the form of jet. the upper edge of the orifice always below the free surface of water	4. Sheet of the water is Napper. Water level is below the upper edge of the notch



Terminology :-

1. Crest or Sill
2. Nappe or Vein
3. Sharp crested weir
4. Broad crested weir
5. Free weir (or) Submerged weir
6. Crest contraction (or) End contraction.



Crest or Sill :-

1. It is the bottom edge of the notch and also it is the top surface of the weir over which liquid touches while flowing.

Nappe or Vein :-

It is the stream ~~fl~~ of the liquid flow in sheet form over the crest of the notch or weir. It is also known as vein (the stream of liquid pass through an orifice is called Jet).

Sharp crested weir :-

The liquid touches in a line due to the edge on the upstream side which is a sharp line

Broad crested weir :-

The top surface of the weir is broad or widen that allows liquid flow in contact with surface (not line contact).

Note:-
For analysis, nothing is indicated for the type of weir than consider sharp crested weir only.

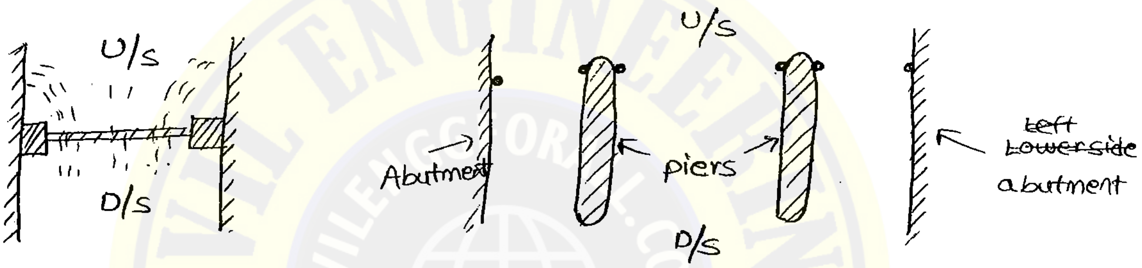
Free weir (or) Submerged weir:-

The liquid flows over the weir is called Free weir. If water level on downstream is lower than the crest level of the weir.

Submerged weir is downstream side of the water level higher than the crest.

Crest contraction

It is observed for the sharp crested weir. At the weir there is certain portion end contracted.



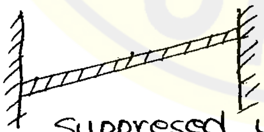
$n=2$

No. of end contractions, $n=6$

Effective length, $L_e = (L - 0.1nH)$

Note:-

1. No end contraction is called suppressed notch or weir.



Suppresed weir
(No end contraction)

2. End contraction is 10% of the Head on either side (0.1 H). There are two sides then effective width (or) breadth (or) length.

$$L_e = (L - 0.1 \times 2H)$$

For 'n' contractions, $L_e = (L - 0.1nH)$.

$$\therefore\ Q = \frac{2}{3} C_d \sqrt{2g}\,(L - 0.1 \times n \times H) H^{3/2} \quad \text{(Rectangular weir or notch)}$$

Approach head, $h_a = \dfrac{V_a^2}{2g}$

$$Q = \frac{2}{3} C_d \sqrt{2g}\,(L - 0.1nH)\left[(H+h_a)^{3/2} - h_a^{3/2}\right]$$

1. Bazins formula

$$Q = m\sqrt{2g}\, L\, H^{3/2}$$

$$m = 0.405 + \frac{0.003}{H}$$

2. Francis formula :-

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H^{3/2}$$

$$C_d = 0.623$$

$$Q = 1.84\, L H^{3/2}$$

3. V-notch formula :-

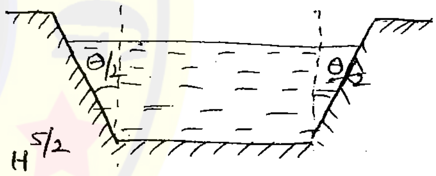
$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\, H^{5/2}$$

$$Q = 1.418\, H^{5/2}$$

$$C_d = 0.6, \quad \theta = 90^\circ$$

Other types of weir :-

1. Trapezoidal weir :-



$$Q = Q_{\square} + Q_{\nabla}$$

$$= \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\, H^{5/2}$$

1 H, 4 V $\longrightarrow$ Trapezoidal weir (or) cipolletti weir.

$$C_d = 0.63, \quad \tan\frac{\theta}{2} = \frac{1}{4}$$

$$\frac{\theta}{2} = 14^\circ$$

$$Q_{\text{cippolleti weir}} = 1.86\, L H^{3/2}$$

Time of empty a tank :-

Rectangular notch :-

$$T = \int_{H_1}^{H_2} \frac{A \cdot dh}{\frac{2}{3} C_d \sqrt{2g} \cdot L \cdot H^{3/2}}$$

$$T = \frac{2A}{\frac{2}{3} C_d \sqrt{2g} \cdot L} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]$$

P.g NO:- 176

15. $Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$

$C_d = 0.62$

$= \frac{2}{3} \times 0.62 \times L \sqrt{2 \times 9.81} \times H^{3/2}$

$= 1.83\, L\, H^{3/2}$

17. $Q = 1.418\, H^{5/2}$

$= 1.418 \times (0.1)^{5/2}$

$= 4.48$ lps

$= 4.48 \times 60$ lpm

$= 269 \frac{\text{lit}}{\text{min}}$

19. $$T = \frac{2A}{\frac{2}{3} C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

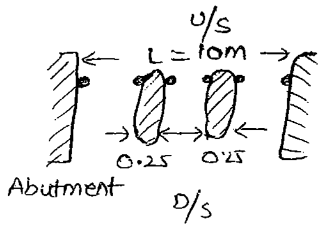
$$T \propto \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

$$\frac{T_1}{T_2} = \frac{\left[\frac{1}{\sqrt{H_2'}} - \frac{1}{\sqrt{H_1'}}\right]}{\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]}$$

$$\frac{200}{T_2} = \frac{\left(\frac{1}{\sqrt{16}} - \frac{1}{\sqrt{25}}\right)}{\left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{25}}\right)}$$

$T_2 = 1200$ sec.

18. Given $L = 10$ m



$L_e = L - 0.1 n H$

$= [(10 - 0.25 - 0.25) - (0.1 \times 6 \times 0.6)]$

$= [9.5 - 0.6 \times 0.6]$

$= 9.14$ m

P.g NO:- 176

2. $\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$

$\frac{dH}{H} = \frac{2}{60} = 3.33\%$

$\frac{dQ}{Q} = 1.5\,(3.33)$

$= 5\%$

12. $\frac{dQ}{Q} = \frac{3}{2} \cdot \frac{dH}{H}$

$= \frac{3}{2} (5\%)$

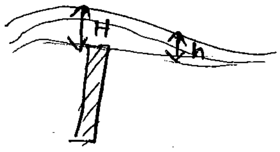
$= 12.5\%$

18. $$\frac{Q_{V\text{-notch}}}{Q_{Rectangle}} = \frac{\frac{8}{15} C_d \sqrt{2g}\, H^{5/2}}{\frac{2}{3} C_d \sqrt{2g}\, L\, H^{3/2}}$$

$= 2.5$

P.g NO:- 178.

27. $Q \propto Hh^2 - h^3$



$$\frac{dQ}{dh} = 0 \qquad \frac{h}{H} = \frac{2}{3}$$

$$0 = H \cdot 2h - 3h^2$$

$$2Hh = 3h^2$$

$$2H = 3h$$

$$\frac{h}{H} = \frac{2}{3}$$

2-12-2014

UNIT - 12

COMPRESSIBLE FLOW

1. Fluid two types with respect to mass density changes if mass density doesn't change @ different points of the control volume is said to be incompressible fluid.
2. If mass density does change is said to be compressible fluid flow.

   Ex:- Gases, and steams.
3. Study of compressible fluid flow is necessary for space applications high speed machinery like compressors and other flow measuring devices for compressible fluid. (Nozzle meter, Orifice meter, pitot tube and anemometer).
4. compressible flow is governed by the following equation.

   Equation of continuity for compressible fluid.

$$m = \rho \cdot Q$$

$$m = \rho A V = \text{constant across the control volume}$$

$$m = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\rho A V = \text{constant}$$

Differentiate both sides

$$d(\rho A V) = d(C)$$

$$d\rho \cdot (AV) + \rho d(AV) = 0$$

$$d\rho AV + \rho[dA\, V + A \cdot dV] = 0$$

$$d\rho AV + \rho dA \cdot V + \rho A dV = 0$$

$$\boxed{\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0}$$

Conservation of mass (or) continuting fluid equation of compressible flow.

Basis for compressiblity consideration:-

$$\text{Mach number } (M) = \frac{\text{Inertia force}}{\text{Elastic force}} = \frac{V}{C}$$

$$C = \sqrt{\frac{K}{\rho}}$$

$$C_{air} = \sqrt{\frac{1 \times 10^5 \ N/m^2}{1.2 \ (kg/m^3)}}$$

$$= 300 \ m/sec$$

If $M < 0.3$ that fluid treated as incompressible fluid. (compressibility effect ignored)

Classification of compressible fluids (based on Mach number):

$0.3 < M < 1.0$ (subsonic flow)

$M = 1.0$ (sonic flow)

$1 < M < 5$ (Super sonic flow)

$M > 5$ (Hyper sonic flow)

Bernoullis energy equation for compressible fluid flow:-

$$\int \frac{dP}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

Gas state equation (or) perfect gas equation (or) Ideal gas equation:-

$$PV = mRT$$

P = absolute pressure (N/m^2)

V = volume of gas (m^3)

m = mass of gas (kg)

R = Gas constant (J/kg)

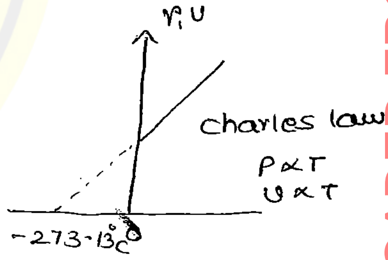
T = absolute temperature ($K = C^\circ + 273$)

$$PV = mRT$$

$$P = \left(\frac{m}{V}\right) RT$$

$$P = \rho \cdot R \cdot T$$

$$\rho = \frac{P}{RT}$$



(ρ is a function of pressure, Temperature)

$$\int \frac{dP}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\int \frac{dP}{\frac{P}{RT}} + gz + \frac{v^2}{2} = \text{constant}$$

$$RT \int \frac{dP}{P} + \overset{gz+\frac{v^2}{2}}{\cancel{(-d_0)}} = \text{constant}$$

$$RT \log_e P + gz + \frac{v^2}{2} = \text{constant}$$

$$RT_1 \log_e P_1 + gz_1 + \frac{v_1^2}{2} = RT_2 \log_e P_2 + gz_2 + \frac{v_2^2}{2}$$

If $T_1 = T_2 = T$

$$RT \log_e \left(\frac{P_1}{P_2}\right) + g(z_1 - z_2) + \left(\frac{v_1^2 - v_2^2}{2}\right) = 0$$

$$\text{Specific volume} = \frac{1}{\text{mass density}} \text{ (volume per unit mass)}$$

Terminology in compressible fluid flow:-

1. Specific heat (constant pressure and volume) C_p & C_v.
2. Ratio of specific heats of a gas ($\gamma = K = C_p/C_v$)
3. Enthalpy of a gas, entralpy of a gas, Adiabatic process, Isentropic process, polytropic process, Isothermal process, Internal energy of a gas, Mach number.

Enthalpy (h):-

It is called total heat of a gas. It is the energy pocessed by certain mass of the gas by virtue of its temperature, ~~hinges~~ higher the temperature, higher the enthalpy value.

Internal energy :-

It is the property of the gas system. It is the function of temperature. The net change equal to zero for a cyclic process.

cyclic process are thermodynamic process. In which the gas undergoes certain thermodynamic process and restore its initial stage.

$$dE = dU = 0$$

$$dE = dU = m \cdot C_v \cdot dT$$

$$dE \propto dT$$

Area of pressure diagram - Volume (U) Diagram = workdone per kg

= Energy by kg of fluid.

$$P \times U = \frac{N}{m^2} \times \frac{m^3}{kg}$$

$$= \frac{N-m}{kg} = \frac{J}{kg}$$

Enthalpy (h) = sum of internal energy and product of pressure and volume

$$h = E \text{ (or) } U + PV$$

$$h = E + PV$$

$$dh = dE + d(P \cdot V)$$

$$= dE + (PdV + VdP)$$

where

PdV is called thermodynamic work for closed system.
VdP is called flow work (open system work)

Open system:-

It is one through which mass and energy both crosses the boundary.

closed system:-

It is one in which mass remains constant and energy mass cross the boundary.

Isolated System:-

It is a system through which neither mass (or) energy crosses the boundary.

Specific heat:-

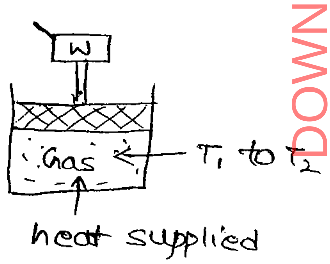
It is the amount of heat required to rise 1°c of one unit mass

Heat supplied ∝ change of temperature

$$Q \propto dT$$

$$Q = C_p \cdot dT$$

$$C_p = \frac{Q}{dT}$$



Specific value has two units:-

1. C_p = sp. heat at constant pressure, it is the amount of heat required to rise one unit mass through 1°c at constant pressure process

$$Q = C_p \times dT \qquad C_p = \left(\frac{Q}{dT}\right)_{\text{constant}}$$

$$\frac{p}{\rho} = c \text{ (constant)}$$

$$\therefore \boxed{\rho = \frac{p}{c}}$$

$$\therefore \int \frac{dp}{\frac{p}{c}} + \int v \cdot dv = \text{constant}$$

$$c \int \frac{dp}{p} + \int v \cdot dv = \text{constant}$$

$p_1 v_1 = p_2 v_2 \leftarrow$ $$c \cdot \log_e p + \frac{v^2}{2} = \text{constant}$$

$$\boxed{\therefore \log_e p (pv) + \frac{v^2}{2} = \text{constant}}$$

Energy equation for Isentropic process:- (Reversible adiabatic process)

$$\int \frac{dp}{\rho} + \int v \cdot dv = \text{constant}$$

Isentropic process

$$p v^{\gamma} = c$$

$$p v^{k} = c$$

$$\frac{p}{\rho^k} = c$$

$$\rho^k = \frac{p}{c}$$

$$\rho = \left(\frac{p}{c}\right)^{1/k}$$

$$\int \frac{dp}{\left(\frac{p}{c}\right)^{1/k}} + \int v \cdot dv = \text{constant}$$

$$(c)^{1/k} \int \frac{dp}{p^{1/k}} + \int v \cdot dv = \text{constant}$$

$$c^{1/k} \int p^{-1/k} \cdot dp + \int v \cdot dv = \text{constant}$$

$$c^{1/k} = \frac{p^{-1/k + 1}}{-\frac{1}{k} + 1} + \frac{v^2}{2} = \text{constant}$$

$$c^{1/k} = \frac{p^{\frac{k-1}{k}}}{\frac{k-1}{k}} + \frac{v^2}{2} = \text{constant}$$

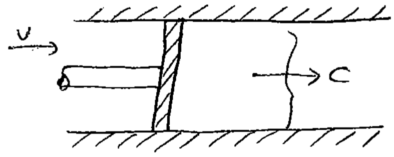
$$\frac{k \cdot c^{1/k} \cdot p^{\frac{k-1}{k}}}{k-1} + \frac{v^2}{2} = \text{constant}$$

$$\left(\frac{k}{k-1}\right) \cdot \left(\frac{p}{\rho^k}\right)^{1/k} (p)^{\frac{k-1}{k}} + \frac{v^2}{2} = \text{constant}$$

$$\boxed{\frac{k}{k-1} \cdot \frac{p}{\rho} + \frac{v^2}{2} = \text{constant}}$$

Propagation of pressure wave (or) compressible wave:-

Let v = velocity of moving body or object or fluid in a compressible media $\left(\frac{m}{sec}\right)$



c = velocity of elastic / pressure / sound wave $= \sqrt{\frac{K}{\rho}}$

For isothermal process

$$c = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{P}{\rho}}$$

$$PV = mRT \qquad \therefore c = \sqrt{RT}$$

$$P = \frac{m}{V} RT$$

$$P = \rho RT$$

$$\frac{P}{\rho} = RT$$

$$\cancel{c = \sqrt{RT}}$$

For Isentropic process (PV^{γ} = constant)

$$c = \sqrt{\frac{K}{\rho}} = \sqrt{\gamma RT}$$

$$\therefore \gamma = \frac{C_p}{C_v}$$

Mach number (M):-

$$M = \frac{V}{C}$$

$M < 0.3$ (or) 0.4

Fluid is incompressible

$0.3 < M < 1$ (Sub sonic fluid)

$M = 1$ (sonic fluid)

$1 < M < 5$ (Super sonic fluid)

$M > 5$ (Hyper sonic)

$1 < M < 1.1$ (Trans sonic).

Ex:- An aeroplane is found to be flying at 720 kmph where the atmospheric temperature is 7°c. Determine the following

a. velocity of sound under isothermal condition.

b. velocity of sound (or) pressure wave under Isentropic conditions ($R = 0.283\ KJ/kg\,kelvin$ and $\gamma = \frac{C_p}{C_v} = k = 1.4$]

c. Mach number

d. Type of compressible air flow of the aeroplane.

A. Given $V = 720$ kmph $\gamma = k = \frac{C_p}{C_v} = 1.4$, $R = 0.283\ \frac{k\cdot Joule}{kg-k}$

$= 720 \times \frac{5}{18}$ $T = 7^\circ c = 273+7 = 280\ K$ $= 283\ \frac{J}{kg-k}$

$= 200\ m/sec$

a) $C = \sqrt{R \cdot T}$

$= \sqrt{283 \times 280}$

$= 281.5\ m/sec$

b) $C = \sqrt{\left(\gamma = k = \frac{C_p}{C_v}\right) \times RT}$

$= \sqrt{1.4 \times 283 \times 280}$

$= 333\ m/sec$

c) $M = \frac{V}{C}$ (under Isentropic process)

$= \frac{200}{333}$

$M \simeq 0.6$ $(0.3 < M < 1)$

d) Body motion in "sub sonic condition".

P.g No:- 189

40. Given $K = 1.4$ $T = 30^\circ c$

$= 30 + 273$

$= 303\ K$

$C = \sqrt{KRT}$

$= \sqrt{1.4 \times 283 \times 303}$

$= 346\ m/sec$

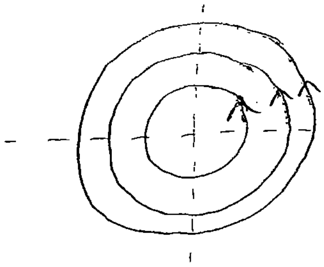
21. $V = \sqrt{RT \cdot \gamma}$

$= \sqrt{283 \times (273+15) \times 1.4}$

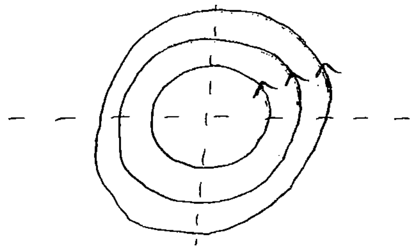
$= 340 \text{ m/s}$

propagation of compressible fluid waves:-

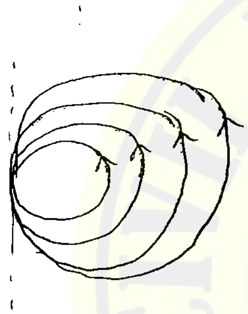
$0 < M < 0.3$



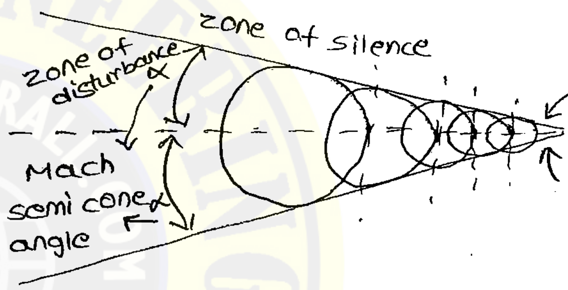
$0.3 < M < 1$



$M = 1$



$M > 1$



P.g No:- ~~188~~ ~~192~~ 189

~~16~~ 25. Given $P = 16.5 \text{ kN/m}^2$

$\rho = 0.2865 \text{ kg/m}^3$

$k = 1.4$

$PV = mRT$

$P = \rho RT$

$T = \frac{P}{\rho R}$

$C = \sqrt{K \cdot R \cdot T}$

$= \sqrt{K \cdot \cancel{R} \cdot \frac{P}{\rho \cancel{R}}}$

$= \sqrt{1.4 \times \frac{16.5}{0.2865}}$

$= 295 \text{ m/sec}$

$\sin \alpha^\circ = \frac{C}{V}$

$\sin \alpha^\circ = \frac{1}{\frac{V}{C}} = \frac{1}{m}$

$\alpha^\circ = \sin^{-1}\left(\frac{1}{m}\right)$

26. $T = -56^\circ C$

$= -56 + 273$ K

$= 217$ K

$k = \gamma = \frac{C_p}{C_v} = 1.4$

$R = 287$ J/kg·K

$$c = \sqrt{KRT}$$

$$= \sqrt{1.4 \times 287 \times 217}$$

$$= 295 \text{ m/s}$$

29. Given $V = 1580 \times \frac{5}{18}$

$= 439$ m/sec

$T = -60^\circ C$

$= -60 + 273$

$= 213$ K

$R = 287$ J/kg-K

$K = 1.4$

$$M = \frac{V}{C}$$

$$= \frac{439}{\sqrt{KRT}}$$

$$= \frac{439}{\sqrt{1.4 \times 287 \times 213}}$$

$$= 1.50$$

P.g NO:- 191

36. $C = 300$ m/sec

$V = 1620 \times \frac{5}{18} = 450$

$\sin\alpha = \frac{C}{V}$

$= \frac{300}{450}$

$\alpha = \sin^{-1}\left(\frac{2}{3}\right)$

$= 41.8^\circ$

8. $C = \sqrt{\gamma RT} = \sqrt{\frac{K_{\text{bulk modulus}}}{\rho}}$

$PV = MRT$

$P = \rho RT$

$\rho = \frac{P}{RT}$

$C = \sqrt{\gamma \cdot R \cdot \frac{P}{\rho R}}$

$C = \sqrt{\frac{\gamma \cdot P}{\rho}} = \sqrt{\frac{K_{bulk}}{\rho}}$

$K_{\text{bulk modulus}} = \gamma P$ (or) KP

P.g NO:- 192

1. $\sin\alpha^\circ = \frac{C}{V}$

$= \frac{340}{393}$

$\sin\alpha = 0.86$

$\alpha = 60^\circ$

7. $V = ?$ m/sec $\alpha = 30°$

$c = 330$ m/sec

$$\sin\alpha = \frac{c}{V}$$

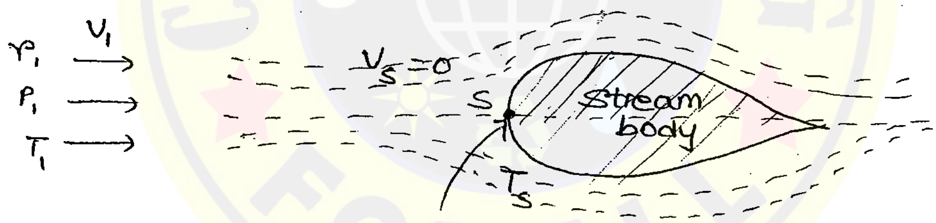
$$\sin 30° = \frac{330}{V}$$

$$V = 660 \text{ m/sec}$$

27. Due to friction between surface and compressible fluid flow the estimated value is to be multiplied with compressibility correction factor.

Stagnation properties:-

In the analysis of compressible fluid flow ($M > 0.3$). The fluid properties like absolute pressure and absolute temperature effected by the velocity of the fluid. When moving fluid brought to rest (approaches to zero velocity) is said to be stagnation point. The properties at stagnation point are called stagnation property.



Stagnation point

$$T_s = \text{stagnation temperature} = T_1\left[1 + \frac{\gamma - 1}{2} M_1^2\right]$$

where $\gamma = K = \frac{C_p}{C_v} = 1.4$

$$= T_1\left[1 + \frac{1.4 - 1}{2} M_1^2\right]$$

$$T_s = T_1\left[1 + 0.2 M_1^2\right]$$

P.g No:- 189

40. $T_s = T_1(1 + 0.2 M_1^2)$

$$(273 + 59.7) = (273 + 30)(1 + 0.2 M_1^2)$$

$$M_1 = 0.7$$

39. $166.7(1 + 0.2 M_1^2) = 30($

$$M_1 = 2.0$$

Pg NO:- 192

17. $T_s = T_1(1+0.2M_1^2)$

$$M_1 = \frac{V}{C} = \frac{0.6C}{C}$$

$M_1 = 0.6$

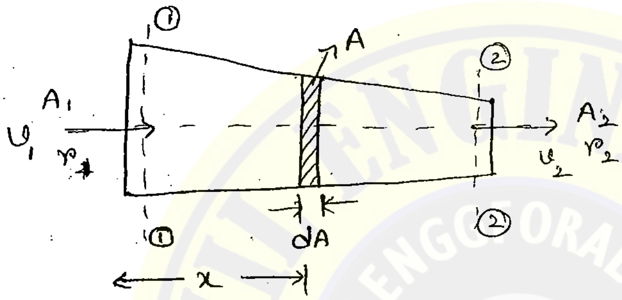
$= (273+16)(1+0.2(0.6)^2)$

$= 305.5\ K$

$= 305.5 - 273^\circ C$

$= 32.5^\circ C$

Compressible flow through variable Area:-



continuity equation:

$$m' = \rho A v = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0$$

$$M = \frac{V}{C} = \sqrt{\frac{F_I}{F_E}}$$

$$C = \sqrt{\frac{dP}{d\rho}}$$

Simplify above equation

$$-\frac{dA}{A} = (1-M^2)\left(\frac{dv}{v}\right)$$

$$\boxed{\frac{dA}{A} = (M^2-1)\frac{dv}{v}}$$

HYDRAULIC MACHINES (30 M)

A3 – civil

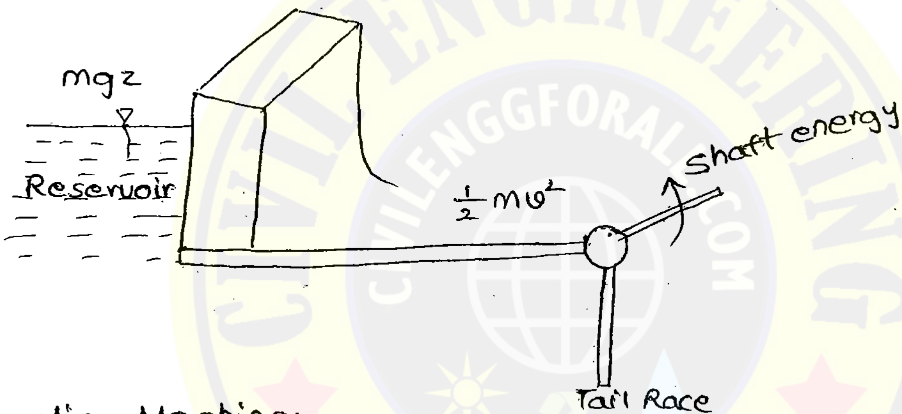
(1)

UNIT – I

INTRODUCTION TO HYDRAULIC MACHINES

Hydraulic means study of water. water at rest mean potential energy (pressure energy stored per unit volume).
Ex:- Reservoir of a dam.

The fluid machine which converts Fluid energy into useful shaft power for which fluid in motion make use of it penstock (long, large pipe) which conveys fluid from dam to turbine. potential energy converted into kinetic energy ($mgz = \frac{1}{2}mv^2$) i.e., $v = \sqrt{2gz}$



Hydraulic Machine:-

Machine is a device consists moving parts like runner, rotating shaft, guide vanes etc. Hydraulic machine mak use of Hydraulic energy (mgz (or) $\frac{1}{2}mv^2$) with water as working substance which passes through runner wheel.

Impulse momentum principle (Newtons second law) use to develop force on the runner ($F = ma = m'\Delta v$). This force multiply with radius of the wheel produces useful torque ($T = F \times R = F \times d/2$). This turbine wheel coupled to electrical generat shaft which converts shaft power into electrical power.

$$T = F \times R$$

$$T = ma \times \frac{D}{2}$$

$$T = m'(\Delta v) \times \frac{D}{2}$$

$$\text{Shaft power} = \frac{2\pi NT}{60} \quad \text{(Watt)}$$

$$\text{Electric power} = \text{volt} \times \text{current} \quad (\text{volt} \times \text{Ampere} = \text{watt})$$

$$N = \frac{120 f}{P}$$

$$N = \frac{60 f}{P_1}$$

P' = No. of pairs of poles

P = No. of poles (North and south pole)

f = electric power supply frequency (50 Hz)

Note:-

1. Rotating element of a turbo machine called Runner for water turbines. In case of pumps it is called Impellar.
2. The impeller is mounted on fixed axis shaft which is coupled to electrical generator shaft.
3. The generator shafts needs certain RPM to cut the magnetic field (EMF of conductor ∝ rate of change of magnetic flux). (webers)

$$\text{emf} = \text{voltage produced} \propto \frac{d\phi}{dt}$$

$$\text{emf} = N \cdot \frac{d\phi}{dt}$$

Hydraulic machine broadly a device, capable to convert available fluid flower into shaft power is called Water turbine.

A Hydraulic machine which converts shaft power into fluid power is called pump.

In generall fluid machine called prime movers and also called Turbo machinery.

Examples for power generating turbo machine.

1. Wind mill.
2. Water turbine.
3. Steam turbine
4. Gas turbine

Examples for power consuming fluid machine.

1. Roto dynamic pumps (centrifugal pump, gear pump, screw pu[mp] axial pump and compressors) ②

Note:-

Fans and bloyers also called Turbo machinery as the consists rotating element called Rotor.

Reciprocating water pump, gear pump, vane pump, lobe pump are not turbo machines, as they have sliding action (fluid motion is linear due to relative motion of the element) Generally these are positive displacement machines (No slip takes place).

Other fluid machines like Hydraulic coupling, Torque convertor, Hydraulic ram, are belongs to energy transmitted devices.

Diesel engines and petrol engines are also called prim[e] movers consists piston which reciprocates. Hence diesel, petr[ol] fluid engines are not turbo machines.

Indian base power depends on Thermal powers (coal water) whose operating cost very high whereas Hydraulic power plant operating cost is very cheap and initial investment is high (dam construction)

Ex:- 1 unit = 1 KW Hr

10 bulbs x 60 watts = 600 watts

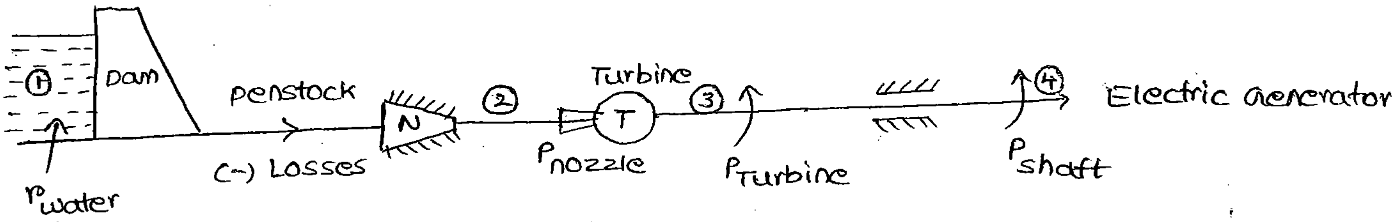
4 Fans x 100 watts = 400 watts

1000 Watts

= 1 KW → Running for 1 hour.

UNIT - II

HYDRO - ELECTRIC POWER PLANT



1. water power $= P_w = P_{I/p} = P = 100$ watts

2. Losses in penstock $= -3$ watts

3. power available at inlet of nozzle $= 97$ watts

$$\eta_{Transmission} = \frac{97}{100} \times 100\,\%$$

$$= 97\,\%$$

4. Power lost in nozzle $= -1$ watt

5. power delievered by nozzle $= \frac{1}{2} m v^2$

$$= 97 - 1 = 96 \text{ watts}$$

$$\eta_{nozzle} = \frac{96}{100} \cancel{\times 100}\,\%$$

$$= \frac{96}{97}\,\%$$

6. power loss in turbine $= 4$ watts

7. $P_{Turbine} = 96 - 4$

$$= 92 \text{ watts}$$

$$\eta_{Turbine} = \frac{P_T}{P_N} = \frac{92}{96} \text{ watts}$$

8. $P_{shaft} = 92 - 2$

$$= 90 \text{ watts}$$

$$\eta_{mechanical} = \frac{P_{shaft}}{P_{turbine}} = \frac{90}{92}\,\%$$

$$\eta_{overall} = \frac{P_{shaft}}{P_{water}} = \frac{90}{100} \times 100\,\% = 90\,\%$$

$$\eta_{overall} = \eta_{Transmission} \cdot \eta_{nozzle} \cdot \eta_{Turbine} \cdot \eta_{mech}$$

$$= \frac{96}{100} \times \frac{92}{96} \times \frac{90}{92} \times \frac{96}{97}$$

3

$$= 90\%$$

Ex:- A small Hydel powerplant consists a single turbine producing shaft power is 10 MW. The available head at the dam is 42 m. The penstock losses 7m. The nozzle efficiency is 98; The turbine efficiency expected 80%, Mechanical losses is 5% of the shaft power. Determine the following

i) Mechanical efficiency

ii) Turbine power

iii) Amount of water flow rate required

iv) Diameter of the nozzle jet.

A) Given $\eta_T = 80\%$ $\quad \eta_N = 98\%$ $\quad P_{shaft} = 10$ MW, $\quad h_{L\,pipe} = -7$m

1) Mechanical power lost $= 5\% \cdot P_{shaft}$

$$= 0.05 \times 10$$

$$= 0.5 \text{ MW}$$

$$\eta_{mech} = \frac{P_{shaft}}{P_{Turbine}} = \frac{10}{10.5} = 95\%$$

2) $P_{Turbine} = 10.5$ MW

$$\eta_{Turbine} = \frac{P_{Turbine}}{P_{nozzle}}$$

$$0.8 = \frac{10.5}{P_{nozzle}}$$

$$P_{nozzle} = 13.125 \text{ MW}$$

Loss of power in turbine $= 13.125 - 10.5$

$= 2.625$ MW

$$P_{nozzle} = 98\% = \frac{P_{\text{input of } \cancel{\text{nozzle}} \text{ Turbine}} = P_{\text{output of nozzle}}}{P_{\text{input of nozzle}}}$$

$$0.98 = \frac{13.125}{P_{input}}$$

$$P_{input} = 13.39 \text{ MW}$$

$$\frac{1}{2} m v^2 = 13 \cdot 39 \times 10^6$$

$$\frac{1}{2} \rho Q v^2 = 13 \cdot 39 \times 10^6$$

$$\frac{1}{2} \times 1000 \times A_{jet} \times V_{jet} \times v_{jet}^2 = 13 \cdot 39 \times 10^6$$

$$\frac{1}{2} \times 1000 \times \frac{\pi d^2}{4} \times \sqrt{2g(H-h_L)} \times 2g(H-h_L) = 13 \cdot 39 \times 10^6$$

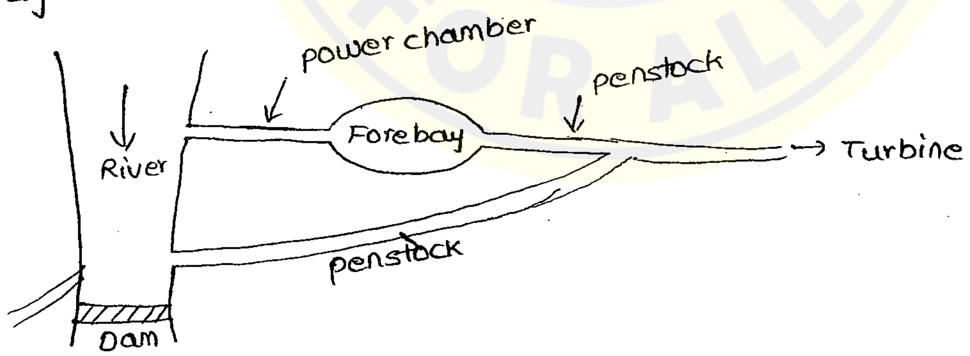
$$H = 42m \qquad h_L = -7$$

$$d =$$

Different components of Hydel power plant:-

1. Reservoir (or) Pond (or) Fore bay
2. Dam (concrete structure to built the head)
3. Penstock (To convey or carry water from reservoir to turbine)
4. Surge tank
5. Nozzle (For impulse turbines only)
6. Water turbine (impulse or reaction type)
7. Electrical generator
8. Draft tube (only for reaction turbines)
9. Tail race

Forebay and its location:-



Forebay is located at the junction of power channel and penstock.

Draft tube:-

Draft tube is a long, tappered pipe connected at exit of the reaction water turbine to the tail race. It is not use for impulse turbines (pelton wheel). Since the pelton wheel operates in open atmosphere. Draft tube converts exit K.E of water from reaction turbine into useful potential energy.

Pumped storage Hydro electric powerplant :- ④

In this plant the same turbine is used as pump. During demand hours of power, turbine produce power for the water from reservoir is collected at turbine at downstream pond. During the non demand hours of power same turbine pump the water into the reservoir. such plant is called peak load power plants.

Hydro graph :-

Hydrograph is a graphical representation of discharge used for power generation with respect to time (q-T graph)

Sequence of elements for Hydel power plant :-

1. Dam and Reservoir ⟶ penstock ⟶ guide wheel ⟶ runner wheel ⟶ draft tube.

Classification of water turbines :-

There are five factors to classify water turbines.

1. Available water head
2. Type of flow through water turbine
3. Range of specific speed.
4. Quantity of water per minute.
5. Type of water action (impulse and reaction type)

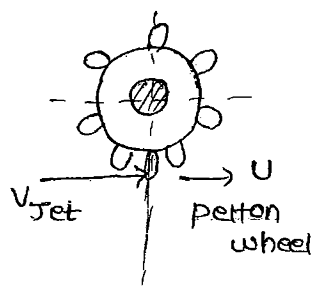
Available water head :-

Ex:-

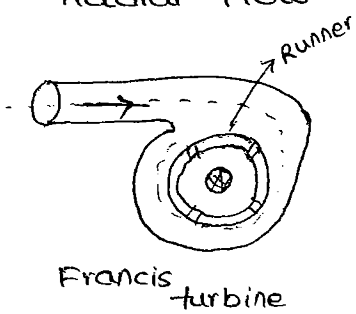
1. High head water turbine (H is more than 300 m) (pelton wheel)
2. Medium head turbine (30m - 300m range). (Francis)
3. Low head turbine (H < 30m) (Keplan and propellar)

Types of flow :-

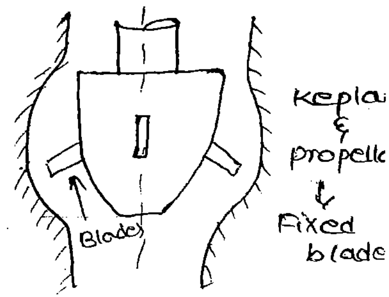
Tangential flow



Radial flow



Axial flow



4-12-

UNIT - 6

SPECIFIC SPEED AND PERFORMANCE OF TURBINES

Specific speed:-

$$N_s = N\sqrt{P}/H^{5/4}$$

N = speed of turbine (rpm)

P = shaft power (KW)

H = Head (m)

Available water head:-

1. High head water turbine ($H > 300$ m) – pelton wheel
2. Medium head water turbine (30 m to 300 m range) – Francis turbine
3. Low head turbine ($H < 300$ m) — kaplan and propellar turbine.

Based on specific speed range values:-

1. Low specific speed turbine (N_s) – $0 < N_s < 30$ → pelton wheel with one jet

   $30 < N_s < 60$ → with multi jet
2. Medium specific speed turbine (N_s) – $60 < N_s < 300$ → Francis turbine
3. High specific speed turbine (N_s) – $N_s > 300$ → Kaplan and propellar

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{\frac{1}{Time}\sqrt{\frac{J}{s} = \frac{N-m}{s}}}{(m)^{5/4}} = \frac{kg - \frac{m}{s^2} \cdot \frac{m}{s}}{m^{5/4}} = T^{-1} \cdot M^{1/2} L^{1} \cdot T^{-3/2} \cdot L^{-5/4}$$

$$= M^{1/2} L^{-1/4} T^{-5/2}$$

Ex:- Identify type of water turbine suitable if N = 540 rpm, P = 8.1 MW and H = 81 m

A.

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{540 \times \sqrt{8.1 \times 1000}}{(81)^{5/4}}$$

$$= \frac{540 \times 90}{(3^4)^{5/4}}$$

$$= \frac{540 \times 90}{\cancel{5}\,720}$$

$$= 200$$

~~Ex~~ Based on discharge passes through turbine:-

$1000 < Q < 10000$ lpm → Medium discharge turbine (~~pelton wheel~~ Francis turbine)

$Q < 1000$ lpm → Low discharge turbine (~~Francis turbine~~ pelton wheel)

$Q > 10000$ lpm → High discharge turbine (kaplon and propello

Based on water action on the runner of the turbine:-

1. Impulse action turbine (Input energy fully is kinematic energy of the water jet).

   Ex:- pelton wheel.

2. Reaction type turbine (I/p energy is partly kinetic energy & mostly potential energy (mgh))

   Ex:- Francis and keplon, propellar.

pelton wheel:-

High head, tangential flow, low specific speed, low discharge and impulse type.

Francis turbine:-

Medium head, inward radial flow, medium specific speed medium discharge and reaction type.

Keplan and propellar turbines:-

Low head, axial flow, high specific speed, high discharge and reaction type turbines.

propellar turbine specific speed is very high $N_s > 800$

keplan, $N_s = 300$ to 800

Modern Francis turbine:-

It is mixed flow turbine (radial + axial flow)

In generall hydroelectric plants also different names given.

1. Micro Hydraulic power plant < 0.1 MW.
2. Mini ideal power plant (0.1 MW to 2 MW)
3. Low ideal power plant (2 MW to 15 MW)
4. Medium ideal power plant (15 MW to 100 MW)
5. Mega power plants (> 1000 MW)

IMPULSE TURBINES

A water turbine is sent to be impulse type. If the following satisfied.

1. High head availability
2. Low discharge sufficient
3. Input energy to the turbine is purely KE $(\frac{1}{2}mv^2)$
4. The turbine is to work in open atmosphere (no pressure drop)
5. It works on impulse momentum principle (Newtons second law)
6. No draft tube is required.
7. Turbine is mounted above the tail race (never be submerged in tail water).
8. Low specific speed range.
9. casing is provided for safety as well as to avoid splashing of water.
10. There is velocity drop across turbine.
11. More overall efficiency. Not suited for part load requirements (keplan is best for part load)
12. Impulse turbine shaft mostly horizontal.
    Ex:- Pelton wheel, Targo wheel
13. Impulse is tangential flow type.

Jet Ratio (m):-

$$m = \frac{D}{d}$$

For pelton wheel Range of jet ratio $10 < m < 20$

No. of buckets (z):-

No. of buckets on pelton wheel, $z = 15 + \frac{D}{2d}$

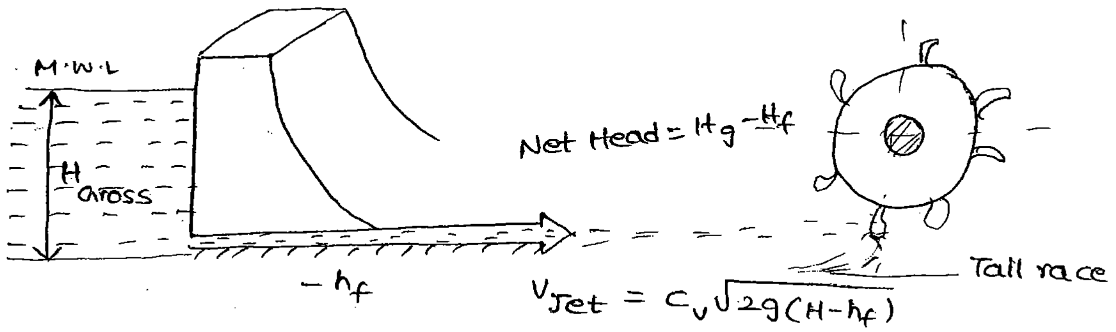
$$= 15 + \frac{m}{2}.$$

$$\text{Speed ratio}(\phi) = \frac{u}{\sqrt{2gH}}$$

Range:- $0.4 < \phi < 0.5$

pelton wheel :-

construction and working :-



$$V_{Jet} = C_v \sqrt{2g(H-h_f)}$$

$$V_{theo} = V_{spouting} = \sqrt{2gH}$$

1. The shape of each bucket cup is Hemispherical.
2. Two Hemispherical cups connected by a splitter.
3. The jet water touches the splitter and divide into two stream and deflected by an angle < 180° (165° - 175°)
4. If deflection angle is 180° then outgoing jet interfere in coming bucket makes wheel retardation. Hence buckets never be flat.

$$\eta_{pelton} = \frac{P_{pelton}}{\frac{1}{2} m v_1^2}$$

$$\eta_{overall\ pelton} = \frac{P_{shaft}}{P_{water}} \rightarrow \rho g Q H$$

P.g NO:- 10

14. $A_{jet} = \frac{\pi d^2}{4} = 0.05\, m^2 = 50\, cm^2$

$H = 400\, m \quad C_v = 0.95$

$$Q = A_{jet} \cdot V_{jet}$$
$$= A_{jet} \times C_v \sqrt{2gH}$$
$$= 0.05 \times 0.95 \sqrt{2 \times 9.81 \times 400}$$
$$Q = 4.2\, m^3/sec$$

15. $Q = 1\, m^3/sec \quad H = 300\, m \quad \eta = 0.85 \quad P_{shaft} = ?$

$$\eta = \frac{P_{shaft}}{P_{water}}$$

$$0.85 = \frac{P_{shaft}\ (watts)}{1000 \times 9.81 \times 1 \times 300}$$

$$P_{shaft} = 2.5 \times 10^6\ watts \Rightarrow 2.5\ MW$$

16. $\eta = \dfrac{P_{shaft}}{\rho g Q H}$

$$0.85 = \frac{16 \times 10^6}{1000 \times 9.81 \times Q \times 800}$$

$$Q = 3.4 \ m^3/sec$$

18. $V_{jet} = 120 \ m/sec$

$Q = 2.95 \ m^3/sec$

No. of nozzle jets = ?

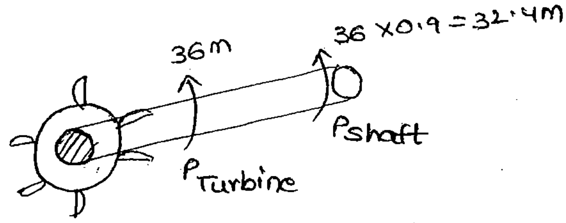
$d = \frac{1}{8} = 0.125m$

$Q_{total} = \text{no. of Jets} \times q_{single\ jet}$

$2.95 = N_{nozzle} \times \frac{\pi}{4}(0.125)^2 \times 120$

$N_{nozzle} = 2$

19.



$\eta_o = \eta_{Hydraulic} \cdot \eta_{mechanical}$

$= \eta_T \cdot \eta_{mech}$

$= \left(\frac{36}{40}\right) \cdot (0.9)$

$= 81\%$

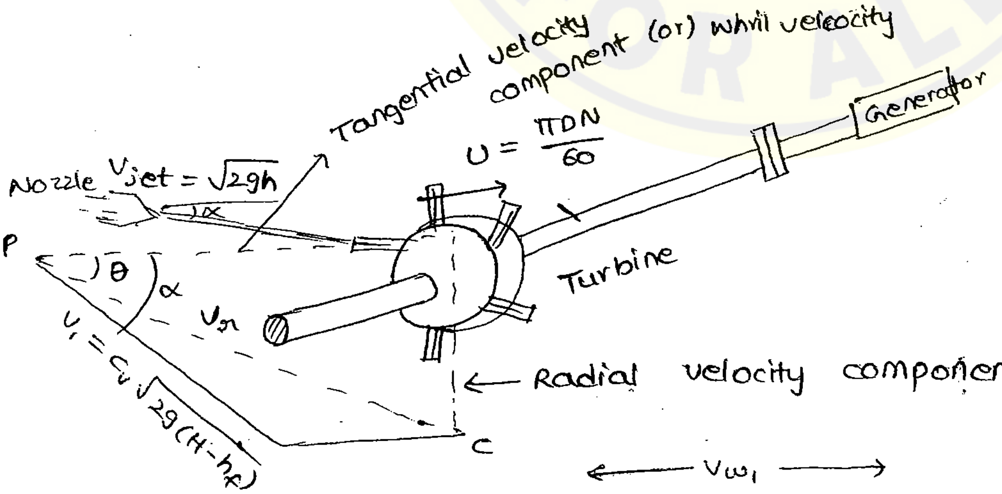
$\eta_o = \frac{32.4}{40} = \frac{P_{shaft}}{P_{Head}}$

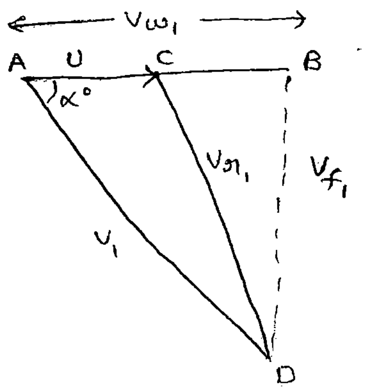
$= 0.81$

$= 81\%$

Velocity diagrams:-

In order to design any turbo machine velocity triangles necessary to fix the turbine blade angles for maximum benefit in terms of maximum power and maximum efficiency.





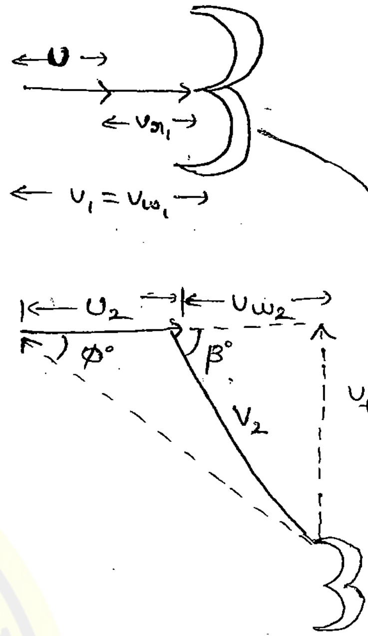
Area of the velocity diagram represents the energy of the fluid converted into useful work of the turbine runner.

Velocity diagram for pelton wheel:-

Pelton wheel is a tangential flow ($\alpha = 0°$) v and u are in same line.

$V_{f_1} = 0$ and $\theta = 0°$, $\alpha = 0°$

$V_1 = V_{w_1}$, $V_{r_1} = V_1 - U$



Inlet diagram of pelton wheel is straight line.

$F_{Jet} = ma$

~~$= F_{Jet} \times U$~~

$= m' \Delta V$

$= \rho Q (\Delta V)_w$

$F_{Jet} = m' (V_{w_1} + V_{w_2})$

$\therefore P_{pelton} = m'(V_{w_1} + V_{w_2}) U$

$$\eta_{pelton} = \frac{m'(V_{w_1} + V_{w_2}) U}{\frac{1}{2} m' v_1^2}$$

Condition for maximum power developed:-

$U = V_1/2$

Let H = head on the pelton wheel

V_1 = absolute velocity of water jet $= C_v \sqrt{2gH}$

U_1 = Turbine runner tangential velocity at inlet $= \frac{\pi D_1 N}{60}$

V_{r_1} = relative velocity of water at inlet

V_{f_1} = radial flow velocity at inlet (radial / verticle component of V_1)

V_{w_1} = whril (or) swirl (or) vortex velocity of water at inlet (tangential velocity component).

Note:-

The bearing reactions developed due to flow velocity components of water. Useful for force of jet components is whirl component. whirl component inline with tangential velocity.

The resultant of V and U is called V_r (Relative velocity).

Resultant V and V_f is called V_w

Efforts are made to increase the power developed by modyfying runner blade angles and size of the wheel.

Tangential force developed by jet on pelton wheel =

$F_d = ma = m'(\Delta V)_w$

α° = nozzle angle at inlet (angle made by V_1 with reference to U_1)

θ° = inlet blade (or) bucket (or) vane angle.

Similarly V_2, U_2, V_{f_2}, V_{w_2}, β°, ϕ° are respective variables at exit of the turbine runner.

power developed by the pelton wheel $= m'(V_{w_1} + V_{w_2})U$ watts

In order to maximise the power developed by pelton wheel replace the velocities in terms of available variables like V_1, U and ϕ°

$$P_{pelton} = m'(V_{w_1} + V_{w_2})\cdot U$$

$V_{r_2} = K \cdot V_{r_1}$

K = bucket friction factor

= 0.9 to 1

$$\cos\phi^\circ = \frac{U_2 + V_{w_2}}{V_{r_2}}$$

$$= \frac{U + V_{w_2}}{K V_{r_1}}$$

$$\cos\phi^\circ = \frac{U + V_{w_2}}{K(V_1 - U)}$$

$$K(V_1 - U)\cos\phi = U + V_{w_2}$$

$$V_{w_2} = K(V_1 - U)\cos\phi - U$$

$$P_{pelton} = m'(V_{w_1} + K(V_1 - U)\cos\phi - U)U$$

$$= m'(V_1 + K(V_1 - U)\cos\phi - U)U$$

$$= m'(V_1 - U)(1 + K\cos\phi)U.$$

$$P_{pelton} = m'(1 + K\cos\phi)[V_1 U - U^2]$$

To maximize power $\frac{d(P_{pelton})}{d(U)} = 0$

$$0 = m'(1 + K\cos\phi)(V_1 \times 1 - 2U)$$

$$V_1 - 2U = 0$$

$$U = \frac{V_1}{2}$$

8

$$\eta_{pelton} = \frac{m(V_{w_1} + V_{w_2})u}{\frac{1}{2} m V_1^2}$$

$$= \frac{2(V_{w_1} + V_{w_2})u}{V_1^2}$$

$$= \frac{2(V_1 + K\cos\phi(V_1 - u) - u)u}{V_1^2}$$

$$= \frac{2(V_1 - u)(1 + K\cos\phi)u}{V_1^2}$$

$$u = \frac{V_1}{2}$$

$$\eta_{pelton_{max}} = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + K\cos\phi)\frac{V_1}{2}}{V_1^2}$$

$$\boxed{\eta_{pelton_{max}} = \frac{1 + K\cos\phi}{2}}$$

Pelton wheel:-

Q. A pelton wheel is to operate under 400 m head with the following specification.

1. Head loss in penstock (ϕ), $h_f = 20m$, Length = 6m with friction factor = 0.0 and diameter is to be determine.
2. coefficient of velocity of the nozzle is 0.98
3. Diameter of the pelton wheel is limited to 2.5m and electric generator synchoronous speed is 360 rpm.
4. The jet ratio is limited to 15.
5. the bucket friction coefficient is 0.92 and jet deflected angle is 165°
6. The mechanical losses limited to 3% of the pelton wheel power developed.

Ex:-Determine the following:-

1. velocity of Jet
2. Diameter of the nozzle jet
3. Diameter of the penstock pipeline
4. power developed by the pelton wheel.

5. Shaft power

6. Overall efficiency of the pelton wheel.

A.) Given $H = 400m$, $h_f = 20m$, $C_v = 0.98$, $D = 2.5m$

$N = 360$ rpm, $m = 15$, $K = 0.92$ Jet deflected $= 165°$

Bucket outlet angle $\phi° = 180° - 165°$

$= 15°$

1. $V_J = C_v \sqrt{2g(H-h_f)}$

$= 0.98\sqrt{2 \times 9.81 \times (400-20)}$

$= 84.62$ m/s

3. Diameter of the penstock (D_p):

$$Q = \frac{\pi}{4} D_p^2 V_p = \frac{\pi}{4} D_{Jet}^2 \cdot V_{Jet}$$

$$\therefore m = \frac{D_p}{D_{jet}} = 15$$

$$= \frac{\pi}{4}(15 D_{jet})^2 \cdot V_p = \frac{\pi}{4} D_{jet}^2 (84.62)$$

~~15~~2

$$V_p \cdot 225 d_{jet}^2 = 84.62 \times d_{jet}^2$$

$$V_p = \frac{84.62}{225} = 0.38 \text{ m/sec}$$

$$h_f = \frac{f L V_p^2}{2 g d_p} = \frac{f l Q^2}{12.1 d_p^5}$$

$$20 = \frac{0.02 \times 6000 \times 0.38^2}{2 \times 9.81 \times d_p}$$

$$d_p = 0.04m$$

4. $P_{pelton} = m'(V_{w_1} + V_{w_2}) \cdot U$

$= \rho Q (V_{jet} + V_{w_2}) U$

$= \rho Q (V_{jet} - U)(1 + k\cos\phi) \cdot U$

$Q = A_{Jet} \cdot V_{Jet}$

$= \frac{\pi}{4}(d_{jet})^2 \cdot V_{jet}$

$= \frac{\pi}{4}(0.003)^2 \cdot (84.62)$

$Q = 6 \times 10^{-4} m^3/sec$

2. $\therefore m = \frac{d_p}{d_{jet}}$

$15 = \frac{0.04}{d_{jet}}$

$d_{jet} = 0.003$

9

$$U = \frac{\pi D N}{60}$$

$$= \frac{\pi \times 2.5 \times 360}{60}$$

$$= 4.71 \text{ m/sec}$$

'U' is limited to $\frac{V_{jet}}{2}$

$$= \frac{84.62}{2}$$

$$= 42.31 \text{ m}$$

∴ Hence dia of wheel should be reduced

$$U = \frac{\pi D N}{60}$$

$$42.31 = \frac{\pi \times D \times 360}{60}$$

$$D = 2.25 \text{ m}$$

$$P_{pelton} = \rho Q \,(V_{jet} - U)\,(1 + K\cos\phi) \cdot U$$

$$= 1000 \times 6\times10^{-4}\,(84.62 - 42.31)\,(1 + 0.92 \times \cos 15^\circ) \cdot 42.31$$

$$= 2 \times 10^3 \text{ watt}$$

$$P_{pelton} = 2 \text{ kW}$$

5. Mechanical loss = 3% of pelton wheel power

$$= 0.03 \times 2$$

$$= 0.06 \text{ kW}$$

$$P_{shaft} = 2 - 0.06$$

$$= 1.94 \text{ kW}$$

6. Overall efficiency $(\eta_o) = \frac{P_{shaft}}{P_{water}}$

$$\cancel{86.7} = \frac{1.94 \times 10^3 \text{ watt}}{\rho \cdot g \cdot Q\,(H - h_f)}$$

$$= \frac{1.94 \times 10^3}{1000 \times 9.81 \times 6\times10^{-4}\,(400 - 20)}$$

$$= 86.7\%$$

UNIT - 4

REACTION TURBINE

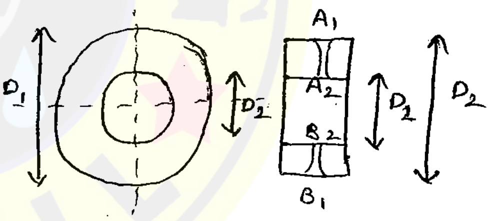
Reaction water turbines basically medium head to the extend of 10 m ($h < 300$ m). The input energy mostly P.E only (mgh). These turbines needs draft tube to convert outlet K.E of water into useful potential head. These turbines works under pressure drop. Guide vanes control the water discharge through governing mechanism. The rotating element called Francis turbine and rotor (for Keplan and propellar turbines) mounted on turbine shaft which is again coupled to electrical unit.

Working principle of reaction turbine:-

It works on Newton third law of motion i.e.) reaction principle. When high pressure water passes through vanes of the runner the P.E converted into K.E. During this change a reactive force developed on the runner. By momentum principle the torque is developed on the shaft.

$$Q = A_f V_f$$

$$= \pi D \cdot B \cdot V_f$$



Runner

1. Flow ratio $(\psi) = \dfrac{V_{f_1}}{\sqrt{2gH}}$

2. Speed ratio $(\phi) = \dfrac{U_1}{\sqrt{2gH}}$

3. $P_{pelton} = m'(V_{w_1} + V_{w_2})U$

$$= m'(V_{w_1}U_1 + V_{w_2}U_2)$$

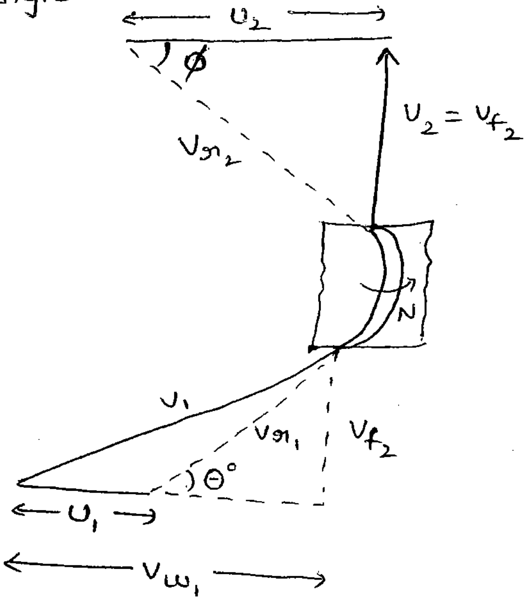
4. $P_{Francis} = m'(V_{w_1}U_1 + 0)$ $\therefore V_{w_2} = 0$ due to water leaves radially from runner

$$= m'(V_{w_1}U_1)$$

5. $\eta_{Francis} = \dfrac{m V_{w_1} \cdot U_1}{m g H}$

$$\eta_{Francis} = \frac{V_{w_1} U_1}{g H}$$

Velocity triangle of Francis turbine:-

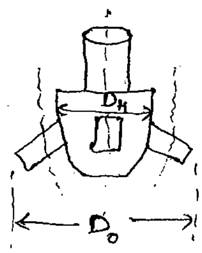


Standard specifications for Francis turbine:-

1. $H = 30 - 300$ m
2. Specific speed $= 60 - 300$
3. Flow ratio $= 0.15 - 0.3$
4. Speed ratio $= 0.6 - 0.9$
5. Water living the turbine radially for which $V_{w_2} = 0$
6. $D_2 = \frac{D_1}{2}$
7. Width, B = 10% to 40% of the diameter of wheel.
8. No. of vanes = 25 to 30
9. Overall efficiency (less than pelton wheel) is about 80% to 85%.
10. Input power $= m' g H$
11. Output power $= m' V_{w_1} U_1$

Kaplan turbine:-

It is low head turbine (below 30 m) and high specific speed, high discharge turbine. It consists 4 to 8 no. of rotors. The discharge through Kaplan turbine, $Q = A_{f_1} \cdot V_f = \frac{\pi}{4}(D_o^2 - D_H^2) V_{f_1}$, where $D_H = \frac{1}{3} D_o$



Note:-

1. Velocity triangles same like Francis turbine.

Specification of Kaplan turbine:-

1. $H = < 30\ m$

2. Flow ratio $(\psi) = 0.7$

3. Speed ratio = above 2

P.g No:- 16

7. $$T_{shaft} = \frac{2\pi N D}{60}$$

$$2515 = \frac{2 \times \pi \times 240 \times D}{60}$$

$$D = 100\ KN-m$$

10. $$\eta_{Reaction\ turbine} = \frac{\text{output}}{\text{Input}}$$

$$= \frac{P_{shaft}}{P_{water}}$$

$$0.8 = \frac{P_{shaft}}{1000 \times 9.81 \times 50 \times 7.5}$$

$$P_{shaft} = 4000\ HP$$

$1\ HP = 0.736\ KW$

$\frac{2943}{0.736} \Rightarrow 4000$

13. $$\eta_{reaction\ turbine} = \frac{P_{shaft}}{P_{water}}$$

$$0.85 = \frac{P_{shaft}}{1000 \times 9.81 \times 1 \times 250}$$

$$P_{shaft} = 2084.6\ kW$$

$$= \frac{2084.6}{0.736}$$

$$P_{shaft} = 2832\ HP$$

14. $$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{180 \times \sqrt{10140 \times 0.736\ KW}}{(24.7)^{5/4}} = 2282$$

15. Given $H = 40\,m$, $Q = 34\ m^3/sec$ $N = 150$ rpm (11)

$$\eta_{Turbine} = \frac{P_{shaft}}{P_{water} = \gamma g Q H}$$

$$100\% = \frac{P_{shaft}}{1000 \times 9.81 \times 34 \times 40}$$

$$P_{shaft} = 13342\ kW$$
$$= 13.34\ MW$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{150\sqrt{13342\ kW}}{(40)^{5/4}}$$

$$= 173 \quad (60 < N_s < 300)$$

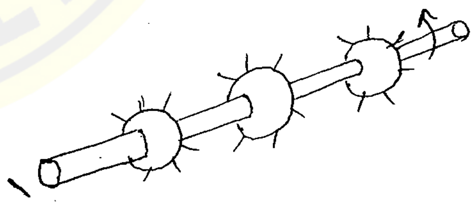
∴ Francis turbine.

Ex:- A pelton wheel consists of 4 nozzles. The total power obtained from a counter shaft where 3 wheels mounted. The power is 60 MW. The available head is 420 m and speed is 180 rpm. Determine specific speed (N_s)?-

A. $P_{total} = 60\ MW$
$= 60,000\ kW$

$$\frac{Power}{runner} = \frac{60,000}{3} = 20,000\ kW$$

$$\frac{Power}{Nozzle} = \frac{20,000}{4} = 5000\ kW$$



$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{180\sqrt{5000\ kW}}{(420)^{5/4}}$$

$$= 6.6$$

$$\boxed{N_{s\ \text{pelton wheel}} \propto \frac{1}{\sqrt{\text{No. of nozzle jets}}}}$$

P.g NO:-17

15. Given $H = 100\ m$, $\psi = 0.2$ $V_{f_1} = ?$

$$\psi = \frac{V_{f_1}}{\sqrt{2gH}}$$

$$0.2 = \frac{V_{f_1}}{\sqrt{2 \times 9.81 \times 100}}$$

$$V_{f_1} = 8.9\ m/s$$

Ex:- A francis turbine overall efficiency is 75% producing 150kw at output shaft. Available head 7.5m speed ratio 0.7 and flow ratio 0.3. Speed of the runner 150 rpm. Water discharge at outlet radially. Hydraulic losses across the runner wheel 20% of the available head. Determine the following.

1. volume flow rate of water
2. Runner pheripheral velocity at inlet.
3. Flow velocity at inlet.
4. Hydraulic efficiency
5. Whirl velocity of water at inlet
6. Guide blade angle
7. Runner vane angle at inlet
8. Diameter of the runner at inlet

A. Given $\eta_o = 0.75$ $P_{shaft} = 150\ kW$, $\psi = 0.3$, $\phi = 0.7$, $N = 150$ rpm

Hydraulic loss = 20% of available head

1. $$\eta_o = \frac{P_{shaft}}{\rho g Q H}$$

$$0.75 = \frac{150000}{1000 \times 9.81 \times Q \times 7.5}$$

$$Q = 0.272\ m^3/sec$$

2. $$\phi = \frac{U_1}{\sqrt{2gH}}$$

$$0.7 = \frac{U_1}{\sqrt{2 \times 9.81 \times 75}}$$

$$U_1 = 26.8\ m/s$$

3. $\varphi = \dfrac{V_{f_1}}{\sqrt{2gH}}$

$$0.3 = \frac{V_{f_1}}{\sqrt{2 \times 9.81 \times 75}}$$

$$V_{f_1} = 11.51 \text{ m/sec}$$

$$U_1 = \frac{\pi D_1 N}{60}$$

$$26.8 = \frac{\pi \times D_1 \times 150}{60}$$

$$D_1 = 3.42 \text{ m}$$

4. $\eta_{hydraulic} = 1 - 0.2 = 0.8 = 80\%$

5. $$\eta_{hyd} = \frac{m' V_{w_1} U_1}{m' g H}$$

$$0.8 = \frac{V_{w_1} \times 26.8}{9.81 \times 75}$$

$$V_{w_1} = 22 \text{ m/s}$$

6. $\tan \alpha^\circ = \dfrac{V_{f_1}}{V_{w_1}}$

$$\tan \alpha^\circ = \frac{11.51}{22}$$

$$\alpha^\circ = 27.59^\circ$$

7. $\tan \theta = \dfrac{V_{f_1}}{V_{w_1} - U_1}$

$$= \frac{11.51}{22 - 26.8}$$

$\theta = -67.3^\circ$ (Inlet velocity triangle modified)

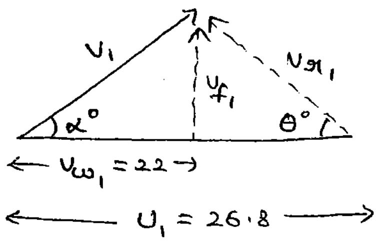
8. Width of the runner at inlet

$$Q = A_{f_1} \cdot V_{f_1}$$

$$Q = \pi D_1 B_1 V_{f_1}$$

$$0.272 = \pi \times 3.42 \times B_1 \times 11.51$$

$$B_1 =$$



EX:- A kaplan turbine developing 9.1 MW, available head is 5.6 m speed ratio ~~to~~ 2, flow ratio is 0.6, overall efficiency is 86%, Hub diameter is $\frac{1}{3}$ of outer dia of runner.

1. Outer dia of runner
2. Discharge pass through runner
3. Speed of the runner
4. Sp. speed of the turbine.

A. Given $P_{shaft\ output} = 9.1\ MW$, $H = 5.6\ m$ $\phi = 2.0$ $\psi = 0.6$

$$D_H = \frac{1}{3} D_o$$

$$\psi = \frac{V_{f_1}}{\sqrt{2gH}}$$

$$0.6 = \frac{V_{f_1}}{\sqrt{2 \times 9.81 \times 5.6}}$$

$$V_{f_1} = 6.8\ m/s$$

$$\phi = \frac{U_1}{\sqrt{2gH}}$$

$$2.0 = \frac{U_1}{\sqrt{2 \times 9.81 \times 5.6}}$$

$$U_1 = 4\ m/s$$

$$\eta_o = \frac{P_{shaft}}{\rho g Q H}$$

$$0.86 = \frac{9.1 \times 10^6}{1000 \times 9.81 \times Q \times 5.6}$$

$$Q = 192.6\ m^3/sec$$

1. $$Q_{kaplan} = \frac{\pi}{4}(D_o^2 - D_H^2) \cdot V_{f_1}$$

$$192.6 = \frac{\pi}{4}\left[D_o^2 - \frac{D_o^2}{9}\right](6.28)$$

$$D_o = 6.63\ m$$

3. $$U_1 = \frac{\pi D_o N}{60}$$

$$4 = \frac{\pi \times 6.63 \times N}{60}$$

$$N = 60 \text{ rpm}$$

4. $$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{60\sqrt{9.81 \times 1000 \text{ kW}}}{(5.6)^{5/4}}$$

$$N_s = 670$$

7-12-2014

UNIT-5

GOVERNING OF TURBINES

Governing means maintain the or keeping the constant speed of water turbine for variation of load on the generator. During governing the flow rate (discharge) is controlled through hydraulic mechanism.

Generally servo mechanism (Hydraulic oil system is used)

$$P = \rho g Q H$$

$$V_{jet} = \sqrt{2gH}$$

Governing of pelton wheel:-

1. The water quantity supplied through nozzle is the point of controlling discharge.

$$Q = AV$$

$V = \sqrt{2gH}$ which is constant at given instant.

Hence to control discharge area of the nozzle is to be manupulated.

2. Along the axis of the nozzle a metal rod (sphere) is used to alter the flow area.
3. To avoid water hammer sphere a tappered shape makes gradual closer.
4. When ~~to~~ controlling force is centrifugal force of the governor (pendulum governor)
5. Run away speed of the governor.

Run away speed of the governor:-

It is the speed of the water turbine without load (discoupled from generator and also governing mechanism not connected). Hence Run away speed is highest speed of water turbine possible when no load governor not working.

Generally pelton wheel designed for run away speed of 185% of main speed ($N_{run} = 1.85 N$)

1. Governing means constant speed maintain.
2. Discharge control
3. Mechanism is called servo mechanicm (Hydraulic oil)
4. Discharge controlled by spear in the nozzle.
5. Run away speed = 1.85 N
6. constant speed = generator speed = $\frac{120f}{p} = \frac{60f}{p'}$

p = No. of poles, p' = No. of pairs of poles

7-12-20

UNIT-6 (6 M)

SPECIFIC SPEED AND PERFORMANCE OF WATER TURBINES

Before manufacture, erection and installation of real turbine, its model turbine with certain scale develop and model turbine tested under different working conditions.

The working conditions called Duty variables like head variable (H), discharge or power and speed. The scale of the model turbine is called scale ratio or model ratio or diameteral ratio.

EX:- 1:1, 1:2, 1:8, 1:10, 1:20 etc.

$$\frac{D_m}{D_p} = 1:5$$

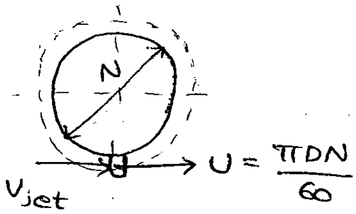
$$\frac{\text{Dia of model turbine}}{\text{Dia of proto type turbine}} = 1:5$$

There are four coefficients equal for model and large turbines.

1. Head coefficient
2. Discharge coefficient
3. power coefficient
4. specific speed

Head coefficient :-

$$U \propto V_{Jet}$$



$$\frac{\pi DN}{60} \propto \sqrt{2gH}$$

$$DN \propto \sqrt{H}$$

$$\boxed{\frac{\sqrt{H}}{DN} = \text{Head coefficient}}$$

* $U = \frac{V}{2}$ for max power and efficiency

$$\text{Speed ratio}(\phi) = \frac{U_1}{\sqrt{2gH}} = \frac{\pi D_1 N/60}{\sqrt{2gH}}$$

Discharge coefficient :-

$$Q = AV$$

$$Q = \frac{\pi d^2}{4} \cdot V \quad (\text{or}) \quad \pi D B \cdot \sqrt{2gH} \quad (\text{or}) \quad \frac{\pi}{4}(D_o^2 - D_B^2)\sqrt{2gH}$$

$$Q \propto D^2 \sqrt{H}$$

$$\boxed{\frac{Q}{D^2\sqrt{H}} = \text{Discharge coefficient}}$$

$$\frac{Q_m}{D_m^2\sqrt{H_m}} = \frac{Q_p}{D_p^2\sqrt{H_p}}$$

Power coefficient :-

$$P = \rho g Q H$$

$$P \propto Q H$$

$$P \propto D^2\sqrt{H} \cdot H$$

$$P \propto D^2 H^{3/2}$$

$$\boxed{\frac{P}{D^2 H^{3/2}} = \text{power coefficient}}$$

P.g NO:- 26

6. $$N_{s\,\text{pelton wheel}} = \frac{N\sqrt{\frac{P}{\text{No. of nozzle jets}}}}{H^{5/4}}$$

$$N_{s\,\text{pelton}} \propto \frac{1}{\sqrt{\text{No. of jets}}}$$

$$1 : \sqrt{N}$$

7. $$N_s \propto \sqrt{P}$$

$$N_s \propto \sqrt{\frac{1}{m^2}}$$

$$N_s \propto \frac{1}{m}$$

13. $$U \propto V_{jet}$$

$$\frac{\pi D N}{60} \propto \sqrt{2gH}$$

$$DN \propto \sqrt{H}$$

$$\left(\frac{D_m}{D_p}\right)\left(\frac{N_m}{N_p}\right) = \sqrt{\frac{H_m}{H_p}}$$

$$\frac{1}{2}\left(\frac{N_m}{400}\right) = \sqrt{\frac{3}{30}}$$

$$\frac{N_m}{800} = \sqrt{\frac{1}{10}}$$

$$N_m = \frac{800}{\sqrt{10}}$$

$$N_m = 253 \text{ rpm}$$

14. $Q \propto D^2\sqrt{H}$

$Q \propto D^2 \cdot N \cdot D$

$Q \propto ND^3$

$P \propto D^2 H^{3/2}$

$\sqrt{H} = ND$

$H = N^2 D^2$

$P \propto D^2 (N^2D^2)^{3/2}$

$P \propto D^2 N^3 D^3$

$P \propto D^5 N^3$

$$\frac{P_m}{N_m^3 D_m^5} = \frac{P_p}{N_p^3 D_p^3}$$

16. $P = \gamma g Q H$

$P \propto Q \cdot H$

$P \propto D^2\sqrt{H} \cdot H$

$P \propto D^2 H^{3/2}$

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^2 \left(\frac{H_1}{H_2}\right)^{3/2}$$

$$\frac{1000}{P_2} = \left(\frac{400}{20}\right)^{3/2}$$

$$= (2)^{3/2}$$

$$P_2 = 353 \text{ KW}$$

Ex:- A large turbine is to generate 10 MW power. A 1:8 model turbine is generated to test the turbine under similar conditions. speed of the both turbine are equal. Determine the following.

1. Head ratio (H_P/H_m):

2. power generated by the model turbine in watt

3. Discharge ratio (Q_p/Q_m)

4. Mass flow rate ratio ($\dot{M}_p/\dot{M}_m$)

A. Given $P_p = 10$ MW $= 10 \times 10^6$ watt

$N_p = N_m$

$$\frac{D_m}{D_p} = \frac{1}{8}$$

1. Head coefficient

$$ND \propto \sqrt{H}$$

$$\frac{\cancel{N_p} \cdot D_p}{\cancel{N_m} \cdot D_m} = \sqrt{\frac{H_p}{H_m}}$$

$$\frac{8}{1} = \sqrt{\frac{H_p}{H_m}}$$

$$\frac{H_p}{H_m} = \frac{64}{1}$$

2. power coefficient

$$P = \rho g Q H$$

$$P \propto Q \cdot H$$

$$P \propto D^2 \sqrt{H} \cdot H$$

$$P \propto D^2 H^{3/2}$$

$$\frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^2 \left(\frac{H_m}{H_p}\right)^{3/2}$$

$$\frac{P_m}{1 \times 10^7} = \left(\frac{1}{8}\right)^2 \left(\frac{1}{64}\right)^{3/2}$$

$$P_m = 305 \text{ watt}$$

3. Discharge coefficient

$$Q = AV$$

$$Q \propto D^2 \sqrt{H}$$

$$\frac{Q_p}{Q_m} = \left(\frac{D_p}{D_m}\right)^2 \sqrt{\frac{H_p}{H_m}}$$

$$= \left(\frac{8}{1}\right)^2 \sqrt{\frac{64}{1}}$$

$$= (8)^3$$

$$= 512$$

4. Mass flow rate

$$\frac{Q_p}{Q_m} = 512$$

$$\frac{\dot{m}_p}{\dot{m}_m} = \frac{\rho Q_p}{\rho Q_m}$$

$$\frac{\dot{m}_p}{\dot{m}_m} = 512$$

Unit Quantities:-

In order to predict the working of particular turbine under different variablies like head, speed, Output power can be expressed in terms of unit quantities applicable to unit head, unit discharge and unit power.

$$U \propto V_{Jet}$$

$$DN \propto \sqrt{H}$$

For same turbine $D = \text{constant}$

$$N \propto \sqrt{H}$$

$$N = N_U \cdot \sqrt{H}$$

Unit speed, $\boxed{N_U = \frac{N}{\sqrt{H}}}$

$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

Unit discharge (Q_U):-

$$Q = AV$$

$$Q = \frac{\pi}{4} d^2 \cdot V \qquad \left[\begin{array}{l} m = \frac{D}{d} \\ d \propto D \end{array}\right]$$

$$Q \propto D^2 \cdot \sqrt{H}$$

For same turbine, $D = \text{constant}$

$$Q \propto \sqrt{H}$$

$$Q = Q_U \sqrt{H}$$

$$\boxed{Q_U = \frac{Q}{\sqrt{H}}}$$

$$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

Unit power (P_U):-

$$P = \rho g Q H$$

$$P \propto Q H$$

$$P \propto \sqrt{H} \cdot H$$

$$P \propto H^{3/2}$$

$$P = P_U \cdot H^{3/2}$$

$$\boxed{P_U = \frac{P}{H^{3/2}}} \qquad \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

Ex:-Find relation between specific speed (N_s) interms of unit speed (N_u) and unit power (P_u). (17)

A. $N_s = \frac{N\sqrt{P}}{H^{5/2}} \Rightarrow \frac{N}{H^{1/2}} \cdot \frac{\sqrt{P}}{H^{3/4}} \Rightarrow \frac{N}{\sqrt{H}} \cdot \sqrt{\frac{P}{H^{3/2}}} \Rightarrow N_u \sqrt{P_u}$

$N_u = \frac{N}{\sqrt{H}}$

$$\boxed{\therefore\ N_s = N_u \cdot \sqrt{P_u}}$$

$P_u = \frac{P}{H^{3/2}}$

Ex:- Derive specific speed of a water turbine.

1. $P = \gamma g Q H$

$P \propto Q H$

2. $Q = AV$

$Q \propto D^2 \cdot \sqrt{H}$

3. $U \propto V_1$

$DN \propto \sqrt{H}$

$P \propto Q \cdot H$

$P \propto D^2 \sqrt{H} \cdot H$

$P \propto D^2 \cdot H^{3/2}$

$P \propto \left(\frac{\sqrt{H}}{N}\right)^2 \cdot H^{3/2}$

$P \propto \frac{H^{5/2}}{N^2}$

By defination of specific speed

$P = 1\ kW \quad H = 1m \quad N = N_s$

$P = \frac{k \cdot H^{5/2}}{N^2} \longrightarrow (1)$

$1 = \frac{k (1)^{5/2}}{N_s^2}$

$k = N_s^2 \longrightarrow (2)$

sub (2) in (1)

$P = \frac{N_s^2 \cdot H^{5/2}}{N^2}$

$N_s^2 = \frac{N^2 P}{H^{5/2}}$

$N_s = \frac{N\sqrt{P}}{H^{5/4}}$

$N_{s\,model} = N_{s\,proto\ type}$

$\frac{N_m \sqrt{P_m}}{(H_m)^{5/4}} = \frac{N_p \cdot \sqrt{P_p}}{(H_p)^{5/4}}$

P.g No:- 27

8. $P = 1\ MW = 1 \times 10^6$ watts

$N = 350$ rpm

$H = 8m$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{350\sqrt{1000}\ kW}{(8)^{5/4}}$$

$N_s = 820$ ~~kW~~

∴ propellar

9. $N_s \propto \frac{1}{m}$

$$m = \frac{D}{d}$$

$$m \propto D$$

$$\uparrow N_s \propto \frac{1}{D} \downarrow$$

specific speed increases, size of the runner decreases

P.g No:- 93

19. Given $\frac{dm}{dp} = \frac{1}{5}$, $P_m = 3$ kW, $N_m = 360$ rpm, $H_m = 2m$

$H_p = 6m$, $P_p = ?$

$P = \rho g Q H$

$P \propto QH$

$P \propto D^2\sqrt{H} \cdot H$

$P \propto D^2 H^{3/2}$

$$\frac{P_p}{P_m} = \left(\frac{D_p}{D_m}\right)^2 \left(\frac{H_p}{H_m}\right)^{3/2}$$

$$\frac{P_p}{3} = \left(\frac{5}{1}\right)^2 \left(\frac{6}{2}\right)^{3/2}$$

$P_p = 390$ kW

20. Given $P = 50$ kW

$V = 400$ m/s

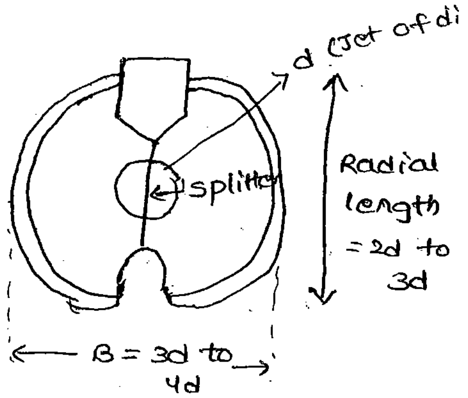
$$\text{Watt} = \frac{N-m}{\text{Sec}}$$

$P = F \times V$

$50000 = F \times 400$

$F = 125$ N

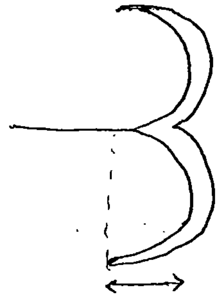
Top view of a bucket of the pelton :-



Radial length = 2d to 3d

Axial width, B = 3d to 5d

Depth (T) = 0.8d to 1.2d



Depth (T) = 0.8d to 1.2d

Difference between impulse turbine and reaction turbine :-

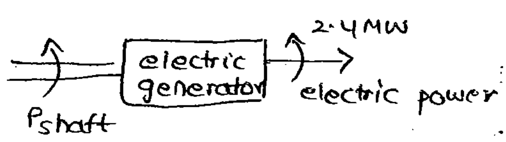
Impulse turbine	Reaction turbine
1. All the available potential energy of water is converted into K.E by a nozzle. (100% KE)	1. The available energy is part converted into K.E and most P.E of water before runner (90% PE + 10% KE)
2. $\eta_{pelton} = \dfrac{m'(V_{w_1}+V_{w_2})u}{\frac{1}{2}m'v_1^2}$	2. $\eta_{reaction} = \dfrac{m' V_{w_1} u_1}{m' g H}$
3. The turbine is installed above the tail race	3. The turbine submerged with water and connected to draft tube otherwise turbine under the tail race.
4. Works on Newton's second law of motion.	4. Works on Newtons third law of motion.
5. Works in open atmosphere	5. Works under pressure
6. No pressure drop across the impulse turbine and only absolute velocity drops $(V_2 - V_1)$	6. pressure drop across the turbine there by exit absolute velocity V_2 more than inlet V_1
7. No draft tube is used	7. Draft tube is necessary.
Ex:- pelton wheel, Girad turbine, Turgo turbine, Banki turbine, Jonual turbine	Ex:- kaplan, propellar and Francis turbine, Thomson turbine, fourneyron turbine

8. Working blades are called buckets.	8. Working blades are called vanes.
9. Water may be allowed partly on other side.	9. Water may be fully allowed
10. Casing has no hydraulic function for safety and avoid splashing of function.	10. Casing has hydraulic function and spiral shape converts P.E into K.E
11. Impulse turbines are high head turbines.	11. Reaction turbines are medium to low.
12. Low discharge	12. Medium and High discharge
13. Low specific speed turbines	13. Medium to High specific Speed turbines.
14. Governing is flow regulation without losses.	14. Governing is always accompanied loss.
15. The relative velocity slightly reduces due to friction ($V_{r_2} = k \cdot V_{r_1}$) k = bucket (or) blade friction coefficient = 0.9	15. Due to pressure drop in runner relative velocity increases $V_{r_2} \geq V_{r_1}$.
16. Flow is tangential mostly	16. Flow is radial, axial, mixed

P.g No:- 93

24. Given $P = 2.4$ MW, $\eta_{generator} = 80\%$.

$$\eta_{electric\ Generator} = \frac{\text{electric power}}{P_{shaft}}$$



$$0.8 = \frac{2.4}{P_{shaft}}$$

$$P_{shaft} = 3 \text{ M.W}$$

Each runner shaft power $= \frac{1}{2}(3) = 1.5$ MW

25. Given $H = 200m$ $Q = 75 \ m^3/min$ $C_v = 0.97$, $d = 0.1 m$

$$Q_{total} = \text{No. of jets} \times q$$

$$1.25 \ \frac{m^3}{sec} = \text{No. of jets} \times \frac{\pi}{4}(0.1)^2 \times 0.97\sqrt{2\times 9.81\times 200}$$

$$\text{No. of jets} = 2.61 \approx 3$$

P.g NO:-98

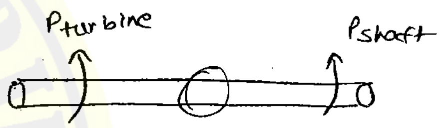
25. $$\varphi \text{ (flow ratio)} = \frac{V_{f_1}}{\sqrt{2gH}}$$

$$0.3 = \frac{V_{f_1}}{\frac{600}{60}}$$

$$V_{f_1} = 3 \ m/s$$

29. Given $P_{shaft} = 625 \ kW$ $\eta_{mech} = 90\%$

$$\eta_{mech} = \frac{P_{shaft}}{P_{turbine}} \times 100$$



$$0.9 = \frac{625}{P_{turbine}}$$

$$P_{turbine} = 694 \ kW$$

$$\text{Mechanical loss} = 694 - 625$$

$$= 69 \ kW$$

$$\approx 70 \ kW$$

30. $P_{shaft} = 440kW$, $H = 40m$, $\eta_{mech} = 0.9$ $H_{useful} = 40m$

$$\eta_o = \eta_H \cdot \eta_m$$

$$= \left(\frac{36}{40}\right) \times 0.9$$

$$= 0.81$$

$$= 81\%$$

31. $$\eta_o = \frac{P_{shaft}}{\rho \cdot g \cdot Q \cdot H}$$

$$0.81 = \frac{440 \times 1000}{1000 \times 9.81 \times Q \times 40}$$

$$Q = 83 \ m^3/sec$$

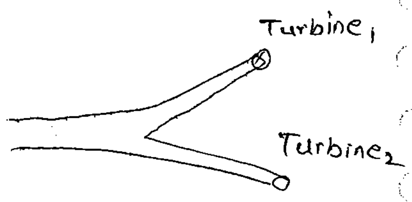
33. Given $Q = 175\ m^3/sec$, $H = 18\ m$, $N = 2.5 \times 60 = 150$ rpm

$\eta_o = 0.82$, $N_s = 460$

$$\eta_o = \frac{P_{shaft\ power}}{\rho g Q H}$$

$$0.82 = \frac{P_{shaft}}{1000 \times 9.81 \times 175 \times 18}$$

$$P_{shaft} = 25.3\ MW$$



$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$460 = \frac{150\sqrt{P}}{(18)^{5/4}}$$

$$P = 12927\ kW \qquad P = \text{power of each turbine}$$

$$= 12.9\ MW$$

$$\text{No. of turbines required} = \frac{25.3}{12.9}$$

$$= 1.96$$

$$\cong 2.0$$

35. Head coefficient $U \propto V_1$ $\qquad \frac{D_m}{D_p} = \frac{1}{10}$, $H_m = 8\ m$

$DN \propto \sqrt{H}$ $\qquad N_m = 400$ rpm

$$\frac{D_p N_p}{D_m N_m} = \sqrt{\frac{H_p}{H_m}} \qquad H_p = 100\ m$$

$N_p = ?$

$$\frac{10 \times N_p}{1 \times 400} = \sqrt{\frac{100}{8}}$$

$$N_p = 141\ rpm$$

36. $P \propto QH$

$P \propto D^2\sqrt{H} \cdot H$

$P \propto D^2 H^{3/2}$

$$\frac{P_p}{P_m} = \left(\frac{D_p}{D_m}\right)^2 \left(\frac{H_p}{H_m}\right)^{3/2}$$

$$= \left(\frac{10}{1}\right)^2 \left(\frac{100}{8}\right)^{3/2}$$

$$= 4400$$

P.g NO:- 104

10. Speed ratio $(\phi) = \frac{U_1}{\sqrt{2gH}}$

$$0.44 = \frac{\frac{\pi DN}{60}}{\sqrt{2gH}}$$

$$0.44 = \frac{\frac{\pi \times D_1 \times 600}{60}}{\sqrt{2 \times 9.81 \times 300}}$$

$$D_1 = 1.07\,m$$

P.g NO:- 105

29. Degree of reaction $= \frac{\Delta P_{runner}}{\Delta P_{guide\ wheel} + \Delta P_{runner}} = \frac{P_2 - P_3}{(P_1 - P_2) + (P_2 -}$

$$= \frac{\Delta P_{runner}}{\Delta P_{stag}} = \frac{P_2 - P_3}{P_1 - P_3}$$

$$D.O.R_{\ pelton\ wheel} = \text{zero } (0)$$

$$D.O.R._{\ Francis} \cong \pm 0.5$$

$$D.O.R._{\ Kaplan} \cong 0.8$$

30. Given $H = 25\,m$ $P = 1\,MW$ $N = 150$

$$P_u = \frac{P}{H^{3/2}}$$

$$= \frac{1000\ KW}{(25)^{3/2}}$$

$$P_u = 8\ KW$$

37. $\eta_{max\ pelton} = \frac{1 + K\cos\phi}{2}$

$$= \frac{1 + 1 \times \cos(180^\circ - 120^\circ)}{2}$$

$$= \frac{1 + 0.5}{2}$$

$$= 0.75$$

$$= 75\%$$

P.g. NO:- 107

4. Given $B = 1.6\ m$ $V = 2\ m/sec$ $H = 0.2\ m$ $\eta_o = 0.7$ $H_1 = 25\ m$

$$\eta_o = \frac{P_{shaft}}{\rho g Q H_1}$$

$$0.7 = \frac{P_{shaft}}{1000 \times 9.81 \times Q \times 0.25}$$

$$P_{shaft} = \cancel{75\%}\ 110\ KW$$

$$Q = AV$$
$$= (B\ H)\ V$$
$$= (1.6 \times 0.2) \times 2$$
$$= 0.64\ m^3/sec$$

P.g NO:- 108

12. Given $D = 0.5\ m$ $B = 0.15\ m$ $V_f = 1.56\ m/sec$

$$Q = \pi D B \cdot V_f$$
$$= \pi \times 0.5 \times 0.15 \times 1.56 \times \left(60\ \frac{m^3}{min}\right)$$
$$= 22\ m^3/min$$

23. $$N_s \propto \frac{1}{\sqrt{No.\ of\ Nozzles}}$$

$$N_s = \frac{8.1}{\sqrt{6}} \Rightarrow 8.1 = \sqrt{6}\ N_s$$ (specific speed of a wheel = $\sqrt{no.\ of\ jet}$ × sp. speed of wheel with single jet)

$$= 3.3$$

Note:-

$$N_s \propto \frac{1}{\text{Jet ratio (m)}}, \quad \uparrow N_s \propto \frac{1}{d} \downarrow$$

$N_s = \sqrt{n}$ times of sp. speed of a single jet

$\downarrow$

no. of jets

Let 'n' be the no. of jets in around the pelton wheel then sp. speed is proportional to $\sqrt{n}$

$$N_s \propto \sqrt{n}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \Rightarrow \frac{N\sqrt{\text{No. of jets} \times \frac{\text{Power}}{\text{Jet}}}}{H^{5/4}} \Rightarrow \sqrt{\text{no. of jet}} \times (N_s)_{\text{single jet}}$$

$$N_{s\ \text{pelton wheel with 'n' Jets}} = \sqrt{n} \times (N_s)_{\text{single jet}}$$

Characteristics of water turbine:-

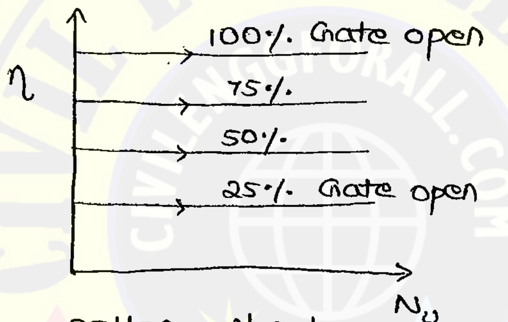
Performance characteristics of water turbine:-

At design stage of water turbines performance curve are used to compare the required parameter change with respect to available parameter to optimise turbine use identify the peak point at which turbine produce best power with minimum discharge and at lowest possible available head.

For turbines the following variables are considered H, Q, P, N and h_f. Mainly three duty variables head (H), Discharge (Q) and N.

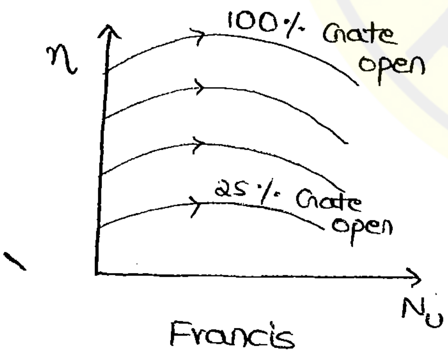
Main characteristic curve:-

Keeping head (H) constant, curve draw interms of unit quantities.

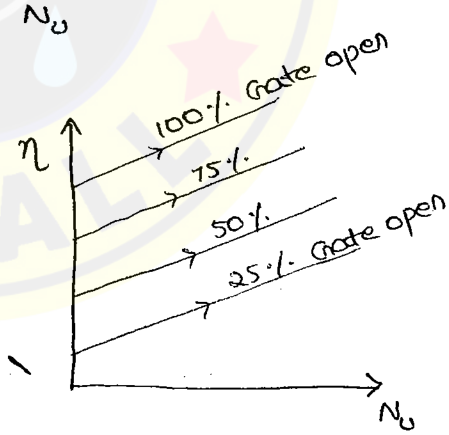


$$N_u = \frac{N}{\sqrt{H}}$$

pelton wheel



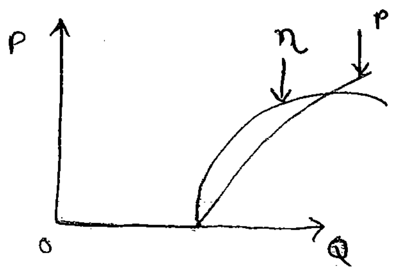
Francis



Kaplan

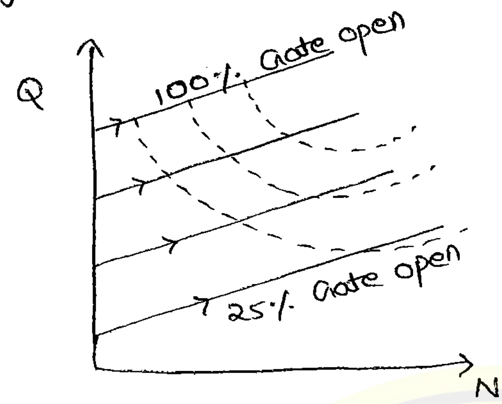
Operating characteristic curve (or) constant speed :-

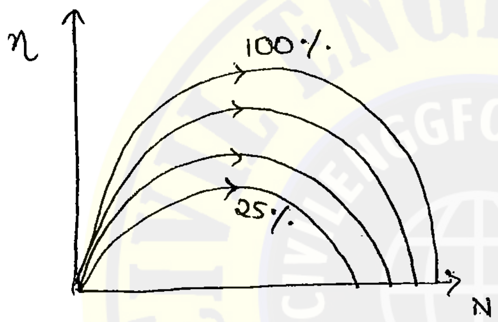
These curves are plotted keeping the governer mechanism in action and speed of the turbine kept constant by changing the discharge Q and power efficiency of different load can be plotted



Constant efficiency curve:-

These curves are plotted with reference to speed efficiency and speed discharge for different gate opening the efficiency of curves as following.





UNIT - 8

CENTRIFUGAL PUMPS

Pump is a fluid machine (Hydraulic machine or prime mover or turbo machine). It needs mechanical energy as inpu and it delievers fluid power (rising the pressure of the fluid i.e., lifting the fluid).

Pump prime element is called Impeller (rotating eleme The pumps used impeller or rotor called centrifugal pump a Axial pump.

Reciprocating pump does not consists rotating elemen instead reciprocating part called Piston or Plunger.

Ex:- Reciprocating pump.

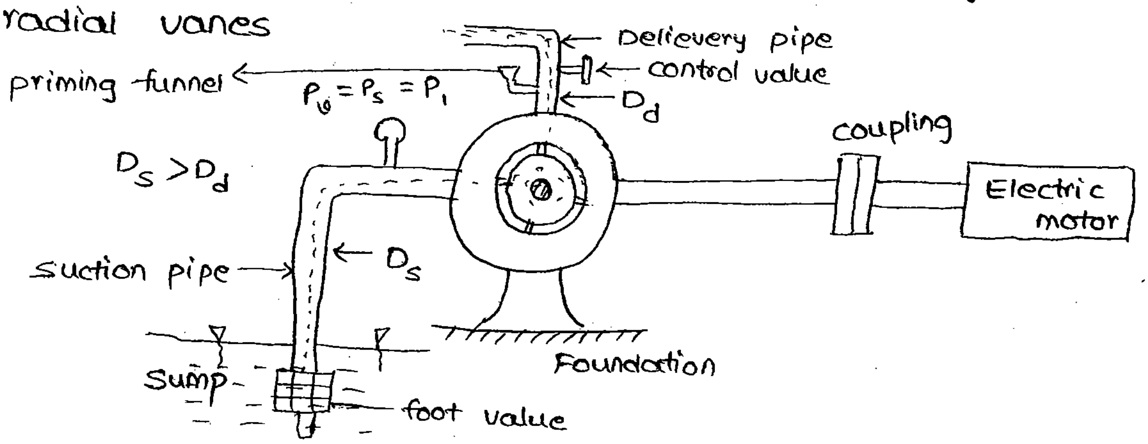
There are special pumps (Miscellaneous pumps), used for a particular application

Ex:- Jet pump, Deep well pump, Submergible pump.

1. Reciprocating pumps
2. Centrifugal pumps
3. Miscellaneous pumps

Centrifugal pumps:-

It is a roto dynamic turbo machine which converts mechanical shaft power into fluid power. It consists a rotating element called Impellar consisting bend backward radial vanes



Ex:- P_s = suction pipe = 60 kpa

P_d = 140 kpa

$$H = \frac{P_d}{\rho_s g} - \frac{P_s}{\rho g}$$

$$= \frac{140 \times 10^3}{1000 \times 10} - \frac{(-60 \times 10^3)}{1000 \times 10}$$

$$= 14 - (-6)$$

$$= 20 \text{ m}$$

Note:-

1. centrifugal pump works on the principle of forced vortex in impeller.
2. Due to centrifugal action the pressure head develops

$$P = \frac{F}{A} = \rho g H$$

$$\frac{F_c = CF}{A} = \rho g H$$

$$\frac{m \omega^2 r}{A} = \rho g H$$

$$\frac{m \frac{v^2}{r}}{A} = \rho g H$$

$$\rho A r \cdot \frac{v^2}{r} = \rho g H$$

$$v^2 = gH$$

$$\frac{v^2}{g} = H.$$

3. centrifugal pump is reverse of Francis turbine i.e., opposite to inward radial flow reaction turbine. Hence centrifugal pumps is an outward radial flow reaction turbo machine.
4. Water enters at small diameter of the impellar (eye portion) and leaves at outer periphery of the impellar.
5. The impellar is rotating part consists no. of backward bend curved vanes.

6. The impellar is mounted on electrical motor shaft (inp power device).

7. Casing is similar to Francis turbine casing which is air tight and it converts K.E into P.E by the shape of three types.
   a. volute casing
   b. vortex casing
   c. Guide wheel casing

Volute casing:-

1. It is a spiral type (area of the flow increases).
2. When area increases velocity decreases and corresponding pressure increases.

Vortex casing:-

1. It is a circular chamber with central impellar force to throw the water towards casing.

Guide wheel casing:-

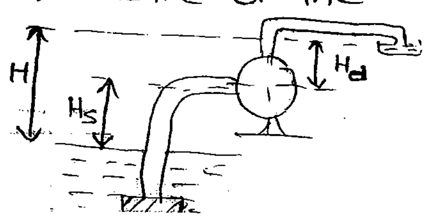
1. It consists a wheel with guide vanes which deflect flow radially delievers.

Terminology:-

1. Suction Head
2. Delievery Head
3. Manometric Head
4. Static Head
5. Manometric efficiency
6. Mechanical efficiency
7. Overall efficiency

Suction head:-

It is the verticle distance between centre of the pump to the sump free water level.



Delicevery head:-

verticle distance between free water surface of the water tank to the centre line of the pump.

Static Head (H):-

Sum of suction head and delievery head

$$H = H_s + H_d$$

Manometric head:-

It is the head against which centrifugal pump has to work.

H_m = manometric head = the impellar developed head

$$H_m = \frac{P_d}{\rho g} - \frac{P_s}{\rho g}$$

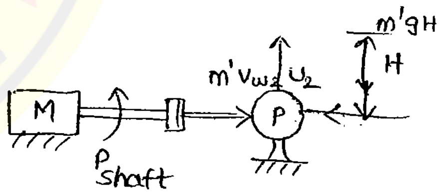
P_s = negative pressure

Manometric efficiency:-

It is the ratio of manometric head developed by the impellar to the head capacity of the runner.

$$\eta_{manometric} = \frac{m' g H_m}{\frac{m' V_{w_2} \cdot U_2}{\cancel{g}}} = \frac{\text{Head lifted}}{\text{Head capacity of impeller}}$$

$$\eta_{mano} = \frac{H_m}{\frac{V_{w_2} \cdot U_2}{g}}$$



$$\eta_{mechanical} = \frac{m' V_{w_2} \cdot U_2}{P_{shaft}}$$

$$\eta_{overall} = \eta_{mechanical} \cdot \eta_{manometric}$$

$$\eta_{overall} = \frac{m' g H}{P_{shaft}}$$

Specific speed of a centrifugal pump:-

It is defined as the speed of a geometrical, similar size and shape pump which delievers 1 m³/sec liquid against 1 m. head.

$$\boxed{N_{s_{pump}} = \frac{N\sqrt{Q}}{H^{3/4}}}$$

N = Impeller r.p.m

Q = Discharge (m^3/sec)

H = Head lifted (m)

proof:-

$$U \propto V_1$$

$$\frac{\pi D N}{60} \propto \sqrt{2gH}$$

$$DN \propto \sqrt{H}$$

$$Q = AU$$

$$Q \propto D^2\sqrt{H}$$

$$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \sqrt{H}$$

$$Q \propto \frac{H}{N^2} \cdot \sqrt{H}$$

$$Q \propto \frac{H^{3/2}}{N^2}$$

$$Q = K \cdot \frac{H^{3/2}}{N^2}$$

By defination

$Q = 1\ m^3/sec$, $H = 1\ m$, $N = N_{s\,pump}$

$$\boxed{N_{s_p} = \frac{N\sqrt{Q}}{(H)^{3/4}}}$$

Classification of centrifugal pump based on specific speed:

Expression for minimum speed of the Impellar, to lift liquid:-

$$\frac{1}{2}\cancel{m'}(u_2^2-u_1^2) \geq \cancel{m'}gH$$

$$u_2^2-u_1^2 \geq 2gH$$

$$u_2^2-u_1^2 = 2gH$$

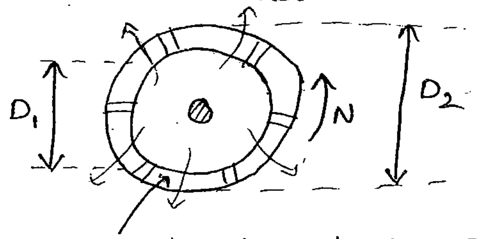
$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 = 2gH$$

$$N^2 = \frac{60^2}{\pi^2(D_2^2-D_1^2)} \, 2gH$$

$$N(rpm) = \frac{60}{\pi\sqrt{D_2^2-D_1^2}}\sqrt{2gH}$$

$$= \left(\frac{60\times\sqrt{2\times9.81}}{\pi}\right)\times\sqrt{\frac{H}{D_2^2-D_1^2}}$$

$$\boxed{N(rpm) = 84.6\sqrt{\frac{H}{D_2^2-D_1^2}}}$$

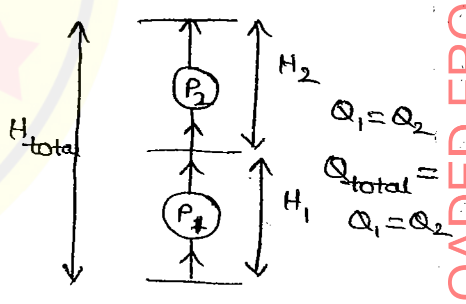


Centrifugal pumps in series and parallel:-

1. Pumps in series (Multistage pumps):-

EX:- $H_1 = 10m$ $\quad Q_1 = 200$ lpm

$H_2 = 15m$ $\quad Q_2 = 250$ lpm



Total head $= H_1 + H_2$

$= 25$ m

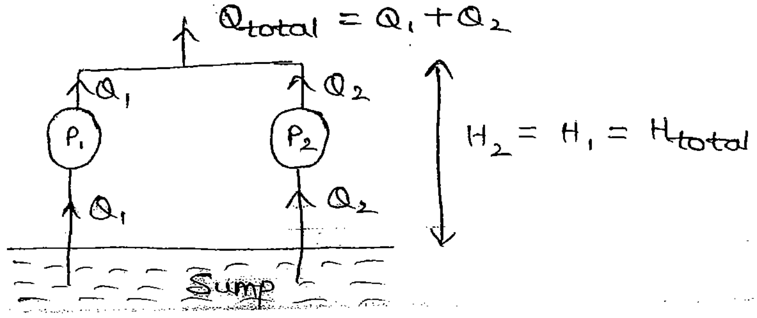
Discharge capacity $= Q_1 \doteq Q_2$

$= 200$ lpm

∴ 25 m and 200 lpm.

∴ More head and same discharge

2. Pumps in parallel :-



Note:-

(25)

1. In parallel pumps more the delievery capacity for the same head that is meant for flood discharge purpose. keeping head same.

P.g NO:- 112

17. Given $N_{s\,pump} = 20$ $\eta_o = 0.8$ $N = 800$ rpm $Q = 0.04\ m^3/s$

$H_{total} = 75\ m$

$$N_{s\,pump} = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$20 = \frac{800\sqrt{0.04}}{H^{3/4}}$$

$$H = 16\ m$$

No. of pumps in series required $= \frac{75}{16} = 4.68 \cong 5$ pumps in series

∴ In options there is no 5 pumps, so we reduce to 4 pumps in series.

21. Given $N = 1414$ rpm $Q = 0.2564\ m^3/sec$ $H = 16\ m$

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$= \frac{1414\sqrt{0.256}}{(16)^{3/4}}$$

$$N_s = 89.4$$

∴ Mixed flow type (80 – 160)

22. Given $N_1 = N_2 = 1000$ rpm

$H_1 = 25\ m$ $H_2 = 16\ m$ $D_1 = 300\ mm$ $D_2 = ?$

$$U \propto U_1$$

$$DN \propto \sqrt{H}$$

$$\frac{D_1 N_1}{D_2 N_2} = \sqrt{\frac{H_1}{H_2}}$$

$$\frac{300\,(1000)}{D_2\,(1000)} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$D_2 = 240\ mm$$

13. $N_1 = N_2$, $Q_1 = Q$ $Q_2 = \frac{Q_1}{2} = \frac{Q}{2}$, $\frac{H_1}{H_2} = ?$

$$Q \propto D^2 \cdot \sqrt{H}$$

$$ND \propto \sqrt{H}$$

$$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \cdot \sqrt{H}$$

$$Q \propto \frac{H}{N^2} \cdot \sqrt{H}$$

$$Q \propto \frac{H^{3/2}}{N^2}$$

$$\frac{Q_1}{Q_2} = \left(\frac{H_1}{H_2}\right)^{3/2} \cdot \left(\frac{N_2}{N_1}\right)^2$$

$$2 = \left(\frac{H_1}{H_2}\right)^{3/2} (1)^2$$

$$\frac{H_1}{H_2} = (2)^{2/3}$$

$$\cancel{\frac{H_2}{H} = (0.5)^{2/3} = \sqrt[3]{0.5}}$$

$$\frac{H_2}{H_1} = (2^{-1})^{2/3}$$

$$= \left(\frac{1}{2}\right)^{2/3}$$

$$= \left(\frac{1}{4}\right)^{1/3}$$

$$= (0.25)^{1/3}$$

$$= \sqrt[3]{0.25}$$

$\left(\sqrt{x} = \sqrt[2]{x}\right)$

$\cancel{(x)}^{1/3}$

$\cancel{\sqrt[3]{x}}$

$\cancel{(x)}^{3/2}$

P.9 NO:- III

8. $$\frac{N_{SA}}{N_{SB}} = \frac{N_A}{N_B} \cdot \sqrt{\frac{Q_A}{Q_B}} \times \left(\frac{H_B}{H_A}\right)^{3/4}$$

$$= \frac{1000}{500} \sqrt{\frac{1}{4}} \times (1)^{3/4}$$

$$= 2 \times \frac{1}{2}$$

$$= 1 : 1$$

6. $$ND \propto \sqrt{H}$$

$$N^2 D^2 \propto H$$

$$P \propto QH$$

$$P \propto D^2 \sqrt{H} \cdot H$$

$$P \propto D^2 \cdot H^{3/2}$$

$$P \propto D^2 (N^2 D^2)^{3/2}$$

$$P \propto D^5 \cdot N^3$$

$$P \propto N^3$$

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3$$

$$\frac{3}{P_2} = \left(\frac{1450}{2900}\right)^3$$

$$P_2 = 24 \text{ kW}$$

P.g NO:- 110

35. $$\eta_{pump} = \frac{\rho g Q H}{P_{shaft}}$$

$$0.75 = \frac{1000 \times 9.81 \times 0.04 \times 30}{P_{shaft}}$$

$$P_{shaft} = 15.69 \text{ kW}$$

P.g NO:- 103

50. Given $D_1 = 0.3$ m $D_2 = 0.6$ m $H = 40$m $N = ?$

$$N = 84.6 \sqrt{\frac{H}{D_2^2 - D_1^2}}$$

$$= 84.6 \sqrt{\frac{40}{0.6^2 - 0.3^2}}$$

$$N = 1000 \text{ rpm}$$

49. i. $ND \propto \sqrt{H}$

For same pump, D = constant

$N \propto \sqrt{H}$

$\boxed{N^2 \propto H}$

ii. $Q \propto D^2 \cdot \sqrt{H}$

D = constant

$Q \propto \sqrt{H}$

$\boxed{Q \propto N}$

iii. $P \propto H^{3/2}$

$P \propto (N^2)^{3/2}$

$\boxed{P \propto N^3}$

45. $$\eta_{pump} = \frac{\rho g Q H}{P_{shaft}}$$

$$= \frac{1000 \times 9.81 \times \left(\frac{0.75}{60}\right) \times 30}{5000}$$

$$\eta_{pump} = 74\,\%$$

35. Given $H = 26.8\ m$, $d = 0.1\,m$, $l = 30\,m$, $Q = \frac{1.8}{60} = 0.03\ m^3/sec$

$f = 0.01$

$$h_f = \frac{f\,l\,Q^2}{12.1\,d^5}$$

$$= \frac{0.01 \times 30 \times 0.03^2}{12.1 \times 0.1^5} \Rightarrow \frac{0.00027}{0.00121}$$

$$h_f = \cancel{29\ m}\ 0.223\ m$$

$$\therefore\ H + h_f$$

$$= 26.8 + 0.223$$

$$= 27.0$$

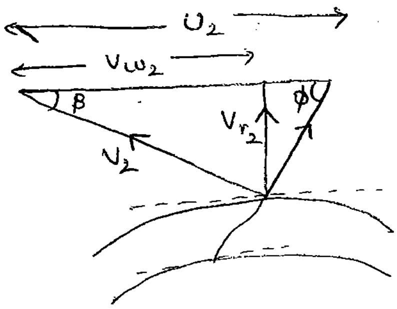
P.g No. 101

24. $$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$30 = \frac{1450\sqrt{0.2}}{H^{3/4}}$$

$$H =$$

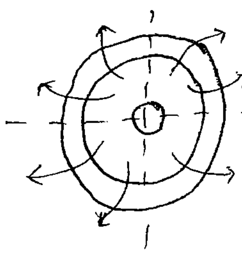
Velocity triangles :-



1. $$\tan\phi = \frac{V_{f_2}}{U_2 - V_{w_2}}$$

2. $$\text{Impellar} = \frac{\text{Workdone}}{\text{sec}}$$

$$= m' V_{w_2} U_2$$



Radially outward

water enters at eye

3. $$\eta_{manometric} = \frac{m' g H}{m' V_{w_2} \cdot U_2}$$

$$= \frac{gH}{V_{w_2} \cdot U_2}$$

P.g NO:- 39.

27

1. $\eta_{manometric} = 0.646$ $V_{w_2} = 23.2$ m/sec $U_2 = 26.18$ m/sec

$H_m = ?$

$$\eta_{mano} = \frac{m' g H}{m' V_{w_2} \cdot U_2}$$

$$0.646 = \frac{9.81 \times H}{23.2 \times 26.18}$$

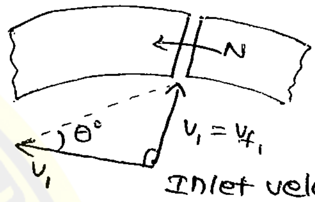
$$H = 40 m$$

2. Given $V_{f_1} = 2.5$ m/sec, $U_1 = 13.09$ m/sec $\theta° = ?$

$$\tan\theta = \frac{V_{f_1}}{U_1}$$

$$\tan\theta = \frac{2.5}{13.09}$$

$$\theta° = 11°$$



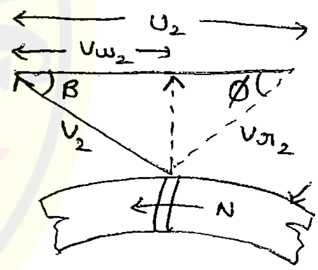
Inlet velocity triang

3. Given $H_{mano} = 18$ m, $\eta_o = 0.9$, $U_2 = 9.81$ m/sec, $V_{w_2} = ?$

$$\eta_o = \frac{m' g H}{m' V_{w_2} \cdot U_2}$$

$$0.9 = \frac{9.81 \times 18}{V_{w_2} \cdot 9.81}$$

$$V_{w_2} = 20 \text{ m/s}$$



Impellar

4. Given $V_{f_2} = 3$ m/sec, $U_2 = 18.9$ m/sec $\phi° = ?$ $V_{w_2} = 17.23$ m/s

$\cancel{\eta_o} = \cancel{m' g H}$

$$\tan\phi° = \frac{V_{f_2}}{U_2 - V_{w_2}}$$

$$= \frac{3}{18.9 - 17.23}$$

$$\phi = 61°$$

5. Given $U_2 = 3.5$ m/s $D_2 = 0.3$ m $N = ?$

$$U_2 = \frac{\pi D_2 N}{60}$$

$$3.5 = \frac{\pi \times 0.3 \times N}{60} \Rightarrow N = 223 \text{ rpm.}$$

6. Given $D_2 = 0.3\ m$, $B_2 = 0.05\ (m)$ $V_{f_2} = 3.556\ (m/sec)$

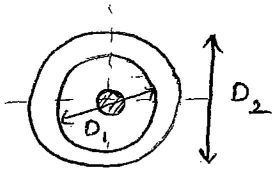
$$Q = A_{f_2} \cdot V_{f_2}$$

$$= \pi D_2 B_2 \cdot V_{f_2}$$

$$= \pi \times 0.3 \times 0.05 \times 3.556$$

$$Q = 0.17\ m^3/sec$$

7. Given $D_2 = 0.3\ m$



$N = 1200\ rpm$

$H = 15\ m$

$\eta_{mano} = 0.95$

$$\frac{W \cdot D}{\text{Newton wt. of water}} = m' \ V_{w_2} \cdot U_2 \quad \text{watt}$$

$$= \frac{w'}{g} \cdot V_{w_2} \cdot U_2 \quad \text{watt}$$

$$= \frac{V_{w_2} \cdot U_2}{g} \left(\frac{N-m}{Newton} \right)$$

$$\eta_{mano} = \frac{m' \cdot g\, H_m}{m' \ V_{w_2} \cdot U_2}$$

$$0.95 = \frac{H_m}{\frac{V_{w_2} \cdot U_2}{g}}$$

$$0.95 = \frac{150}{\frac{V_{w_2} \cdot U_2}{g}}$$

$$\frac{V_{w_2} \cdot U_2}{g} = \frac{W \cdot D}{N}$$

$$= \frac{150}{0.95}$$

$$= 157.9 \ \frac{N-m}{N}$$

$$\frac{V_{w_2} \cdot U_2}{g} = 0.1579 \ \frac{KN-m}{N}$$

$$V_{w_2} \cdot U_2 = g \times 0.1579$$

$$= 1.579 \ \cancel{\frac{KN-m}{N}}$$

8. $$\eta_o = \frac{\dot{m} g H}{P_{shaft}}$$

$$= \frac{\rho g Q H}{P_{shaft}}$$

$$0.75 = \frac{\gamma Q H}{P_{shaft}}$$

$$0.75 = \frac{10 \times 0.03 \times 25}{P_{shaft}}$$

$$P_{shaft} = 10 \text{ kW}$$

P.9 No. 44

62. $$N = \frac{120}{\pi} \frac{\sqrt{gH}}{\sqrt{D_2^2 - D_1^2}}$$

$$1450 = \frac{120}{\pi} \sqrt{\frac{9.81 \times H}{0.3^2 - 0.15^2}}$$

$$H = 19.8 \text{ m}$$

63. $$\frac{D_m}{D_p} = \frac{1}{2}, \quad N_m = N_p, \quad \frac{P_m}{P_p} = ?$$

$$P \propto Q H$$

$$P \propto D^2 \sqrt{H} \cdot H$$

$$P \propto D^2 \cdot H^{3/2}$$

$$P \propto D^2 (N^2 D^2)^{3/2}$$

$$ND \propto \sqrt{H}$$

$$N^2 D^2 \propto H$$

$$P \propto D^5 N^3$$

$$\frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^5 \left(\frac{N_m}{N_p}\right)^3$$

$$= \left(\frac{1}{2}\right)^5 (1)^3$$

$$\frac{P_m}{P_p} = \frac{1}{32}$$

28

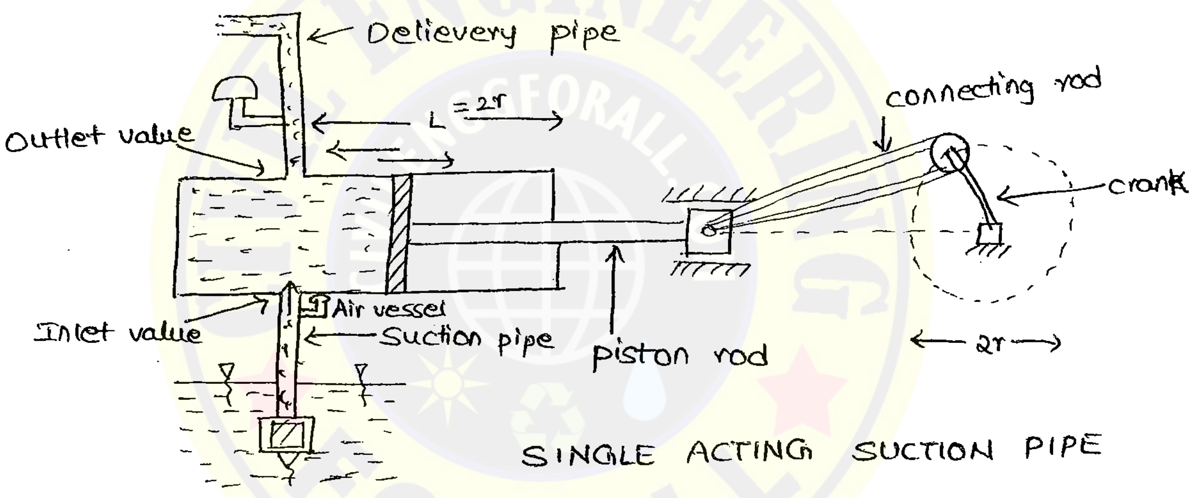
1-12-2014

UNIT - 9

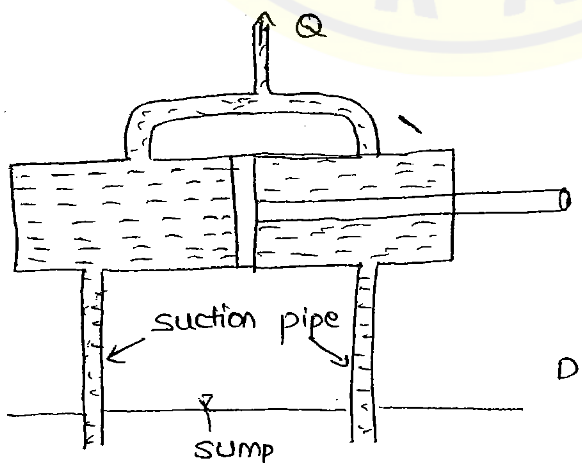
RECIPROCATING PUMP

Reciprocating pump is positive displacement pump and converts mechanical power into fluid power. It is characterised by low discharge and High head (centrifugal pump is for high discharge and low head). It works on the principle of reciprocating motion of a piston. Its maintainence cost is more compared to centrifugal pump. It cannot handle dirty water due to valved operations. It cannot handle high viscous fluids. Only one advantage high head without electrical power with human effort we can lift the water.

Construction of Reciprocating pump :-



SINGLE ACTING SUCTION PIPE



DOUBLE ACTING SUCTION PIPE

piston speed (m/min) = 2 · L · N (m/min)

Discharge through pump, $Q_{theo} = A \cdot V_{piston}$

$$Q = ALN \ (m^3/min)$$

$$Q = \frac{ALN}{60} \ (m^3/sec) \quad \text{For single acting}$$

$$Q = \frac{2ALN}{60} \ (m^3/sec) \quad \text{For double acting}$$

$\therefore$ Slip discharge, $Q_{slip} = Q_{theor} - Q_{actual}$

$$S = slip = \frac{Q_{slip}}{Q_{theo}} = \frac{Q_{theo} - Q_{actual}}{Q_{theo}}$$

$$S = 1 - \frac{Q_{actual}}{Q_{theoretical}}$$

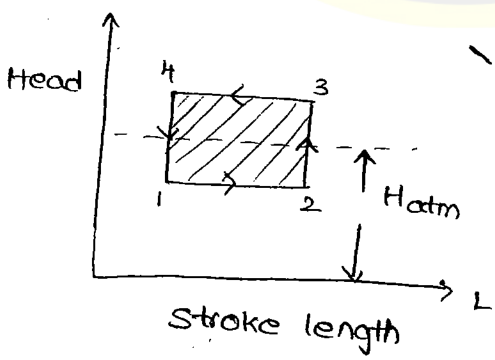
$$\boxed{S = 1 - C_d}$$

Note:-

Slip generally positive but one case like delievery pipe is not provided or very short length when compared to suction pipe then actual discharge is more than theoretical discharge. In that case negative slip observed ($Q_{actual} > Q_{theo}$)

Due to high speed of the crank negative slip also possible.

Indicator diagram of Reciprocating pump:-



① → ② suction

② → ③ work on fluid

③ → ④ Delievery

Area of indicator diagram represents workdone during the one cyclic operation. More the area, more the head develope

Function of air vessel:-

Air vessel is a closed chamber filled with water partly remaining space filled with compressed air. Air vessel connected suction as well as delievery pipe.

1. Workdone on the pump saved.
2. Ensure continuous fluid supply (uniform flow rate)
3. It reduces frictional resistance in suction and delievery pipe.
4. It permits the crank higher speeds without flow separation in cylinder.

Note:-

1. Work saved in single acting reciprocating pump about 84.8%.
2. Work saved in double acting reciprocating pump about 39.2%.

P.g No:- 66

164. Given $H = 10\,m$ $\quad Q = 0.1\,m^3/sec$ $\quad h_f = 5\,m$

$$\text{Power (S.I)} = \frac{\rho g Q H}{\eta_{pump}}$$

$$\text{power (M.K.S)} = \frac{\rho Q H}{\eta_{pump}}$$

$$= 1000 \times 0.1 \times (10+5)$$

$$= 1500\ \frac{kg(f)-m}{sec}$$

P.g No:- 67

177. $P = \rho g Q H$

$$7.5 = 1000 \times 9.81 \times 0.05 \times H$$

$$H = 15.32\,m$$

Model testing and unit quantities :-

1. Head coefficient :-

$$\sqrt{H} \propto ND$$

$$\frac{ND}{\sqrt{H}} = \text{Head coefficient}$$

$$\frac{N_m D_m}{\sqrt{H_m}} = \frac{N_p D_p}{\sqrt{H_p}}$$

2. Discharge coefficient :-

$$AV \propto Q$$

$$D^2\sqrt{H} \propto Q$$

$$\frac{D^2\sqrt{H}}{Q} = \text{Discharge coefficient}$$

$$\frac{D_m^2 \sqrt{H_m}}{Q_m} = \frac{D_p^2 \sqrt{H_p}}{Q_p}$$

3. Power coefficient :-

$$P \propto QH$$

$$P \propto D^2\sqrt{H} \cdot H$$

$$P \propto D^2 H^{3/2}$$

$$\frac{D_m^2 H_m^{3/2}}{P_m} = \frac{D_p^2 H_p^{3/2}}{P_p}$$

Ex:- A pump is to be designed for which the head is to be raised 40m and the larger mortor speed 1440 r.p.m. The model pump developed with scale ratio 1:4 with speed 720 rpm. The discharge expected from the large pump is 100 lit/sec. The model pump power is 0.5 KW. Determine the relavant variables for both the turbine

A.

Model pump	Large pump
$\frac{D_m}{D_p} = \frac{1}{4}$	$H_p = 40\text{ m}$
$P_m = 0.5\text{ KW}$	$Q_p = 0.1\text{ m}^3/\text{sec}$
$N_m = 720\text{ rpm}$	$N_p = 1440\text{ rpm}$
$H_m = ?$ $Q_m = ?$	$P_p = ?$

Head coefficient :-

$$ND \propto \sqrt{H}$$

$$\frac{N_m D_m}{N_p \cdot D_p} = \sqrt{\frac{H_m}{H_p}}$$

$$\frac{720 \times 1}{1440 \times 4} = \sqrt{\frac{H_m}{40}}$$

$$H_m = 0.63\ m$$

Discharge coefficient :-

$$Q \propto D^2 \sqrt{H}$$

$$\frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2 \sqrt{\frac{H_m}{H_p}}$$

$$\frac{Q_m}{0.1} = \left(\frac{1}{4}\right)^2 \sqrt{\frac{0.63}{40}}$$

$$Q_m = 0.008\ m^3/sec$$

$$= 8\ lit/sec$$

Power coefficient :-

$$P \propto D^2 \sqrt{H} \cdot H$$

$$\frac{P_p}{P_m} = \left(\frac{D_p}{D_m}\right)^2 \left(\frac{H_p}{H_m}\right)^{3/2}$$

$$\frac{P_p}{0.5} = (4)^2 \left(\frac{40}{0.63}\right)^{3/2}$$

$$P_p = 4047\ KW$$

$$P_p = 4\ MW$$

P.g NO :- 102

27. Given $NPSH = 3.5\ m$

$$\sigma = 0.1$$

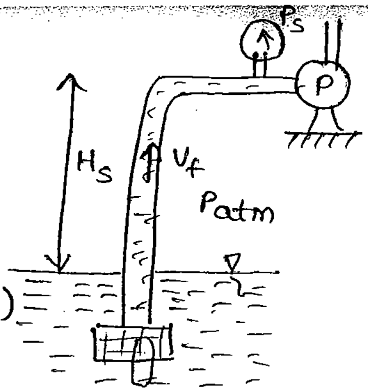
$$\sigma = \frac{NPSH}{H_m}$$

$$H_m = \frac{35}{0.1}$$

$$= 35\ m$$

$$NPSH = \frac{P_{atm}}{\rho g} - H_s - H_v - h_{fs} - \frac{V_s^2}{2g}$$

$$\boxed{NPSH = H_{atm} - H_s - H_v - h_{fs} - \frac{V_s^2}{2g}}$$



Cavitation number (or) Thomas number (σ_c)

$$\boxed{\sigma_c = \frac{NPSH}{H_{manometric}}}$$

$\therefore (NPSH)_{calculated} > (NPSH)_{Manufacture}$.

P.g No:- 67

183. $NPSH = 9.8 - 0.4 - 5 - 0.6$

$= 3.8$ m

Ex:- A centrifugal pump is to operate at a particular place where the atmospheric pressure is 10.33 m of water (101.03 Kpa). The companies data book reavels that the cavitation factor is 0.15 and NPSH is 2.5 m of water. The head on the pump is to be decided and ascertain vertical distance between pump and sump if vapour pressure of water at that room temperature is 2.3 m of water and head loss expected is 1.5 m.

A. Given $H_{atm} = 10.3$ m, $h_{fs} = 1.5$ m $H_v = 2.3$ m, $H_{mano} = ?$

$\sigma_c = 0.15$ $H_s = ?$ $NPSH = 2.5$

$$\sigma_c = \frac{NPSH}{H_{mano}}$$

$$0.15 = \frac{2.5}{H_{mano}}$$

$$H_{mano} = 16.67 \text{ m}$$

$$NPSH = H_{atm} - H_s - H_v - H_{fs}$$

$$2.5 = 10.3 - H_s - 2.3 - 1.5$$

$$H_s = 4 \text{ m}$$

P.g NO:- 101.

22. $ND \propto \sqrt{H}$

$N^2D^2 \propto H$

$P \propto QH$

$P \propto D^2\sqrt{H} \cdot H$

$P \propto D^2 H^{3/2}$

$P \propto D^2 (N^2D^2)^{3/2}$

$P \propto D^5 . N^3$

$$\frac{P_p}{P_m} = \left(\frac{D_p}{D_m}\right)^5 \left(\frac{N_p}{N_m}\right)^3 \qquad \therefore N_p = N_m$$

$$\frac{P_p}{1\,KW} = \left(\frac{2}{1}\right)^5 \left(\frac{N_p}{N_p}\right)^3$$

$P_p = 1 \times 2^5$

$P_p = 32\ KW$

17. $$Ns_{pump} = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$= \frac{750\sqrt{(1.5/2)}}{(16)^{3/4}} \qquad \therefore \text{Double suction}$$

$Ns_{pump} = 82$

P.g NO:- 95

9. Given, $N_1 = 1250$ rpm $\quad P_1 = 4$ HP $\quad N_2 = 1000$ rpm $\quad P_2 = ?$

$P \propto D^5 N^3$

For same pump, discharge is constant

$$\frac{P_2}{P_1} = \left(\frac{D_2}{D_1}\right)^5 \left(\frac{N_2}{N_1}\right)^3$$

$$\frac{P_2}{P_1} = \left(\frac{1000}{1200}\right)^3$$

$P_2 = 4 \times \left(\frac{4}{5}\right)^3$

$= 2$ HP

8. Given $Q = 0.15\ m^3/sec$, $H_{total} = 75\ m$ $N = 1200\ rpm$

$N_{s_{pump}} = 60$, No. of pump impellars = ?

$$N_{s_p} = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$60 = \frac{1200\sqrt{0.15}}{(H)^{3/4}}$$

$$H = 15\ m$$

No. of pumps required $= \frac{75}{15} = 5$

Methods to avoid cavitation:-

1. Pressure of the flowing fluid should not be allowed below its vapour pressure.

   Ex:- Absolute pressure head of water should not be below 2.5 m of water.

2. The material effecting cavitation should be coating with special metal paints. (mixing of alluminium, bronze, stainless steel particles to the paint).

3. provide air vessels near the cavitation prone areas.

4. Use vacuum pumps to remove unwanted gases at the high velocity places.

5. Provide a pump to increase the pressure of the flow

Performance Characteristics:-

1. In order to develop new pump the similar pump under use should be check under variable working conditions.

2. Keeping one parameter constant change the other two parameters then draw the graph for the given pump called characteristic curves of turbo machines (Turbines or pumps)

3. There are three types of characteristic curves.

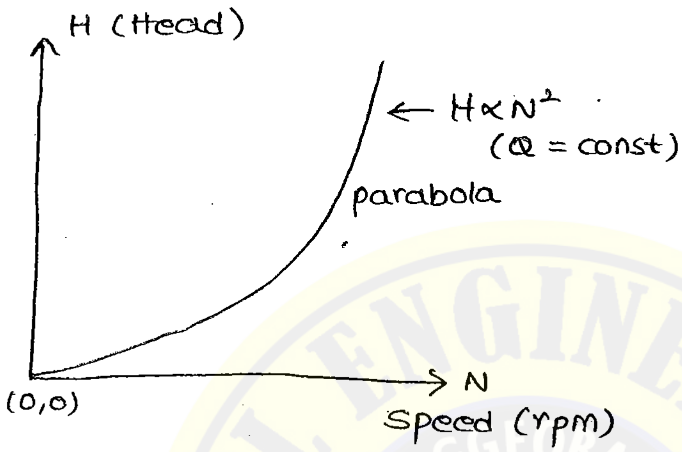
   a. Main characteristic curves

   b. Operating characteristic curves

   c. Constant efficiency curves.

Main characteristic curve of pump :-

1. M.C. curves of a centrifugal pump consists variation of the head, power and discharge deliever w.r.t the speed of the pump.
2. x-axis, speed of the pump and y-axis, (Head, powe Discharge).



$ND \propto \sqrt{H}$

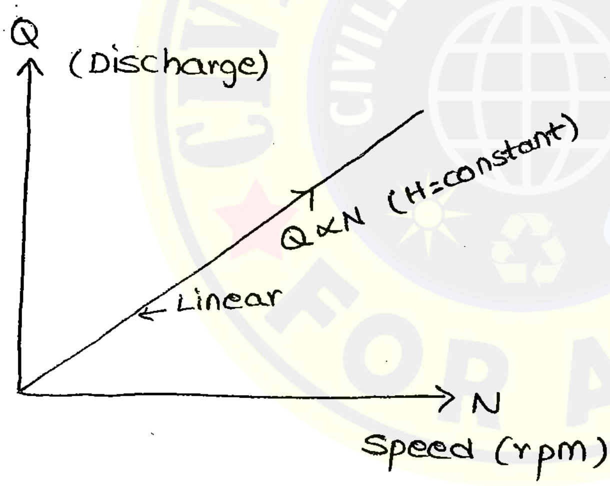
For same pump, $D = $ con

$N \propto \sqrt{H}$

$N^2 \propto H$

$\boxed{H = yN^2}$

$y = my^2$



$Q \propto D^2 \cdot \sqrt{H}$

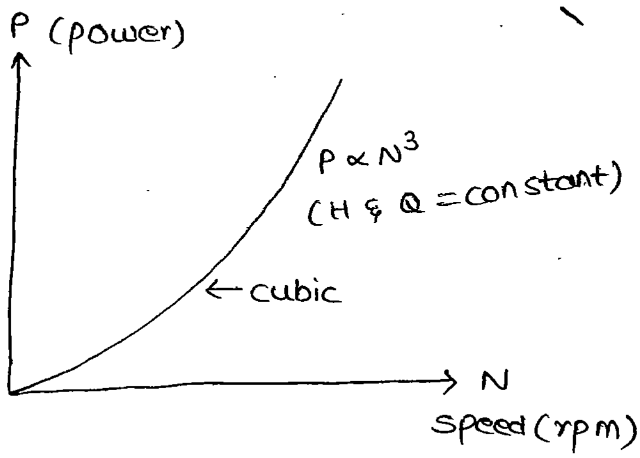
$Q \propto \sqrt{H}$

$D = $ constant

$Q \propto ND$

$\boxed{Q \propto N}$

$Q = kN$



$P \propto QH$

$P \propto D^2 \sqrt{H} \cdot H$

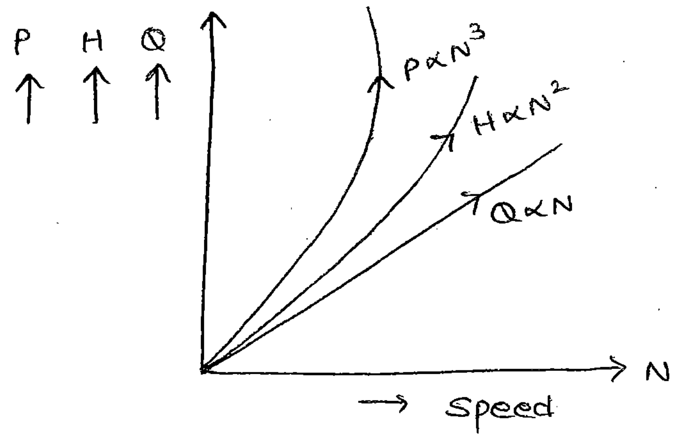
$P \propto D^2 H^{3/2}$

$P \propto D^2 (N^2 D^2)^{3/2}$

$P \propto D^5 N^3$

$D = $ constant

$\boxed{P \propto N^3}$



Operating characteristic curve of a pipepump:-

1. These curves are drawn for a given pump and speed is constant.
2. Along the x-axis - speed of the impeller and along the y-axis - Head, power and efficiency.